Overview

- Introduction
- Modelling parallel systems
- Linear Time Properties
- Regular Properties
- Linear Temporal Logic (LTL)
- Computation-Tree Logic
- **Equivalences and Abstraction**
  - bisimulation
  - CTL, CTL*-equivalence
  - computing the bisimulation quotient
  - abstraction stutter steps ⟵
  - simulation relations

Classification of implementation relations

STUTTER5.4-CL

Classification of implementation relations

- linear vs. branching time
  - ∗ linear time: trace relations
  - ∗ branching time: (bi)simulation relations
- (nonsymmetric) preorders vs. equivalences:
  - ∗ preorders: trace inclusion, simulation
  - ∗ equivalences: trace equivalence, bisimulation
- strong vs. weak relations
  - ∗ strong: reasoning about all transitions
  - ∗ weak: abstraction from stutter steps

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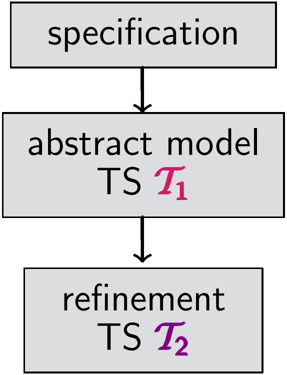
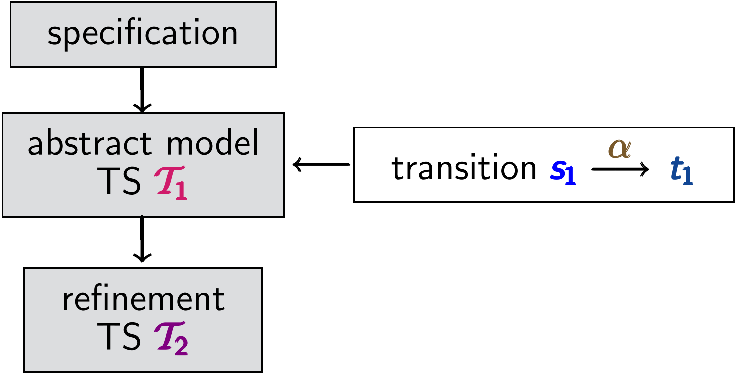
Design by stepwise refinement


specification

abstract model
TS $\mathcal{T}_1$

refinement
TS $\mathcal{T}_2$

Design by stepwise refinement



specification

abstract model
TS $\mathcal{T}_1$

transition $s_1 \xrightarrow{\alpha} t_1$

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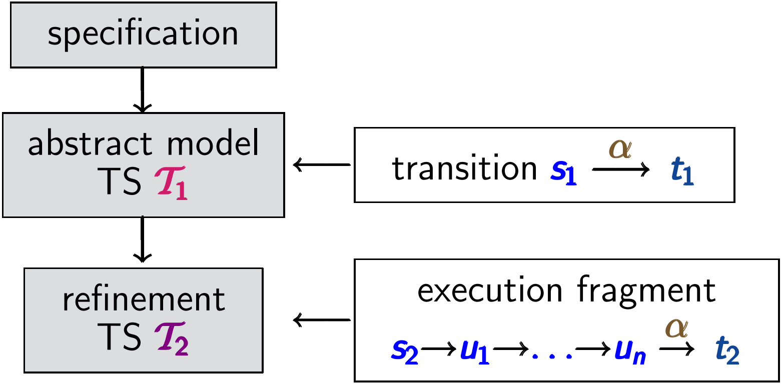
Design by stepwise refinement

STUTTER5.4-1



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abstract model
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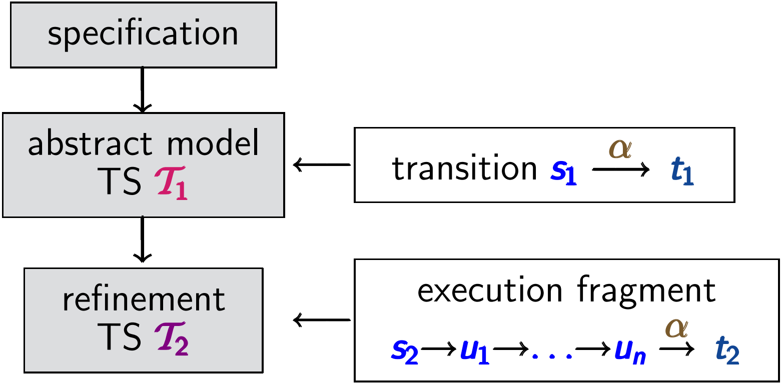
execution fragment
$s_2 \longrightarrow u_1 \longrightarrow \ldots \longrightarrow u_n \xrightarrow{\alpha} t_2$

Design by stepwise refinement



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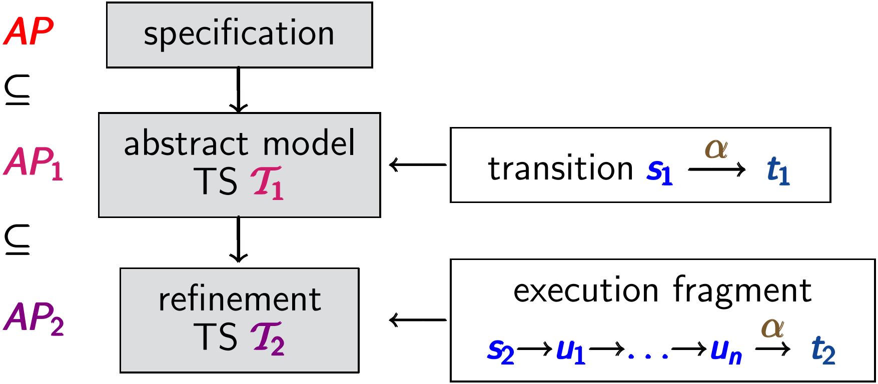
internal computation prior to the execution of action α

- access on auxiliary variables of $\mathcal{T}_2$
- no access on variables of $\mathcal{T}_1$

Design by stepwise refinement



AP

$\subseteq$

AP_1

$\subseteq$

AP_2

specification

abstract model TS $\mathcal{T}_1$

refinement TS $\mathcal{T}_2$

transition $s_1 \xrightarrow{\alpha} t_1$

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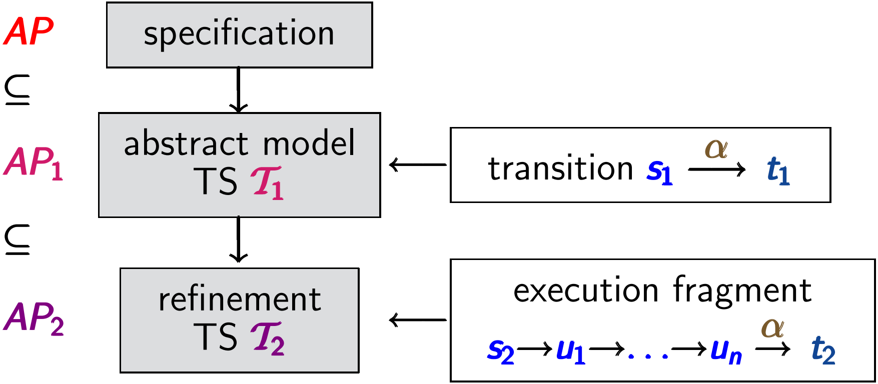

AP specification

$\subseteq$

AP_1 abstract model TS $\mathcal{T}_1$ ← transition $s_1 \xrightarrow{\alpha} t_1$

$\subseteq$

AP_2 refinement TS $\mathcal{T}_2$ ← execution fragment $s_2 \to u_1 \to \ldots \to u_n \xrightarrow{\alpha} t_2$

internal computation prior to the execution of action α

- access on auxiliary variables of $\mathcal{T}_2$
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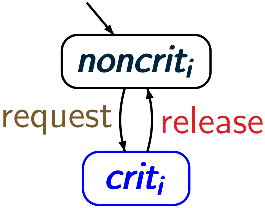
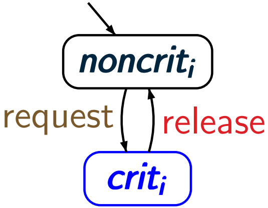
$s_2 \to u_1 \to \ldots \to u_n$: stutter steps w.r.t. AP_1 (or AP)

Mututal exclusion (with arbiter)



abstract representation
for process P_i

Mututal exclusion (with arbiter)



abstract representation
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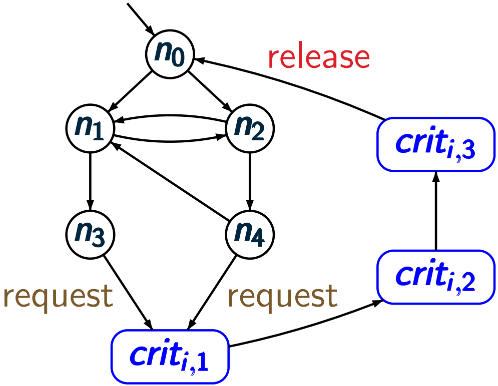

refined representation
for process P_i

Example: abstraction from stutter steps

process P

```
LOOP FOREVER
    $x := y \text{ MOD } 3$
    $y := (x + y) \text{ MOD } 3$
    $z := (2y - x) \text{ DIV } 3$
END LOOP
```

Example: abstraction from stutter steps

process $P \rightsquigarrow$ transition system $\mathcal{T}_P$

```
$\ell_0$  LOOP FOREVER
$\ell_1$          $x := y$ MOD 3
$\ell_2$          $y := (x + y)$ MOD 3
$\ell_3$          $z := (2y - x)$ DIV 3
$\ell_4$  END LOOP
```

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CTL* property: does $\mathcal{T}_P \models \forall\Box\Diamond(z = 1)$ hold ?

Example: abstraction from stutter steps

process $P \rightsquigarrow$ transition system $\mathcal{T}_P$ over $AP = Eval(z)$

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$\ell_0$   LOOP FOREVER
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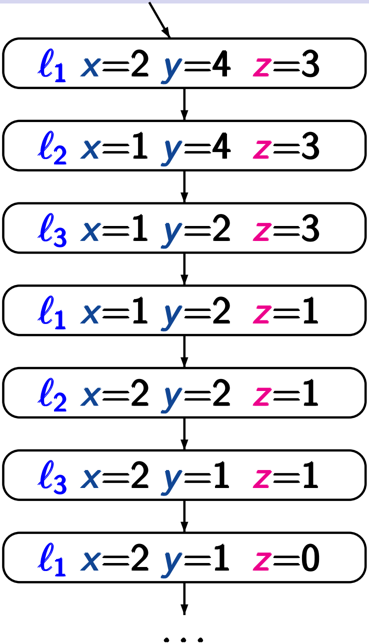
```
$\ell_0$  LOOP FOREVER
$\ell_1$      $x := y$ MOD 3             ⟵ stutter step
$\ell_2$      $y := (x + y)$ MOD 3       ⟵ stutter step
$\ell_3$      $z := (2y - x)$ DIV 3      ⟵ visible action
$\ell_4$  END LOOP
```

CTL* property: does $\mathcal{T}_P \models \forall\Box\Diamond(z = 1)$ hold ?

Transition system for process P



ℓ_1 $x=2$ $y=4$ $z=3$

ℓ_2 $x=1$ $y=4$ $z=3$

ℓ_3 $x=1$ $y=2$ $z=3$

ℓ_1 $x=1$ $y=2$ $z=1$

ℓ_2 $x=2$ $y=2$ $z=1$

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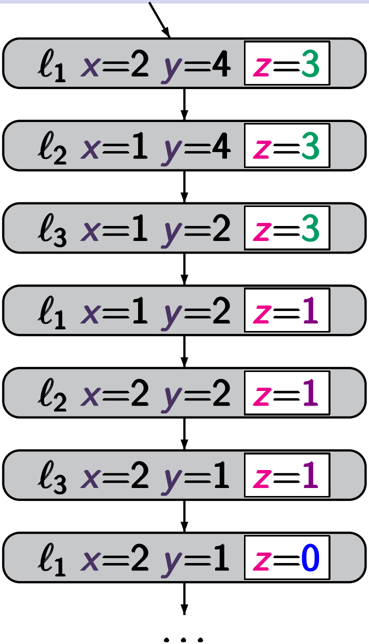
ℓ_1 $x=2$ $y=1$ $z=0$

$\dots$

Analysis by abstraction from stutter steps



ℓ_1 $x{=}2$ $y{=}4$ $\boxed{z{=}3}$

ℓ_2 $x{=}1$ $y{=}4$ $\boxed{z{=}3}$

ℓ_3 $x{=}1$ $y{=}2$ $\boxed{z{=}3}$

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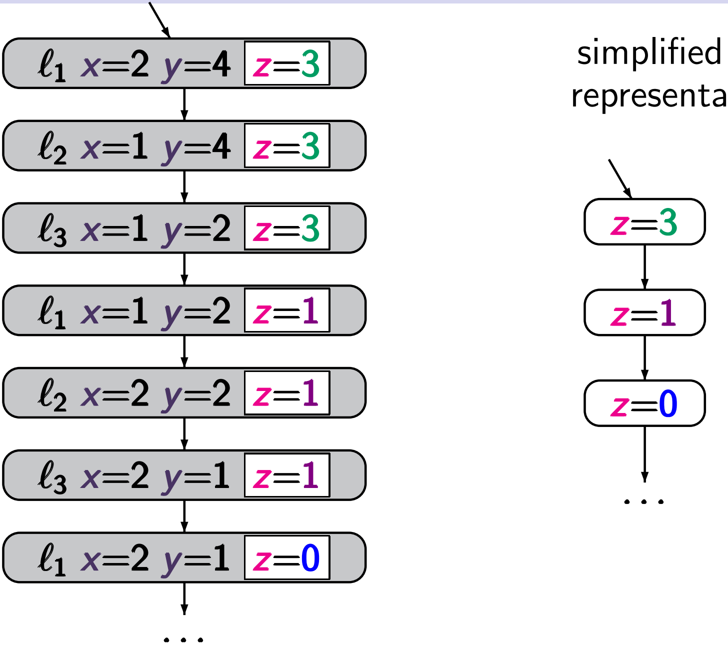
ℓ_3 $x{=}2$ $y{=}1$ $\boxed{z{=}1}$

ℓ_1 $x{=}2$ $y{=}1$ $\boxed{z{=}0}$

$\cdots$

Analysis by abstraction from stutter steps



simplified TS representation

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  - computing the bisimulation quotient
  - abstraction stutter steps
    - stutter LT relations ⟵
    - stutter bisimulation
  - simulation relations

Remind: trace relations

STUTTER5.4-5-REMIND

Remind: trace relations

trace equivalence for paths

π_1, π_2 are trace equivalent iff $\mathit{trace}(\pi_1) = \mathit{trace}(\pi_2)$

Remind: trace relations

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trace inclusion for TS: $\mathit{Traces}(\mathcal{T}_1) \subseteq \mathit{Traces}(\mathcal{T}_2)$

$\forall \pi_1 \in \mathit{Traces}(\mathcal{T}_1)\ \exists \pi_2 \in \mathit{Traces}(\mathcal{T}_2)$

s.t. π_1, π_2 are trace equivalent

Remind: trace relations

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trace equivalence for TS:

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$\mathcal{T}_2 \models E$ implies $\mathcal{T}_1 \models E$

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↑

trace equivalent TS satisfy the same **LTL** formulas

Stutter equivalence for paths

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stutter equivalence for infinite path fragments:

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$\pi_1 \triangleq \pi_2$ iff there exists an infinite word $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$ s.t. the traces of π_1 and π_2 are of the form

$$A_0 \ldots A_0\ A_1 \ldots A_1\ A_2 \ldots A_2 \ldots$$

Stutter equivalence for paths

stutter equivalence for infinite path fragments:

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$$A_0^{n_0}\ A_1^{n_1}\ A_2^{n_2} \ldots$$

where $n_0, n_1, n_2, \ldots$ are natural numbers ≥ 1

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$$A_0{}^{+}\ A_1{}^{+}\ A_2{}^{+} \ldots$$

stutter equivalence for finite path fragments:

$\hat{\pi}_1 \triangleq \hat{\pi}_2$ iff there exists a finite word $A_0\, A_1\, A_2 \ldots A_n \in \left(2^{AP}\right)^{+}$ s.t. the traces of $\hat{\pi}_1$ and $\hat{\pi}_2$ are in

$$A_0{}^{+}A_1{}^{+}A_2{}^{+} \ldots A_n{}^{+}$$

Stutter trace relations for TS

stutter equivalence for infinite path fragments:

$\pi_1 \triangleq \pi_2$ iff there exists an infinite word $A_0 \, A_1 \, A_2 \ldots \in \left(2^{AP}\right)^\omega$ s.t. the traces of π_1 and π_2 are of the form

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Stutter trace relations for TS

stutter equivalence for infinite path fragments:

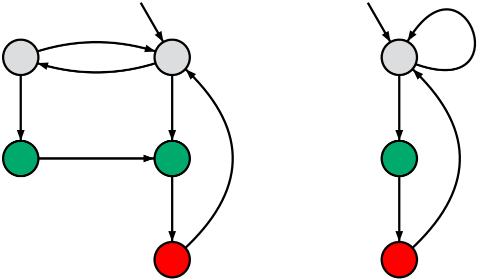
$\pi_1 \triangleq \pi_2$ iff there exists an infinite word $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$ s.t. the traces of π_1 and π_2 are of the form $A_0^{\,+}\, A_1^{\,+}\, A_2^{\,+} \ldots$

stutter trace inclusion for transition systems:

$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2$ iff for all paths π_1 of $\mathcal{T}_1$ there exists a path π_2 of $\mathcal{T}_2$ s.t. $\pi_1 \triangleq \pi_2$

Example: stutter trace inclusion ⊴

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \quad \text{iff} \quad \forall \pi_1 \in \mathit{Paths}(\mathcal{T}_1)\ \exists \pi_2 \in \mathit{Paths}(\mathcal{T}_2)$$
$$\text{s.t. } \pi_1 \triangleq \pi_2$$



⚪ = ∅

🔴 = $\{a\}$

🟢 = $\{b\}$

Example: stutter trace inclusion $\trianglelefteq$

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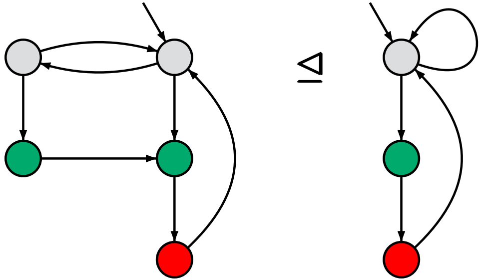

⚪ = ∅

🔴 = {*a*}

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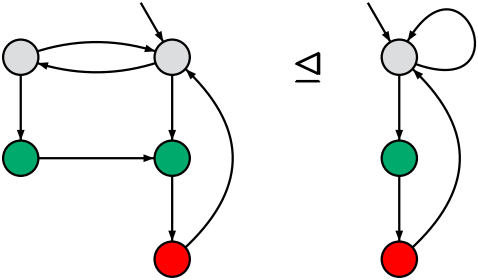

⚪ $= \varnothing$

🔴 $= \{a\}$

🟢 $= \{b\}$

all traces have the form $(\varnothing^+\{b\}^+\{a\}^+)^\omega$

or $(\varnothing^+\{b\}^+\{a\}^+)^*\varnothing^\omega$

Stutter trace inclusion and LTL

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \quad \text{iff} \quad \forall \pi_1 \in \mathit{Paths}(\mathcal{T}_1)\ \exists \pi_2 \in \mathit{Paths}(\mathcal{T}_2)$$
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Does stutter trace inclusion preserve **LTL** properties?

Stutter trace inclusion and LTL

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Does stutter trace inclusion preserve **LTL** properties?

↑

i.e., for all **LTL** formulas φ:

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \ \wedge\ \mathcal{T}_2 \models \varphi \quad \text{implies} \quad \mathcal{T}_1 \models \varphi$$

Stutter trace inclusion and LTL

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answer: **no**

Stutter trace inclusion and LTL

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answer: **no**

Example: **LTL** formulas of the form $\bigcirc a$

Stutter trace inclusion and $\mathbf{LTL}_{\setminus\bigcirc}$

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \quad \text{iff} \quad \forall \pi_1 \in \mathit{Paths}(\mathcal{T}_1)\ \exists \pi_2 \in \mathit{Paths}(\mathcal{T}_2)$$
$$\text{s.t. } \pi_1 \triangleq \pi_2$$

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS without terminal states and φ an $\mathbf{LTL}_{\setminus\bigcirc}$ formula. Then:

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \quad \wedge \quad \mathcal{T}_2 \models \varphi \quad \text{implies} \quad \mathcal{T}_1 \models \varphi$$

Stutter trace inclusion and $\mathbf{LTL}_{\setminus\bigcirc}$

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \quad \text{iff} \quad \forall \pi_1 \in \mathit{Paths}(\mathcal{T}_1)\ \exists \pi_2 \in \mathit{Paths}(\mathcal{T}_2)$$
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where $\mathbf{LTL}_{\setminus\bigcirc} = \mathbf{LTL}$ without the next operator $\bigcirc$

Stutter trace equivalence $\triangleq$ for TS

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stutter trace inclusion $\mathcal{T}_1 \trianglelefteq \mathcal{T}_2$

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stutter trace equivalence

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Stutter trace equivalence $\triangleq$ for TS

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↑

kernel of $\trianglelefteq$, i.e.,
coarsest equivalence that refines $\trianglelefteq$

Stutter trace equivalence $\triangleq$ for TS

stutter trace inclusion $\mathcal{T}_1 \trianglelefteq \mathcal{T}_2$

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For all $\mathbf{LTL}_{\setminus \bigcirc}$ formulas φ:

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \ \wedge \ \mathcal{T}_2 \models \varphi \ \text{ implies } \ \mathcal{T}_1 \models \varphi$$

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Stutter trace equivalence $\triangleq$ for TS

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$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \ \wedge\ \mathcal{T}_2 \models \varphi \ \text{ implies } \ \mathcal{T}_1 \models \varphi$$

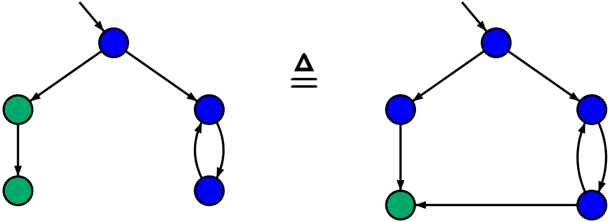
stutter trace equivalence

$$\mathcal{T}_1 \triangleq \mathcal{T}_2 \ \text{ iff } \ \mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \ \text{ and } \ \mathcal{T}_2 \trianglelefteq \mathcal{T}_1$$

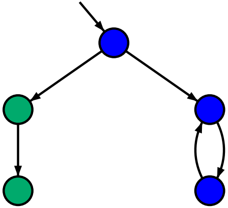
If $\mathcal{T}_1 \triangleq \mathcal{T}_2$ then $\mathcal{T}_1$ and $\mathcal{T}_2$ are $\mathbf{LTL}_{\setminus\bigcirc}$ equivalent.

Correct or wrong?

STUTTER5.4-13A



Correct or wrong?



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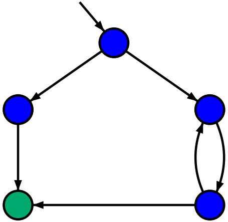
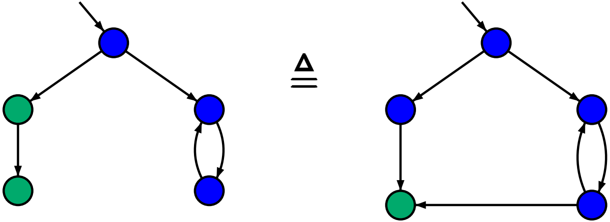


correct

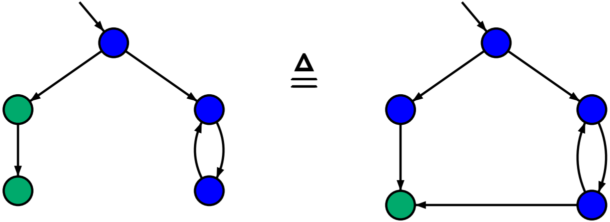
Correct or wrong?



correct

The traces of $\mathcal{T}_1$ and $\mathcal{T}_2$ have the form 🔵$^+$🟢$^+$ or 🔵$^\omega$

Correct or wrong?



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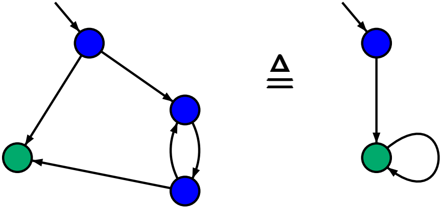

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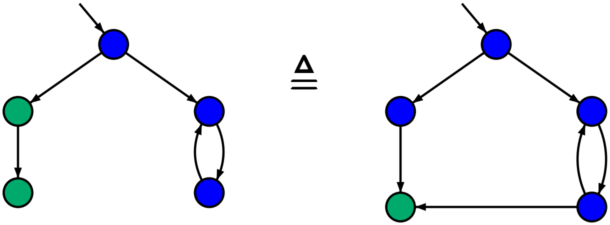

$\triangleq$

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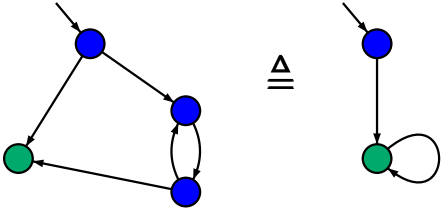

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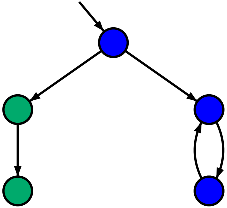
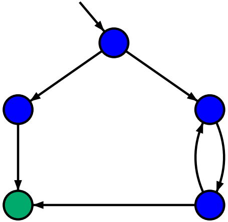


$\triangleq$



correct

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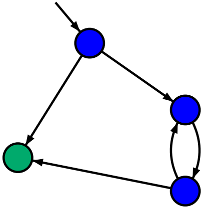
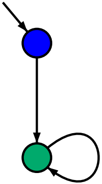


$\triangleq$



wrong

$\mathcal{T}_1$ has a finite trace ●$^+$●, while $\mathcal{T}_2$ has not

Correct or wrong?

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then:

$$\mathcal{T}_1 \sim \mathcal{T}_2 \quad \text{implies} \quad \mathcal{T}_1 \triangleq \mathcal{T}_2$$

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bisimulation equivalence

stutter trace equivalence

Correct or wrong?

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correct, as

- $\mathcal{T}_1 \sim \mathcal{T}_2$ implies $\mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$
- trace equivalent paths are stutter trace equivalent

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obviously: $\mathit{Traces}(\mathcal{T}_1) \subseteq \mathit{Traces}(\mathcal{T}_2)$ implies $\mathcal{T}_1 \trianglelefteq \mathcal{T}_2$

Stutter-insensitive LT properties

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stutter equivalence for infinite words

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Let $E \subseteq \left(2^{AP}\right)^\omega$ be an LT property. E is called stutter-insensitive iff for all $\sigma_1, \sigma_2 \in \left(2^{AP}\right)^\omega$:

if $\sigma_1 \in E$ and $\sigma_1 \triangleq \sigma_2$ then $\sigma_2 \in E$

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Example: if φ is an $\mathbf{LTL}_{\setminus\bigcirc}$ formula then $E = \mathit{Words}(\varphi)$ is stutter-insensitive

Stutter-insensitive LT properties

Let $\mathcal{T}_1$, $\mathcal{T}_2$ be two TS and E a stutter-insensitive LT-property. Then:

$$\mathcal{T}_1 \trianglelefteq \mathcal{T}_2 \text{ and } \mathcal{T}_2 \models E \quad \text{implies} \quad \mathcal{T}_1 \models E$$

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remind: if φ is an $\mathbf{LTL}_{\setminus \bigcirc}$ formula then

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Overview

- Introduction
- Modelling parallel systems
- Linear Time Properties
- Regular Properties
- Linear Temporal Logic (LTL)
- Computation-Tree Logic (CTL)
- **Equivalences and Abstraction**
  - bisimulation, CTL/CTL*-equivalence
  - computing the bisimulation quotient
  - abstraction stutter steps
    - stutter LT relations
    - stutter bisimulation ⟵
  - simulation relations

Stutter bisimulation

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(1) labeling condition

(2) simulation condition up to stuttering
"s_2 can mimick all transitions of of s_1"

(3) simulation condition up to stuttering
"s_1 can mimick all transitions of of s_2"

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$\vdots$ $\qquad\qquad$ $\vdots$

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$$\begin{tikzcd}
s_1 \arrow[r, phantom, "-\mathcal{R}-"] \arrow[d] & s_2 \\
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s_1 \arrow[r, dash, "\mathcal{R}"] \arrow[d] & s_2 \\
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STUTTER5.4-DEF-STUTTER-BIS

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$$\begin{tikzcd}
s_1 \arrow[r, phantom, "\text{-}\mathcal{R}\text{-}"] \arrow[d] & s_2 \\
s_1' &
\end{tikzcd}$$

can be completed to

$$\begin{tikzcd}
s_1 \arrow[r, phantom, "\text{-}\mathcal{R}\text{-}"] \arrow[dddd] & s_2 \arrow[d] \\
& u_1 \arrow[d] \\
& \vdots \arrow[d] \\
& u_n \arrow[d] \\
s_1' \arrow[r, phantom, "\text{-}\mathcal{R}\text{-}"] & s_2'
\end{tikzcd}$$

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$\vdots$ $\vdots$

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s_1 \arrow[r, dash, "\mathcal{R}" description] \arrow[d] & s_2 \\
s_1' &
\end{tikzcd}$$

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s_1 \arrow[r, dash, "\mathcal{R}" description] \arrow[dddd] & s_2 \arrow[d] & \\
 & u_1 \arrow[d] \arrow[dr, dashed, no head] & \\
 & \vdots \arrow[d] & s_1 \text{ -}\mathcal{R}\text{- } u_i \\
 & u_n \arrow[d] \arrow[ur, dashed, no head] & \\
s_1' \arrow[r, dash, "\mathcal{R}" description] & s_2' &
\end{tikzcd}$$

Stutter bisimulation for a TS

Let $\mathcal{T}$ be a transition system wih state space S.

A *stutter bisimulation* for $\mathcal{T}$ is a binary relation $\mathcal{R}$ on S such that for all $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

(2) for each transition $s_1 \rightarrow s_1'$ with $(s_1', s_2) \notin \mathcal{R}$ there exists a path fragment $s_2\, u_1\, u_2\, \ldots\, u_n\, s_2'$ s.t. ...

(3) ...

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(3) symmetric condition

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there exists a path fragment $s_1\, v_1\, v_2\, \ldots\, v_n\, s_1'$
s.t. $n \geq 0$ and $(v_i, s_2) \in \mathcal{R}$ for $1 \leq i \leq n$

Stutter bisimulation equivalence $\approx_{\mathcal{T}}$

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A *stutter bisimulation* for $\mathcal{T}$ is a binary relation $\mathcal{R}$ on S such that for all $(s_1, s_2) \in \mathcal{R}$:

(1) labeling condition

(2) and (3) mutual simulation condition

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stutter bisimulation equivalence $\approx_{\mathcal{T}}$:

$s_1 \approx_{\mathcal{T}} s_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $\mathcal{T}$ s.t. $(s_1, s_2) \in \mathcal{R}$

$\approx_{\mathcal{T}}$ is an equivalence

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proof:

if $\mathcal{R}$ is a stutter bisimulation with $(s_1, s_2) \in \mathcal{R}$ then

$$\mathcal{R}^{-1} = \{(t_2, t_1) : (t_1, t_2) \in \mathcal{R}\}$$

is a stutter bisimulation that contains (s_2, s_1).

$\approx_{\mathcal{T}}$ is an equivalence

symmetry: if $s_1 \approx_{\mathcal{T}} s_2$ then $s_2 \approx_{\mathcal{T}} s_1$

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proof:

$\mathcal{R} = \{(s, s) : s \in S\}$ is a stutter bisimulation

$\approx_{\mathcal{T}}$ is an equivalence

symmetry: if $s_1 \approx_{\mathcal{T}} s_2$ then $s_2 \approx_{\mathcal{T}} s_1$

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Proof: Let $\mathcal{R}_{1,2}$ and $\mathcal{R}_{2,3}$ be stutter bisimulations s.t.

$$(s_1, s_2) \in \mathcal{R}_{1,2}, \ (s_2, s_3) \in \mathcal{R}_{2,3}$$

Show that $\mathcal{R} = \mathcal{R}_{1,2} \circ \mathcal{R}_{2,3}$ is a stutter bisimulation.

$$\begin{tikzcd}
s_1 \arrow[r, dash] \arrow[d] & \mathcal{R}_{1,2} \arrow[r, dash] & s_2 \arrow[r, dash] & \mathcal{R}_{2,3} \arrow[r, dash] & s_3 \\
s_1' & & & &
\end{tikzcd}$$

$$\begin{tikzcd}
s_1 \arrow[r, dash] \arrow[dddddddddd] & \mathcal{R}_{1,2} \arrow[r, dash] & s_2 \arrow[r, dash] \arrow[d] & \mathcal{R}_{2,3} \arrow[r, dash] & s_3 \\
 & & u_1 \arrow[d] & & \\
 & & \cdots \arrow[d] & & \\
 & & u_{j-1} \arrow[d] & & \\
 & & u_j \arrow[d] & & \\
 & & \cdots \arrow[d] & & \\
 & & u_{k-1} \arrow[d] & & \\
 & & u_k \arrow[d] & & \\
 & & \cdots \arrow[d] & & \\
 & & u_m \arrow[d] & & \\
s'_1 \arrow[r, dash] & \mathcal{R}_{1,2} \arrow[r, dash] & s'_2 & &
\end{tikzcd}$$

$$\begin{tikzcd}
s_1 \arrow[r, dash] \arrow[dddddddddd] & \mathcal{R}_{1,2} \arrow[r, dash] & s_2 \arrow[r, dash] \arrow[d] & \mathcal{R}_{2,3} \arrow[r, dash] & s_3 \\
 & & u_1 \arrow[d] & & \\
 & & \cdots \arrow[d] & & \\
 & & u_{j-1} \arrow[d] & & \\
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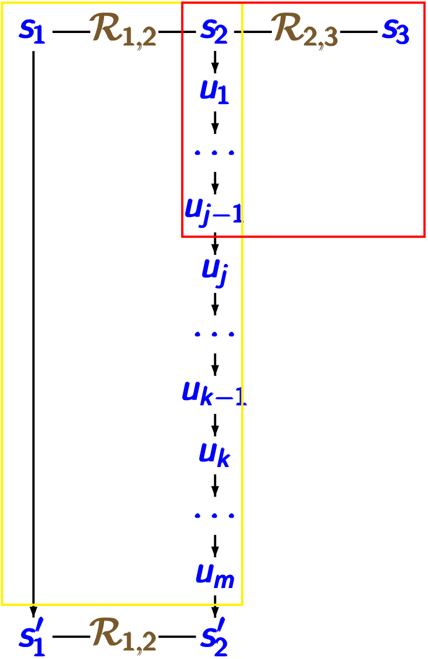
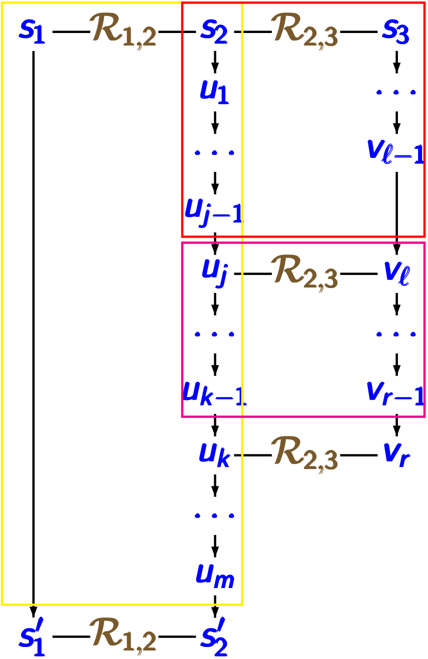
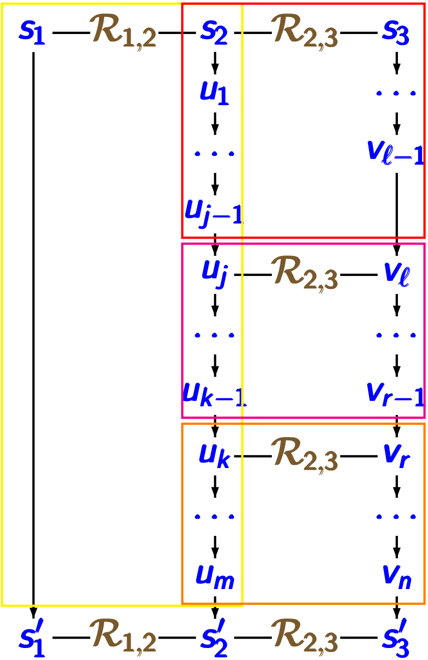
 & & \cdots \arrow[d] & & \\
 & & u_m \arrow[d] & & \\
s'_1 \arrow[r, dash] & \mathcal{R}_{1,2} \arrow[r, dash] & s'_2 & &
\end{tikzcd}$$


s_1 — $\mathcal{R}_{1,2}$ — s_2 — $\mathcal{R}_{2,3}$ — s_3
u_1
$\dots$
u_{j-1}
u_j
$\dots$
u_{k-1}
u_k
$\dots$
u_m
s_1' — $\mathcal{R}_{1,2}$ — s_2'

$$\begin{tikzcd}
s_1 \arrow[r, dash] \arrow[dddddddddd] & \mathcal{R}_{1,2} \arrow[r, dash] & s_2 \arrow[r, dash] \arrow[d] & \mathcal{R}_{2,3} \arrow[r, dash] & s_3 \arrow[d] \\
 & & u_1 \arrow[d] & & \cdots \arrow[d] \\
 & & \cdots \arrow[d] & & v_{\ell-1} \arrow[dd] \\
 & & u_{j-1} \arrow[d] & & \\
 & & u_j \arrow[r, dash] \arrow[d] & \mathcal{R}_{2,3} \arrow[r, dash] & v_\ell \\
 & & \cdots \arrow[d] & & \\
 & & u_{k-1} \arrow[d] & & \\
 & & u_k \arrow[d] & & \\
 & & \cdots \arrow[d] & & \\
 & & u_m \arrow[d] & & \\
s_1' \arrow[r, dash] & \mathcal{R}_{1,2} \arrow[r, dash] & s_2' & &
\end{tikzcd}$$


s_1 — $\mathcal{R}_{1,2}$ — s_2 — $\mathcal{R}_{2,3}$ — s_3
u_1
$\dots$
u_{j-1}
$\dots$
$v_{\ell-1}$
u_j — $\mathcal{R}_{2,3}$ — v_ℓ
$\dots$
$\dots$
u_{k-1}
v_{r-1}
u_k — $\mathcal{R}_{2,3}$ — v_r
$\dots$
u_m
s_1' — $\mathcal{R}_{1,2}$ — s_2'

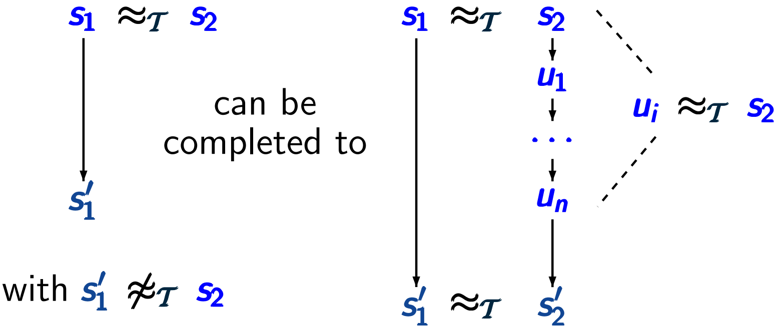

s_1
$\mathcal{R}_{1,2}$
s_2
$\mathcal{R}_{2,3}$
s_3
u_1
$\dots$
u_{j-1}
$\dots$
$v_{\ell-1}$
u_j
$\mathcal{R}_{2,3}$
v_ℓ
$\dots$
$\dots$
u_{k-1}
v_{r-1}
u_k
$\mathcal{R}_{2,3}$
v_r
$\dots$
$\dots$
u_m
v_n
s'_1
$\mathcal{R}_{1,2}$
s'_2
$\mathcal{R}_{2,3}$
s'_3

Stutter bisimulation equivalence

$\approx_{\mathcal{T}}$ is an equivalence on state space S of $\mathcal{T}$ such that for all states s_1, s_2 with $s_1 \approx_{\mathcal{T}} s_2$:

(1) $L(s_1) = L(s_2)$

(2) simulation condition up to stuttering



$s_1 \approx_{\mathcal{T}} s_2$, $s_1 \to s_1'$

can be completed to

with $s_1' \not\approx_{\mathcal{T}} s_2$

$s_1 \approx_{\mathcal{T}} s_2$, $s_2 \to u_1 \to \cdots \to u_n \to s_2'$, $s_1 \to s_1'$

$u_i \approx_{\mathcal{T}} s_2$

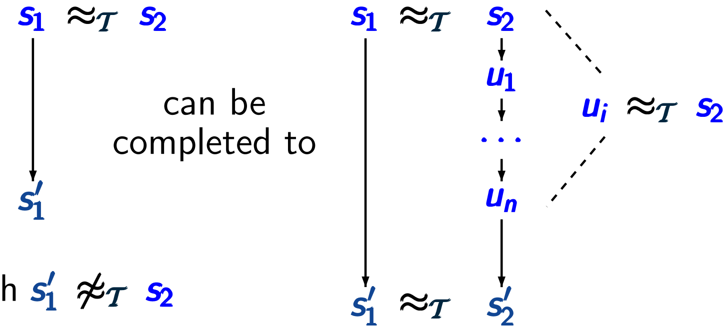
$s_1' \approx_{\mathcal{T}} s_2'$

Stutter bisimulation equivalence

$\approx_{\mathcal{T}}$ is the coarsest equivalence on state space S of $\mathcal{T}$ such that for all states s_1, s_2 with $s_1 \approx_{\mathcal{T}} s_2$:

(1) $L(s_1) = L(s_2)$

(2) simulation condition up to stuttering

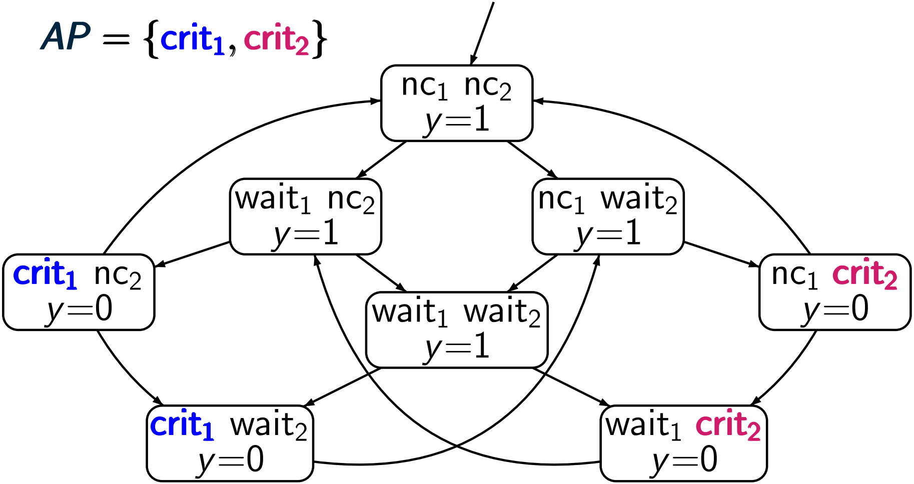


can be completed to

with $s_1' \not\approx_{\mathcal{T}} s_2$

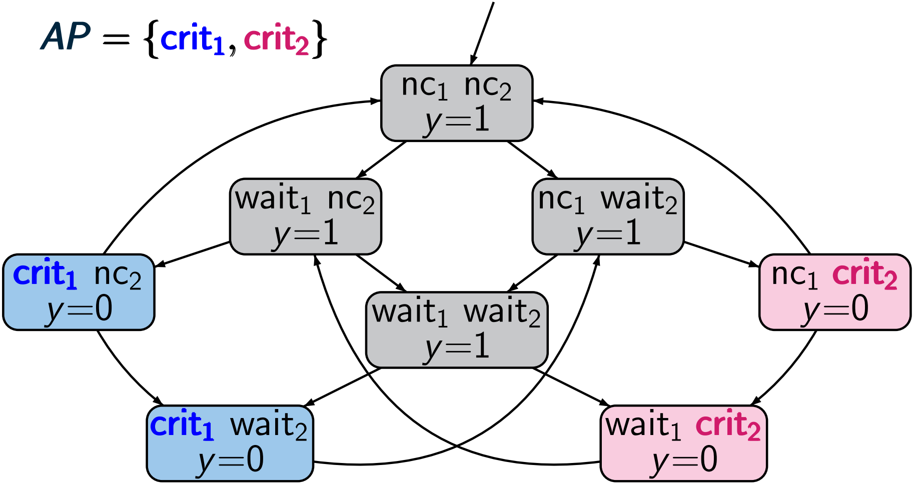
Example: mutual exclusion with semaphore

$AP = \{crit_1, crit_2\}$



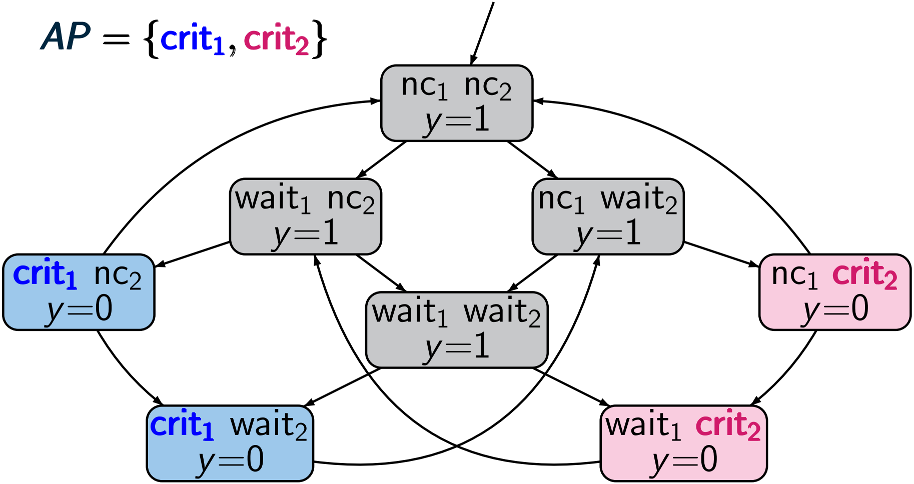
Example: mutual exclusion with semaphore

$AP = \{crit_1, crit_2\}$



Example: mutual exclusion with semaphore

$AP = \{\mathbf{crit_1}, \mathbf{crit_2}\}$



stutter bisimulation with three equivalence classes

Peterson algorithm

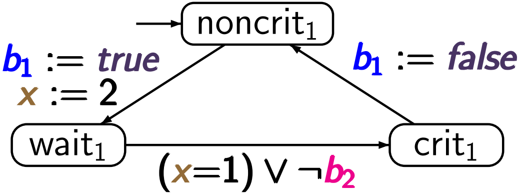
protocol for P_1

```
LOOP FOREVER
    noncritical section
    $b_1 := true$; $x := 2$
    AWAIT $(x{=}1) \lor \neg b_2$
    critical section
    $b_1 := false$
END LOOP
```

Peterson algorithm

protocol for P_1

```
LOOP FOREVER
    noncritical section
    $b_1 := true$; $x := 2$
    AWAIT $(x=1) \vee \neg b_2$
    critical section
    $b_1 := false$
END LOOP
```



Peterson algorithm

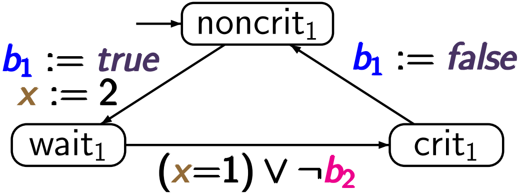
STUTTER5.4-7

protocol for P_1

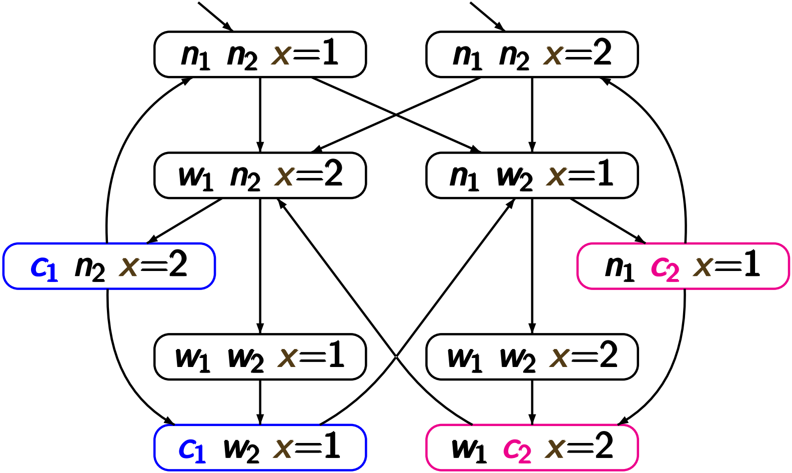
```
LOOP FOREVER
    noncritical section
    $b_1 := true$; $x := 2$
    AWAIT $(x=1) \lor \neg b_2$
    critical section
    $b_1 := false$
END LOOP
```

protocol for P_2

```
LOOP FOREVER
    noncritical section
    $b_2 := true$; $x := 1$
    AWAIT $(x=2) \lor \neg b_1$
    critical section
    $b_2 := false$
END LOOP
```

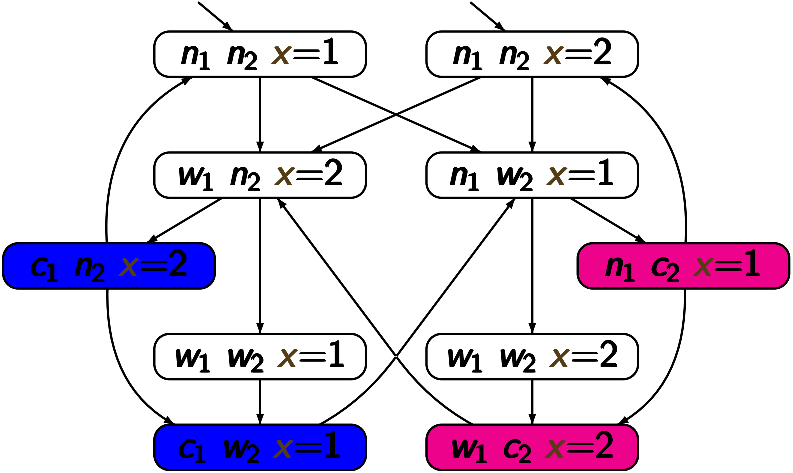


TS for the Peterson algorithm



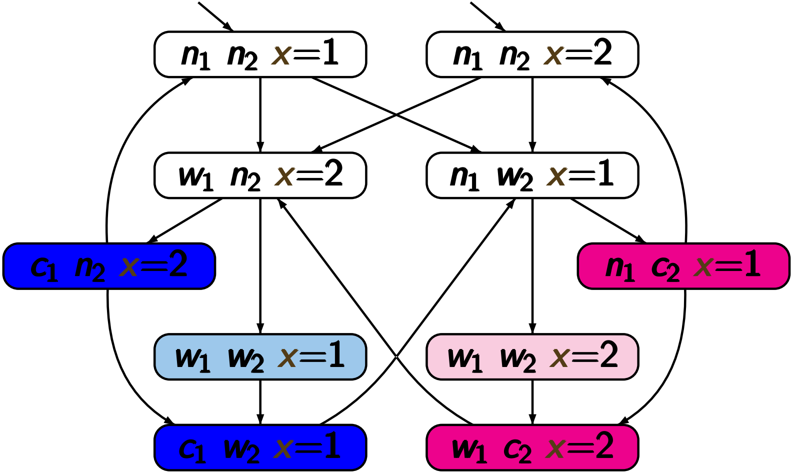
$AP = \{crit_1, crit_2\}$

TS for the Peterson algorithm



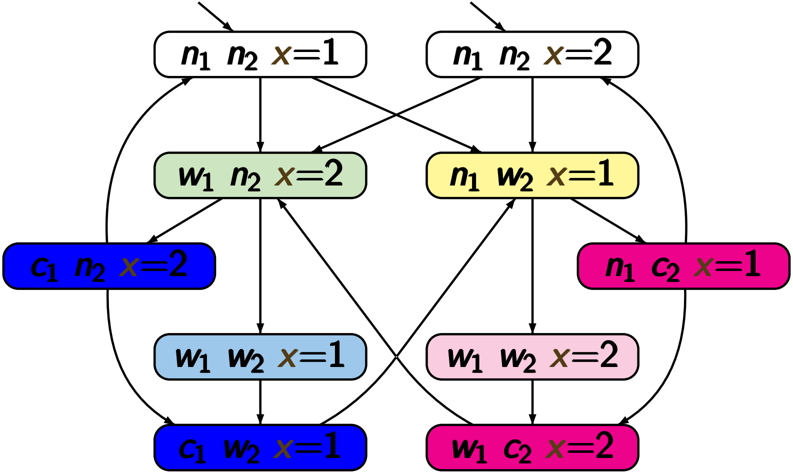
$AP = \{crit_1, crit_2\}$

TS for the Peterson algorithm



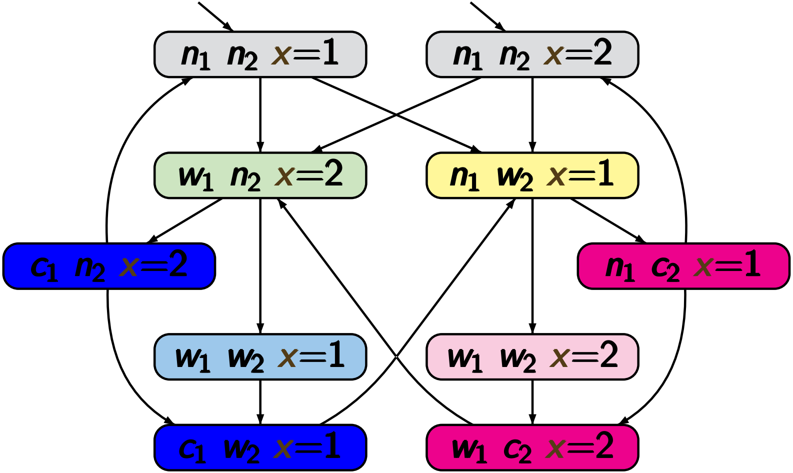
$AP = \{crit_1, crit_2\}$

TS for the Peterson algorithm



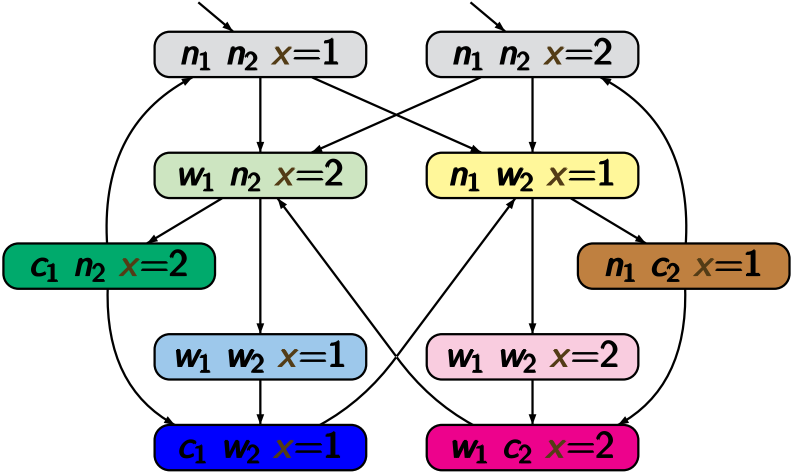
$AP = \{\text{crit}_1, \text{crit}_2\}$

TS for the Peterson algorithm



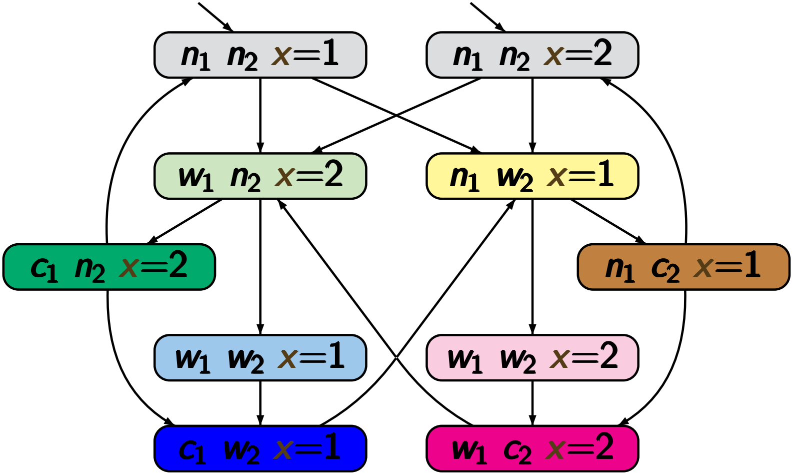
$AP = \{crit_1, crit_2\}$

TS for the Peterson algorithm



$AP = \{\text{crit}_1, \text{crit}_2\}$

TS for the Peterson algorithm



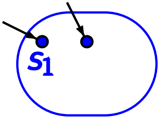
$AP = \{\text{crit}_1, \text{crit}_2\}$

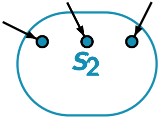
9 stutter bisimulation equivalence classes

Stutter bisimulation equivalence for two TS

Stutter bisimulation equivalence for two TS

transition system $\mathcal{T}_1$ transition system $\mathcal{T}_2$

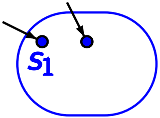


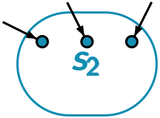


state space S_1 state space S_2

Stutter bisimulation equivalence for two TS

transition system $\mathcal{T}_1$ transition system $\mathcal{T}_2$





state space S_1 state space S_2

$\mathcal{T}_1 \approx \mathcal{T}_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$ such that

Stutter bisimulation equivalence for two TS

transition system $\mathcal{T}_1$ transition system $\mathcal{T}_2$

state space S_1 state space S_2

$\mathcal{T}_1 \approx \mathcal{T}_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$ such that

$\forall$ initial states s_1 of $\mathcal{T}_1$ $\exists$ initial state s_2 of $\mathcal{T}_2$ s.t. $s_1 \approx_{\mathcal{T}} s_2$

$\forall$ initial states s_2 of $\mathcal{T}_2$ $\exists$ initial state s_1 of $\mathcal{T}_1$ s.t. $s_1 \approx_{\mathcal{T}} s_2$

Stutter bisimulation equivalence for two TS

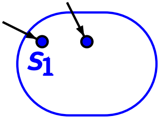
transition system $\mathcal{T}_1$ transition system $\mathcal{T}_2$

state space S_1 state space S_2

$\mathcal{T}_1 \approx \mathcal{T}_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $(\mathcal{T}_1, \mathcal{T}_2)$

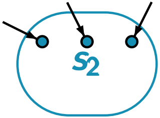
Stutter bisimulation equivalence for two TS

transition system $\mathcal{T}_1$



state space S_1

transition system $\mathcal{T}_2$



state space S_2

$\mathcal{T}_1 \approx \mathcal{T}_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $(\mathcal{T}_1, \mathcal{T}_2)$, i.e., $\mathcal{R} \subseteq S_1 \times S_2$ s.t.

Stutter bisimulation equivalence for two TS

transition system $\mathcal{T}_1$ transition system $\mathcal{T}_2$

state space S_1 state space S_2

$\mathcal{T}_1 \approx \mathcal{T}_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $(\mathcal{T}_1, \mathcal{T}_2)$, i.e., $\mathcal{R} \subseteq S_1 \times S_2$ s.t.

(1) if $(s_1, s_2) \in \mathcal{R}$ then $L_1(s_1) = L_2(s_2)$

Stutter bisimulation equivalence for two TS

transition system $\mathcal{T}_1$ transition system $\mathcal{T}_2$

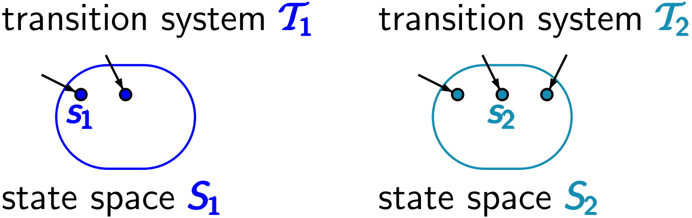
state space S_1 state space S_2

$\mathcal{T}_1 \approx \mathcal{T}_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $(\mathcal{T}_1, \mathcal{T}_2)$, i.e., $\mathcal{R} \subseteq S_1 \times S_2$ s.t.

(1) if $(s_1, s_2) \in \mathcal{R}$ then $L_1(s_1) = L_2(s_2)$

(2) and (3) $\ldots$

Stutter bisimulation equivalence for two TS



$\mathcal{T}_1 \approx \mathcal{T}_2$ iff there exists a stutter bisimulation $\mathcal{R}$ for $(\mathcal{T}_1, \mathcal{T}_2)$, i.e., $\mathcal{R} \subseteq S_1 \times S_2$ s.t.

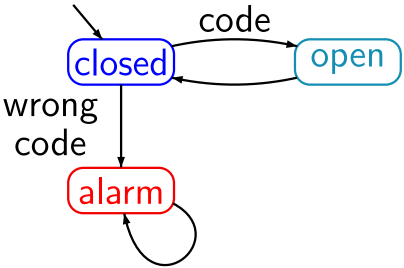
(1) if $(s_1, s_2) \in \mathcal{R}$ then $L_1(s_1) = L_2(s_2)$

(2) and (3) ...

(I) $\forall$ initial state s_1 of $\mathcal{T}_1$ $\exists$ initial state s_2 of $\mathcal{T}_2$ with $(s_1, s_2) \in \mathcal{R}$, and vice versa

Example: door opener

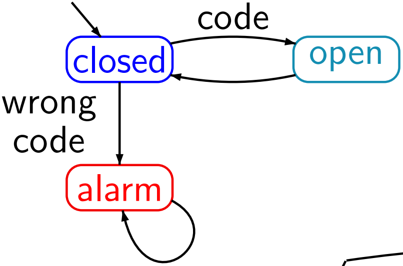
abstract model $\mathcal{T}_1$



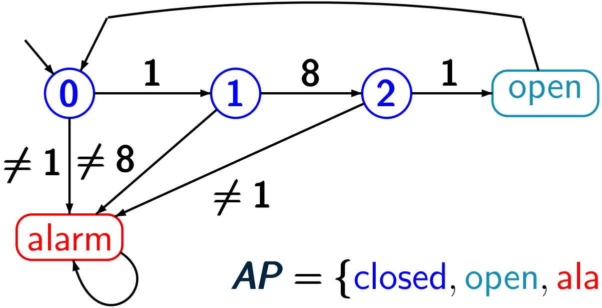
$AP = \{\text{closed}, \text{open}, \text{alarm}\}$

Example: door opener with code no. 181

abstract model $\mathcal{T}_1$



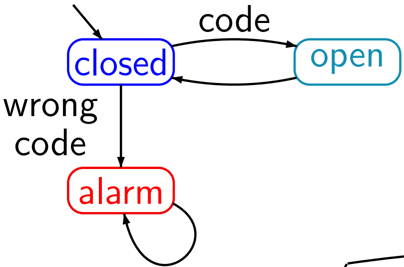
refinement TS $\mathcal{T}_2$



$AP = \{\text{closed}, \text{open}, \text{alarm}\}$

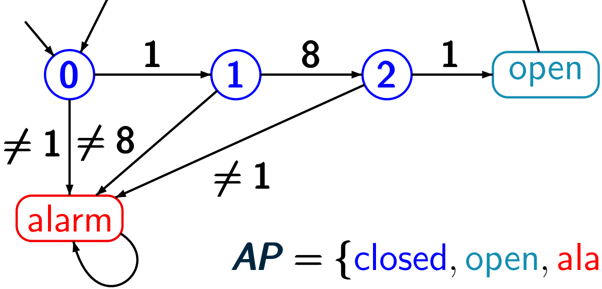
Example: door opener with code no. 181

abstract model $\mathcal{T}_1$



$\mathcal{T}_1 \not\approx \mathcal{T}_2$

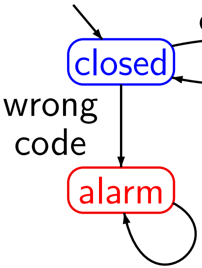
refinement TS $\mathcal{T}_2$



$AP = \{\text{closed}, \text{open}, \text{alarm}\}$

Example: door opener with code no. 181

abstract model $\mathcal{T}_1$

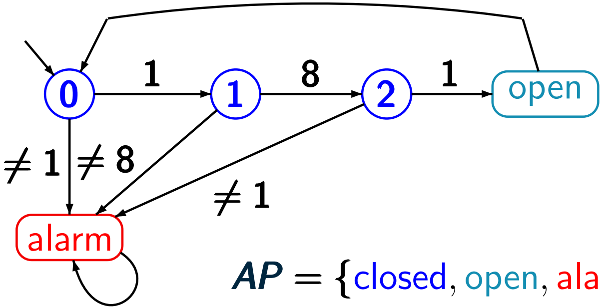


$$\mathcal{T}_1 \not\sim \mathcal{T}_2$$

abstraction from stutter steps:

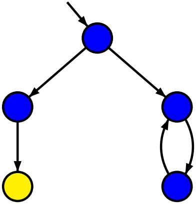
$$\mathcal{T}_1 \approx \mathcal{T}_2$$

refinement TS $\mathcal{T}_2$

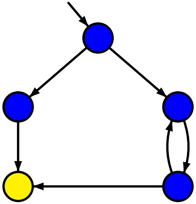


$AP = \{$closed, open, alarm$\}$

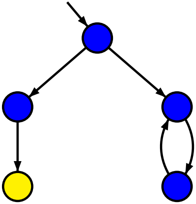
Correct or wrong?



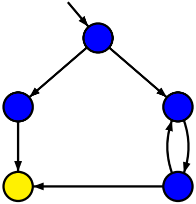
$\mathcal{T}_1 \approx \mathcal{T}_2$



Correct or wrong?

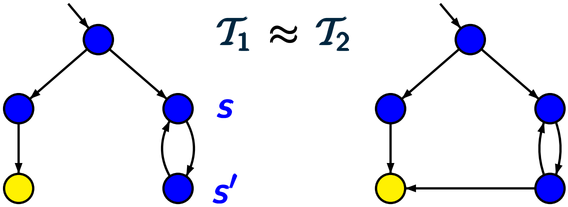


$\mathcal{T}_1 \approx \mathcal{T}_2$



wrong

Correct or wrong?

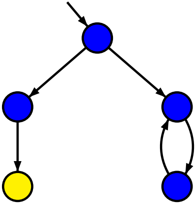


$\mathcal{T}_1 \approx \mathcal{T}_2$

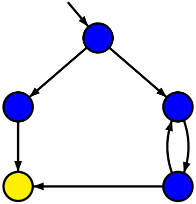
wrong

$\mathcal{T}_2$ does not contain an equivalent state to s and s'

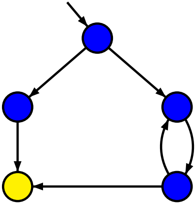
Correct or wrong?



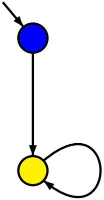
$\mathcal{T}_1 \approx \mathcal{T}_2$



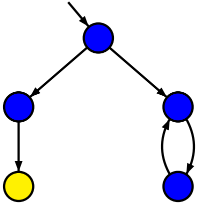
wrong



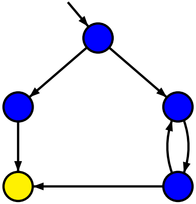
$\mathcal{T}_1 \approx \mathcal{T}_2$



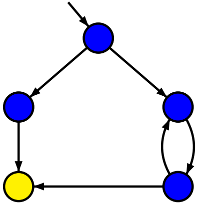
Correct or wrong?



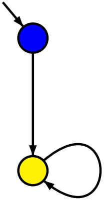
$\mathcal{T}_1 \approx \mathcal{T}_2$



wrong

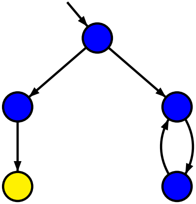


$\mathcal{T}_1 \approx \mathcal{T}_2$

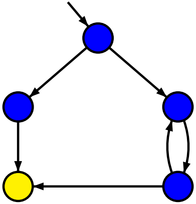


correct

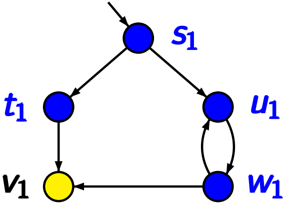
Correct or wrong?



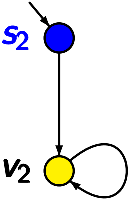
$\mathcal{T}_1 \approx \mathcal{T}_2$



wrong



$\mathcal{T}_1 \approx \mathcal{T}_2$



correct

stutter bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$:

$$\{(s_1, s_2), (t_1, s_2), (u_1, s_2), (w_1, s_2), (v_1, v_2)\}$$

Correct or wrong?

If $s_1 \sim_{\mathcal{T}} s_2$ then $s_1 \approx_{\mathcal{T}} s_2$

remind: $\sim_{\mathcal{T}}$ bisimulation equivalence for $\mathcal{T}$

$\approx_{\mathcal{T}}$ stutter bisimulation equivalence for $\mathcal{T}$

Correct or wrong?

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correct

remind: $\sim_{\mathcal{T}}$ bisimulation equivalence for $\mathcal{T}$

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as $\sim_{\mathcal{T}}$ is a stutter bisimulation for $\mathcal{T}$

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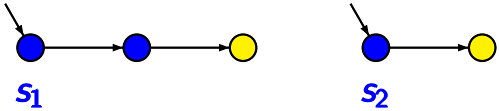

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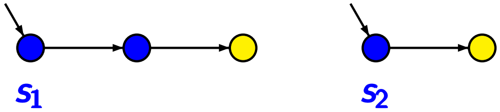

$s_1 \approx_{\mathcal{T}} s_2$

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Then $s_1 \approx_{\mathcal{T}} s_2$ implies $s_1 \sim_{\mathcal{T}} s_2$

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$\Longrightarrow$ $m=0$. Hence: $s_2 \rightarrow s_2'$ and $s_1' \approx_{\mathcal{T}} s_2'$

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- state space: $S/\!\approx_{\mathcal{T}}$ ⟵ set of stutter bisimulation equivalence classes

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equivalence class of state s

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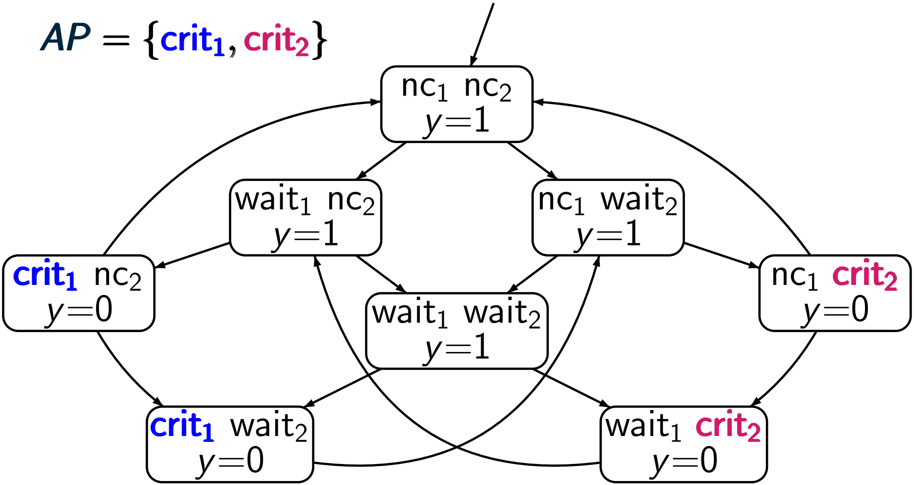
proof: $\mathcal{R} = \big\{ (s, [s]) \, : \, s \in S \big\}$

is a stutter bisimulation for $(\mathcal{T}, \mathcal{T}/{\approx})$

Example: mutual exclusion with semaphore

$AP = \{\mathsf{crit}_1, \mathsf{crit}_2\}$



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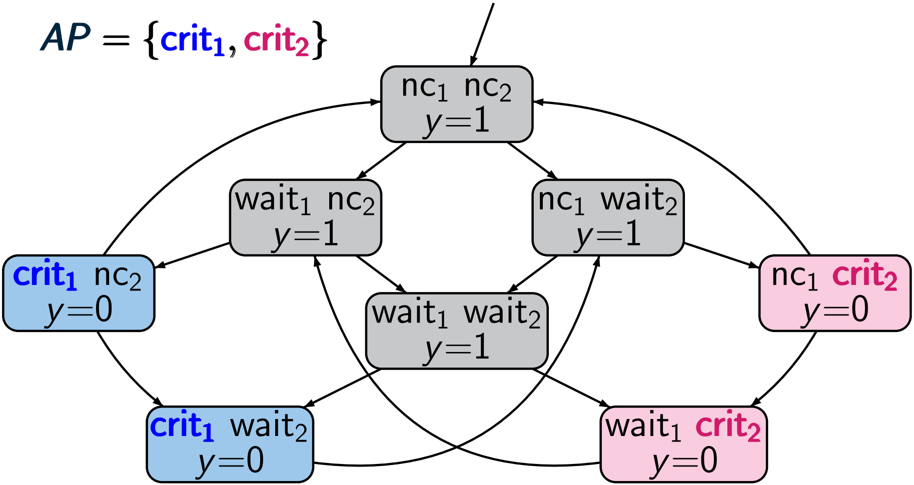

stutter bisimulation with three equivalence classes

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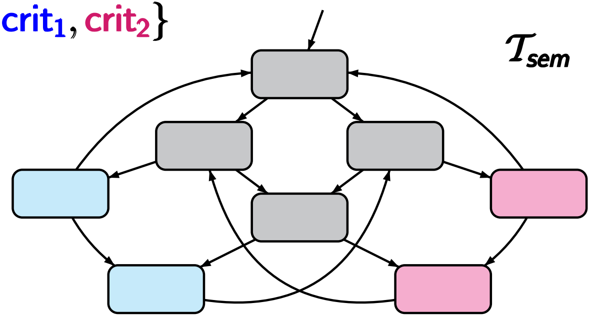

Example: mutual exclusion with semaphore STUTTER5.4-16B

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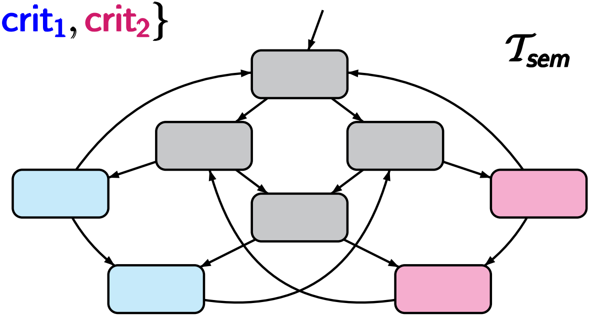
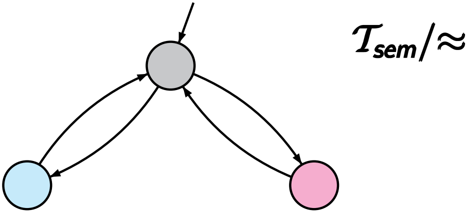
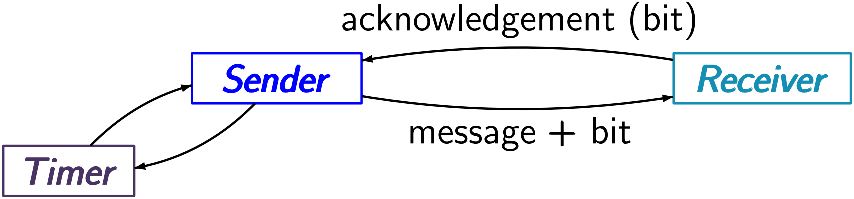




Alternating bit protocol



acknowledgement (bit)

Sender

Receiver

message + bit

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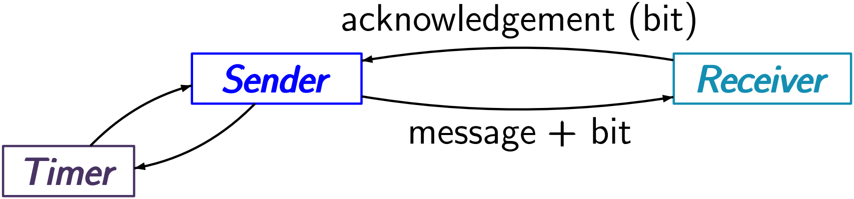

- formalization by a closed channel system

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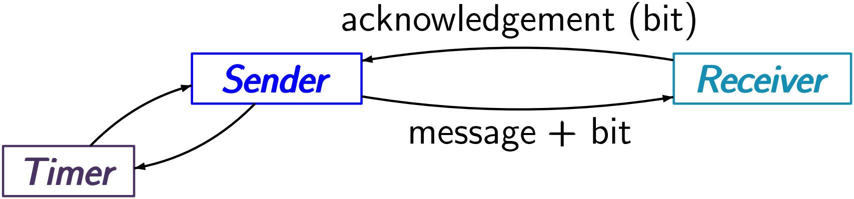
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- TS with about $\mathbf{2^{30}}$ states
  for channels of capacity **10**

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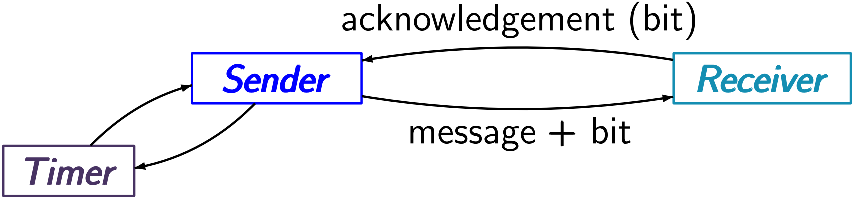
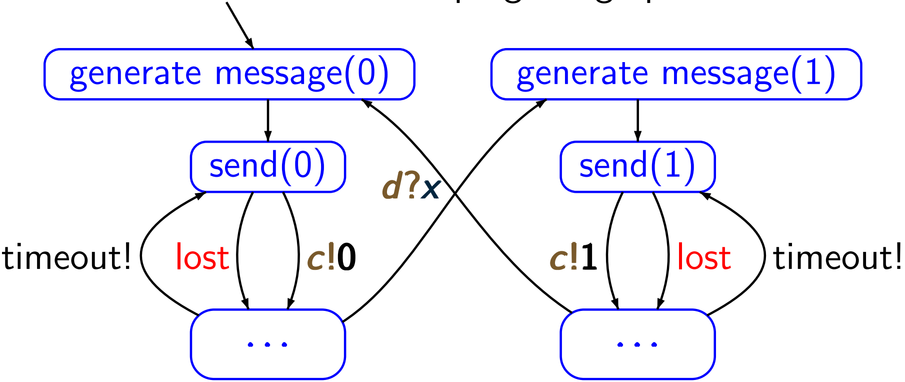


program graph for sender



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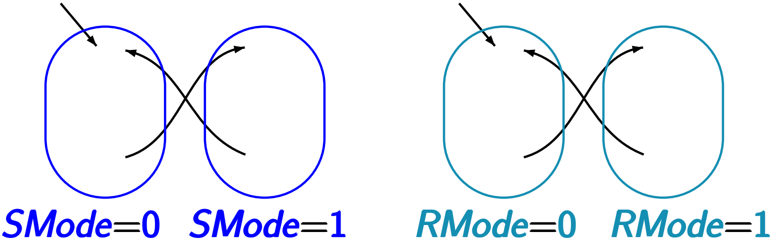

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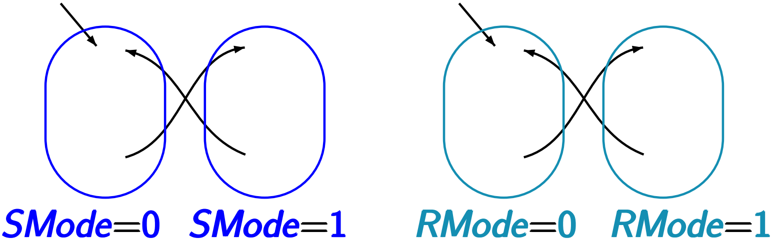

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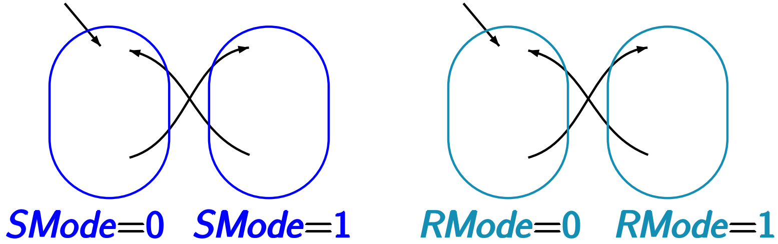

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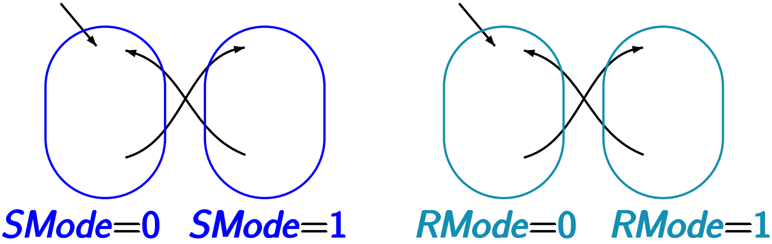

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stutter bisimulation quotient

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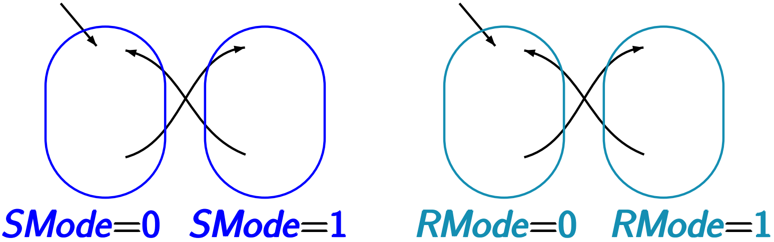

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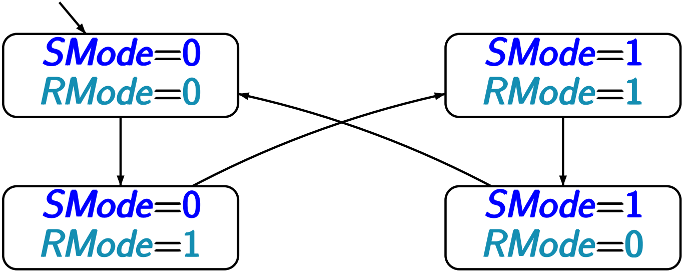

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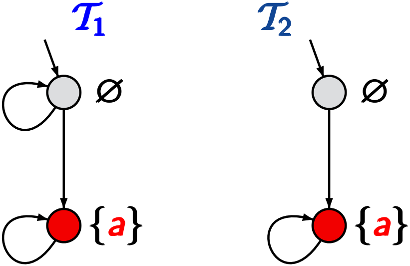

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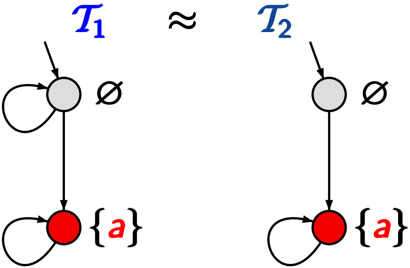

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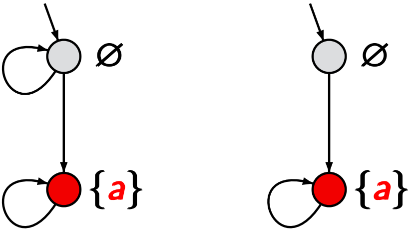


$\mathcal{T}_1 \not\models \Diamond a$ $\qquad$ $\mathcal{T}_2 \models \Diamond a$

$AP = \{a\}$

$\varnothing^\omega \in \mathit{Traces}(\mathcal{T}_1)$

$\varnothing^\omega \notin \mathit{Traces}(\mathcal{T}_2)$

Abstraction from stuttering: LT vs. BT

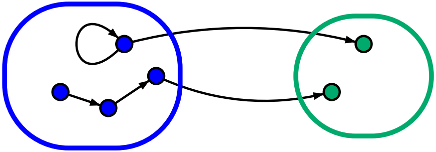
Stutter bisimulation/stutter trace equivalence

stutter trace equivalence: $\mathcal{T}_1 \triangleq \mathcal{T}_2$ iff

$$\forall \pi_1 \in Paths(\mathcal{T}_1)\ \exists \pi_2 \in Paths(\mathcal{T}_2) \text{ s.t. } \pi_1 \triangleq \pi_2$$

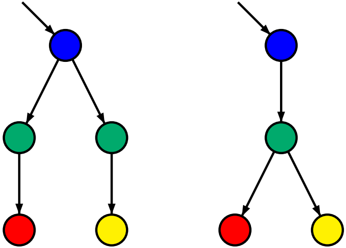
$$\forall \pi_2 \in Paths(\mathcal{T}_2)\ \exists \pi_1 \in Paths(\mathcal{T}_1) \text{ s.t. } \pi_1 \triangleq \pi_2$$

stutter bisimulation equivalence $\approx_{\mathcal{T}}$



$B \in S/\approx_{\mathcal{T}}$ $\qquad$ $C \in S/\approx_{\mathcal{T}}$

Stutter bisimulation/stutter trace equivalence



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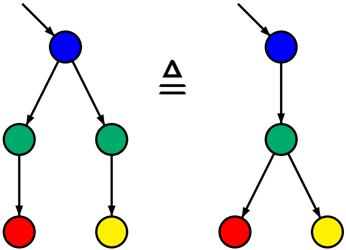
$\approx$ stutter bisimulation equivalence

Stutter bisimulation/stutter trace equivalence



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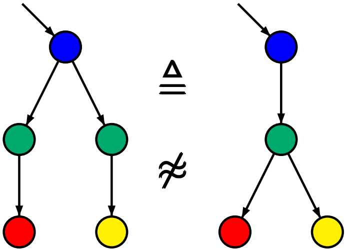

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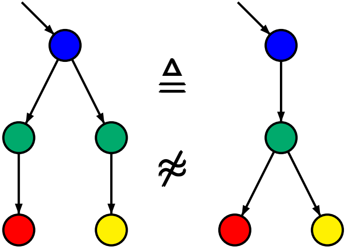

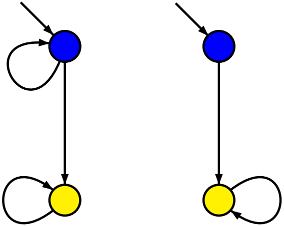

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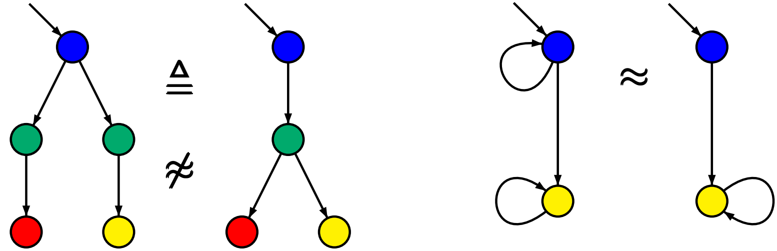

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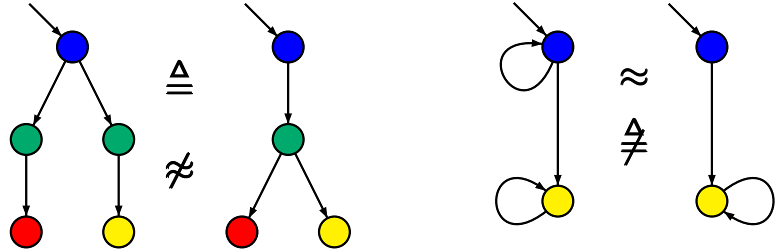

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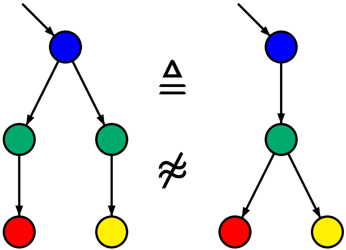
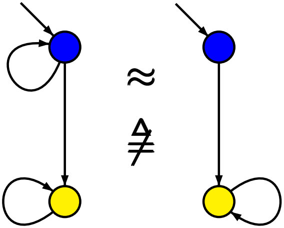
$\approx$ stutter bisimulation equivalence

Stutter bisimulation/stutter trace equivalence





$\triangleq$ stutter trace equivalence

$\approx$ stutter bisimulation equivalence

$\approx$ and $\triangleq$ are incomparable