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- strong vs. weak relations
  * strong: reasoning about all transitions
  * weak: abstraction from stutter steps

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation ⟵
- CTL, CTL*-equivalence
- computing the bisimulation quotient
- abstraction stutter steps
- simulation relations

Bisimulation for two transition systems

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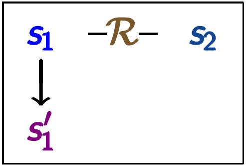
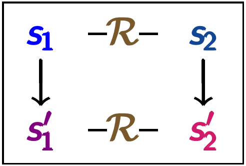


can be completed to



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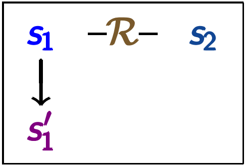

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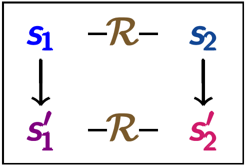

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>
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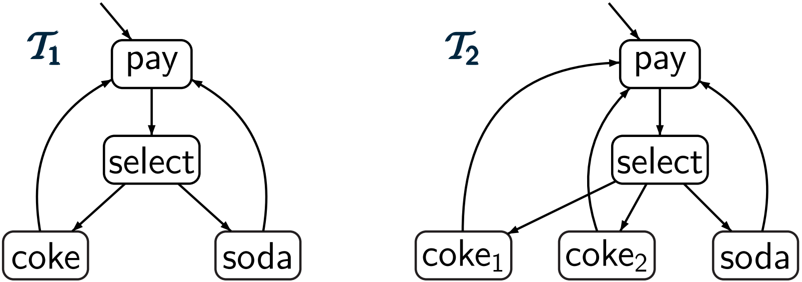
Two beverage machines



$$AP = \{pay, coke, soda\}$$

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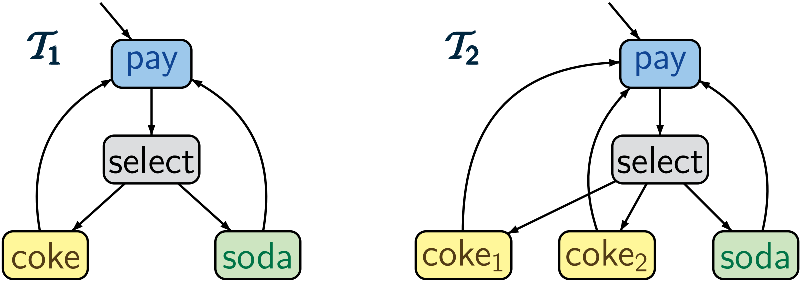

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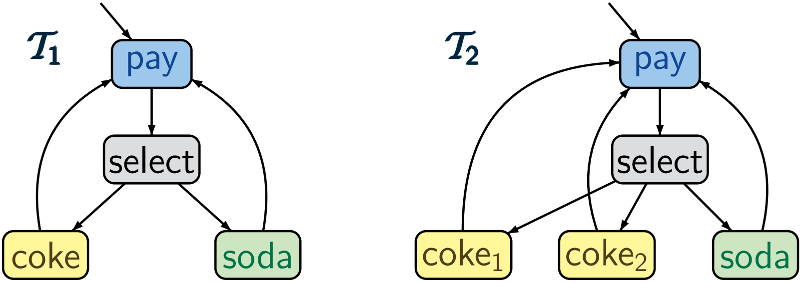

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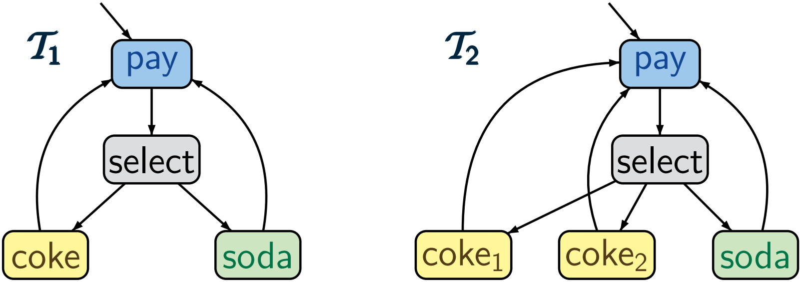

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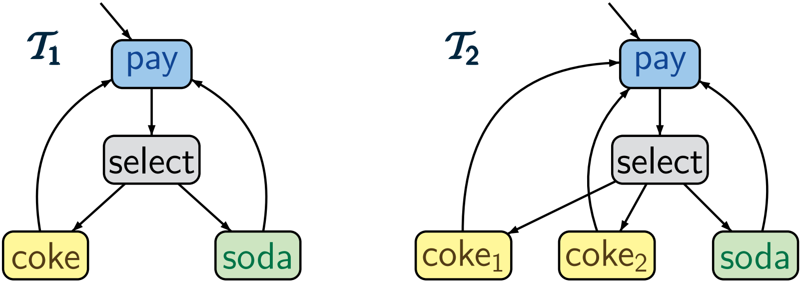

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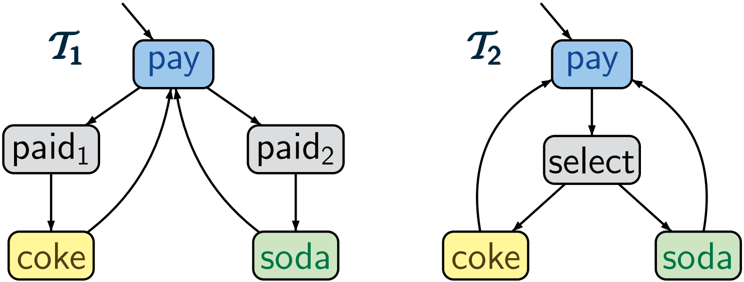
{ (pay,pay), (select,select), (soda,soda)
(coke,$coke_1$), (coke,$coke_2$) }

Two beverage machines



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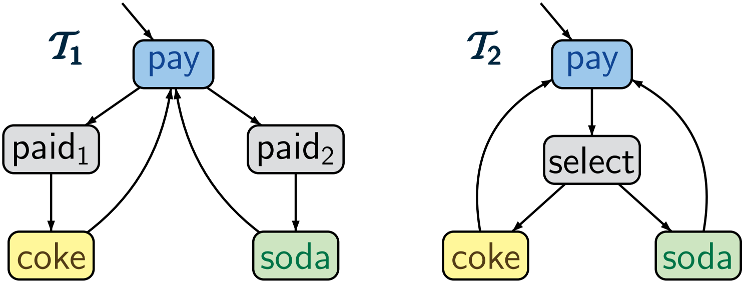

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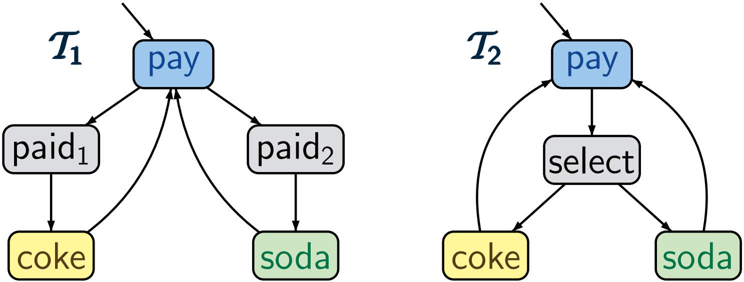

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$\mathcal{T}_1 \not\sim \mathcal{T}_2$

because there is no state in $\mathcal{T}_1$ that has both

- a successor labeled with coke and
- a successor labeled with soda

Simulation condition of bisimulations

BSEQOR5.1-9-BIS

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \\ s_1' & & \end{array}$$

can be completed to

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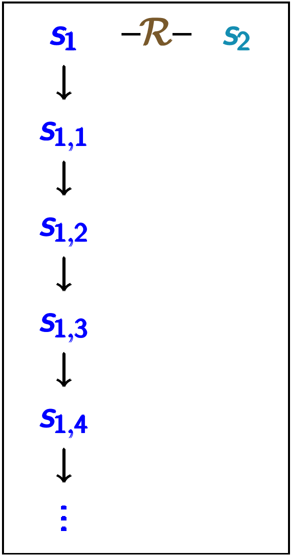
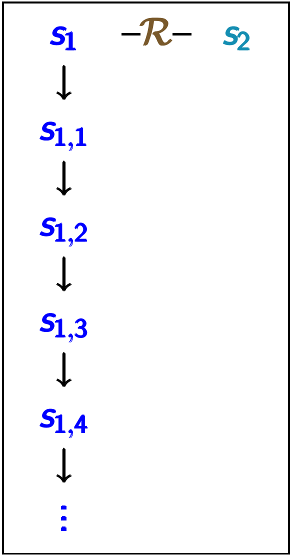
Path lifting for bisimulation $\mathcal{R}$



Path lifting for bisimulation $\mathcal{R}$



can be
completed to

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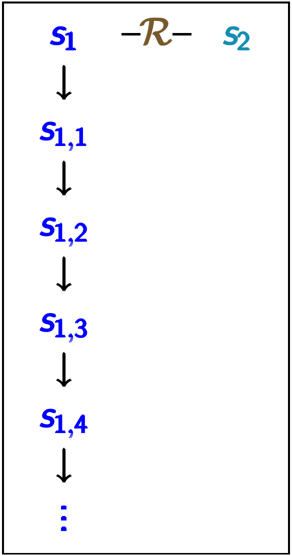

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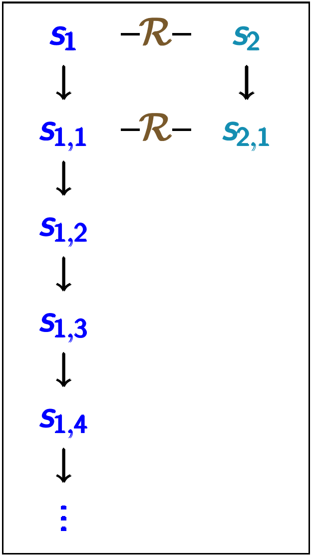

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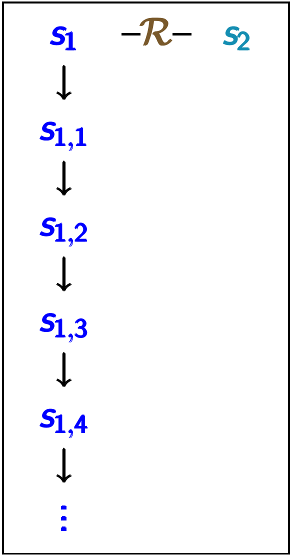

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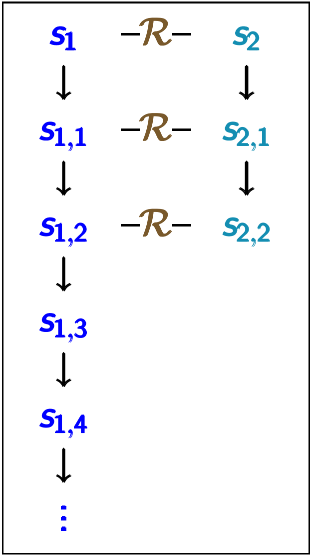

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$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \downarrow \\
s_{1,3} & -\mathcal{R}- & s_{2,3} \\
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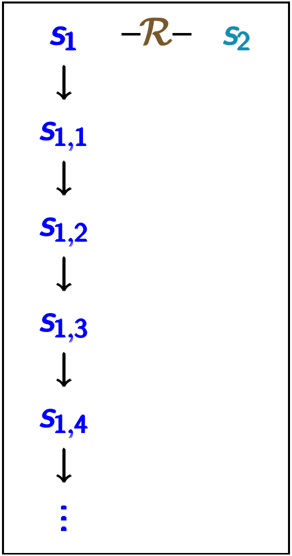

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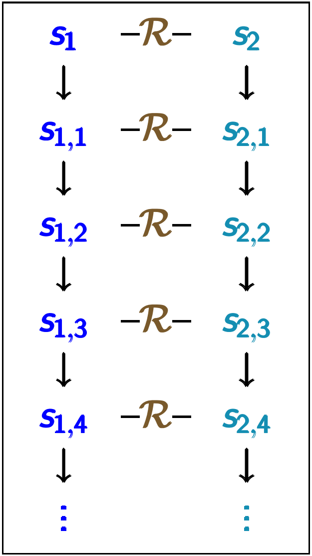

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If $\mathcal{R}$ is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$ then

$$\mathcal{R}^{-1} = \{(s_2, s_1) : (s_1, s_2) \in \mathcal{R}\}$$

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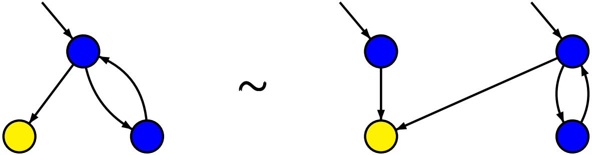
$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_3) : \exists s_2 \text{ s.t. } (s_1, s_2) \in \mathcal{R}_{1,2} \text{ and } (s_2, s_3) \in \mathcal{R}_{2,3} \}$$

is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_3)$

Correct or wrong?



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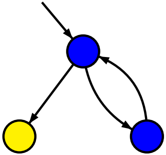

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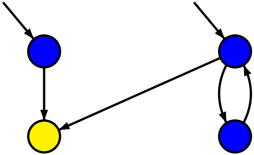

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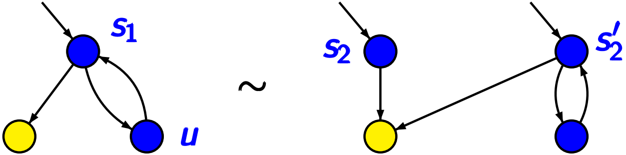

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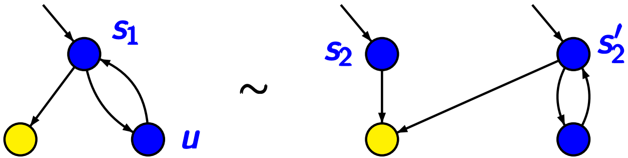

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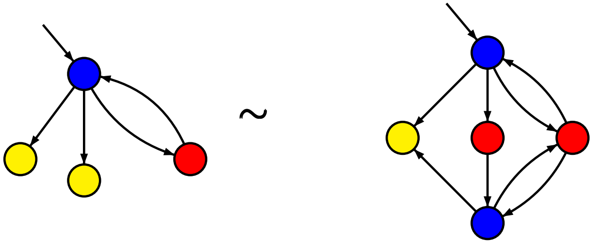

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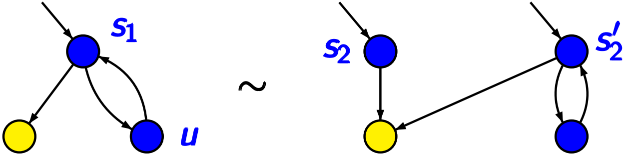

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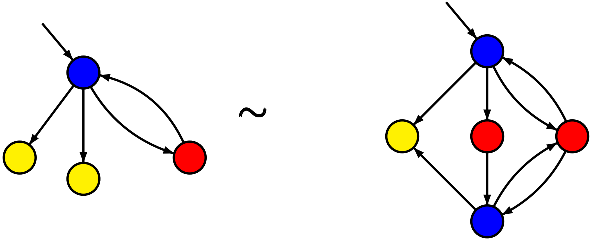

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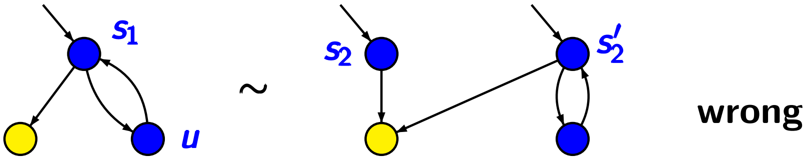

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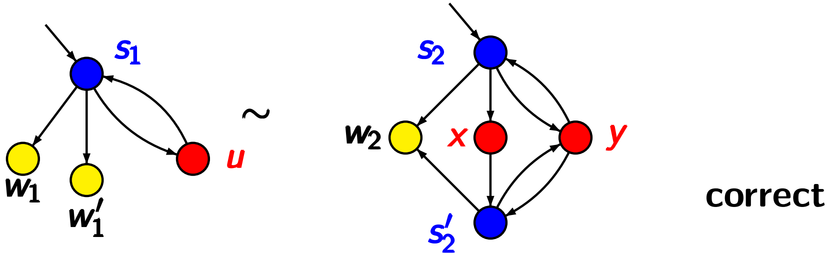

bisimulation:

$\{(w_1, w_2), (w_1', w_2), (s_1, s_2), (s_1, s_2'), (u, x), (u, y)\}$

Bisimulation vs. trace equivalence

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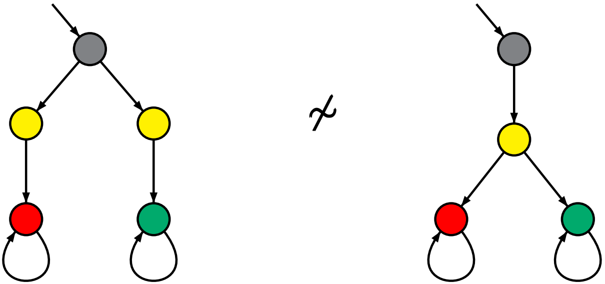

trace equivalent, but not bisimulation equivalent

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Trace equivalence is strictly coarser than bisimulation equivalence.

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Trace equivalence is strictly coarser than bisimulation equivalence.

Bisimulation equivalent transition systems satisfy the same **LT properties** (e.g., **LTL** formulas).

Bisimulation equivalence ...

- as a relation that compares 2 transition systems

Bisimulation equivalence ...

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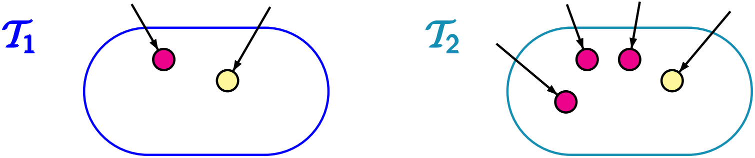

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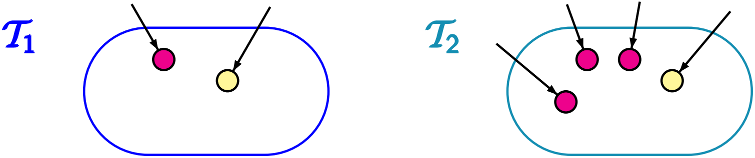

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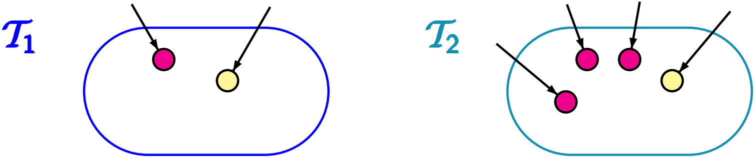

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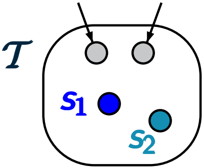

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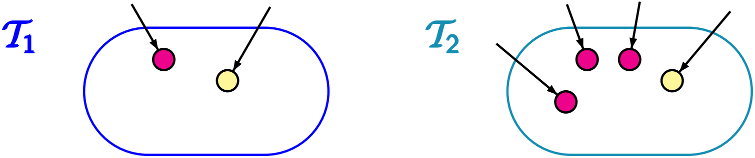

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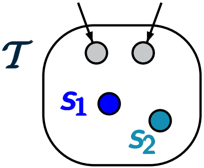

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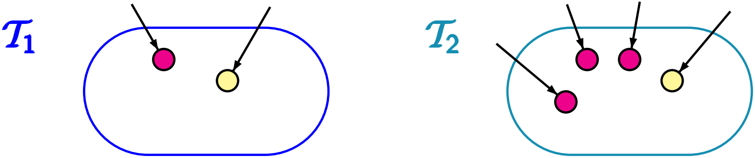

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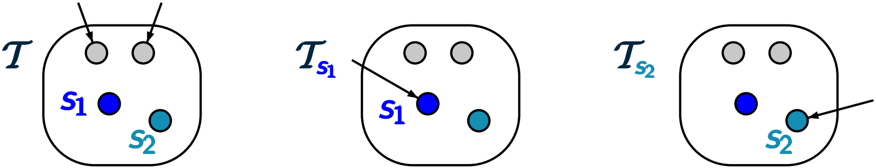

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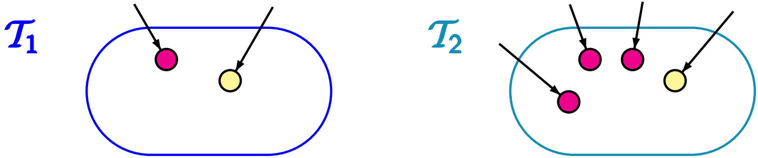

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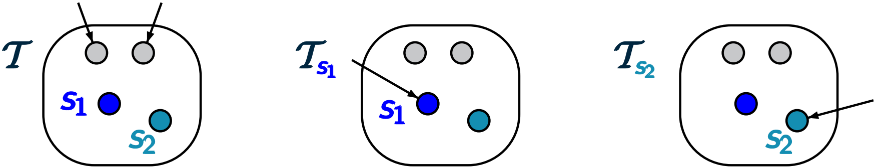

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Bisimulations on a single TS

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A bisimulation for $\mathcal{T}$ is a binary relation $\mathcal{R}$ on the state space of $\mathcal{T}$ s.t. for all $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

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(3) $\forall\, s_2' \in Post(s_2)\ \exists\, s_1' \in Post(s_1)$ s.t. $(s_1', s_2') \in \mathcal{R}$

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coinductive definition of $\sim_{\mathcal{T}}$:

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$$\begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \\ s_1' & & \end{array} \qquad \text{can be completed to} \qquad \begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \downarrow \\ s_1' & \sim_{\mathcal{T}} & s_2' \end{array}$$

Two variants of bisimulation equivalence

$\sim$ relation that compares **2** transition systems

$\sim_{\mathcal{T}}$ equivalence on the state space of a single TS $\mathcal{T}$

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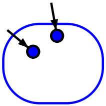
2. $\sim$ can be derived from $\sim_{\mathcal{T}}$

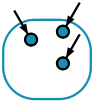
Derivation of $\sim$ from $\sim_{\mathcal{T}}$

given two transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$

$\mathcal{T}_1$ with state space S_1



$\mathcal{T}_2$ with state space S_2



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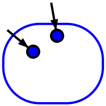

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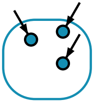

consider $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$
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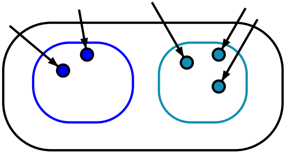

Derivation of $\sim$ from $\sim_{\mathcal{T}}$

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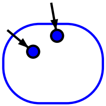
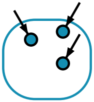


$\mathcal{T}_2$ with state space S_2



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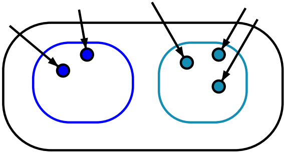

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Derivation of $\sim$ from $\sim_{\mathcal{T}}$

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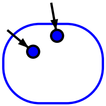

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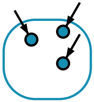

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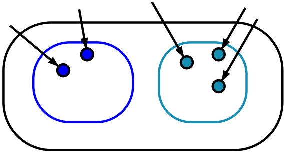

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Bisimulation quotient

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Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

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bisimulation quotient $\mathcal{T}/\!\sim$ arises from $\mathcal{T}$
by collapsing bisimulation equivalent states

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$$\mathcal{T}/\!\sim\; = (S', Act', \rightarrow', S'_0, AP, L')$$

- state space: $S' = S/\!\sim_{\mathcal{T}}$

↑ set of bisimulation equivalence classes

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↑

well-defined

by the labeling condition of bisimulations

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action labels irrelevant

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bisimulation quotient:

$$\mathcal{T}/{\sim} = (S', \{\tau\}, \rightarrow', S'_0, AP, L')$$

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$$\mathcal{T} \sim \mathcal{T}/\sim$$

Example: interleaving of *n* printers

parallel system $\mathcal{T} = \underbrace{\mathit{Printer} \,|||\, \mathit{Printer} \,|||\, \ldots \,|||\, \mathit{Printer}}_{n \text{ printers}}$

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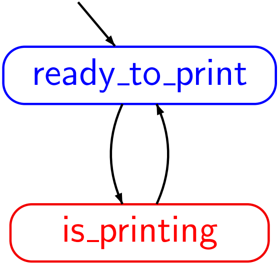
transition system for each printer



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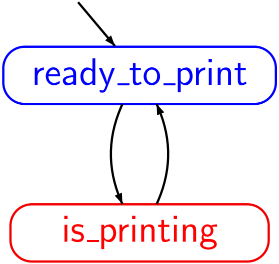
$AP = \{0, 1, \ldots, n\}$ "number of available printers"

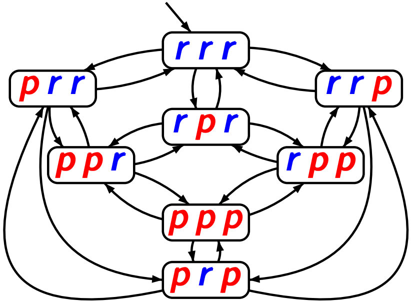
transition system for each printer



Example: n=3 printers

parallel system $\mathcal{T} = \underbrace{\mathit{Printer} \mid\mid\mid \mathit{Printer} \mid\mid\mid \ldots \mid\mid\mid \mathit{Printer}}_{n \text{ printers}}$

$AP = \{0, 1, 2, 3\}$



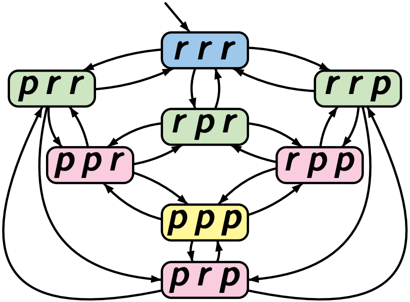
p: is printing

r: ready to print

Example: n=3 printers

parallel system $\mathcal{T} = \underbrace{\textit{Printer} \,|||\, \textit{Printer} \,|||\, \ldots \,|||\, \textit{Printer}}_{\boldsymbol{n} \text{ printers}}$

$AP = \{0, 1, 2, 3\}$



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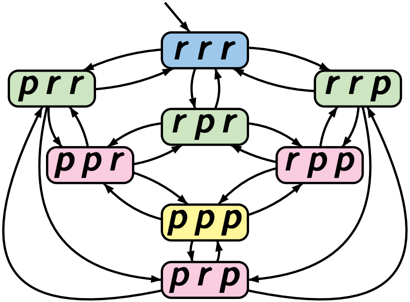

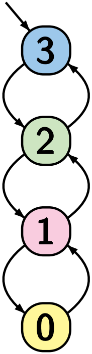

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bisimulation quotient

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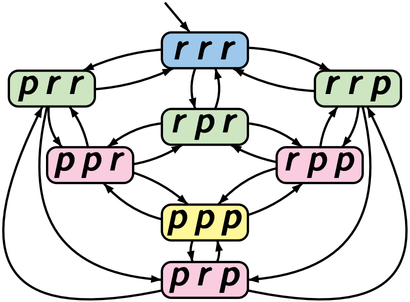

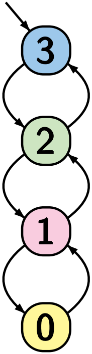

2^n states

$n+1$ states

Mutual exclusion

Mutual exclusion

solutions for mutual exclusion problems:

- semaphore
- Peterson's algorithm

Mutual exclusion: Bakery algorithm

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↑

given two concurrent processes P_1 and P_2

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- two additional shared variables: $x_1, x_2 \in \mathbb{N}$

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  if $x_1 < x_2$ then P_1 enters its critical section

  if $x_2 < x_1$ then P_2 enters its critical section

Mutual exclusion: Bakery algorithm

solutions for mutual exclusion problems:

- semaphore
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given two concurrent processes P_1 and P_2

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- if P_1 and P_2 are waiting then:

  if $x_1 < x_2$ then P_1 enters its critical section

  if $x_2 < x_1$ then P_2 enters its critical section

  $x_1 = x_2$: cannot happen

Bakery algorithm

protocol for P_1:

```
LOOP FOREVER
    noncritical actions
    x1 := x2+1
    AWAIT (x1 < x2) ∨ (x2=0);
    critical section;
    x1 := 0
END LOOP
```

symmetric protocol for P_2

Bakery algorithm

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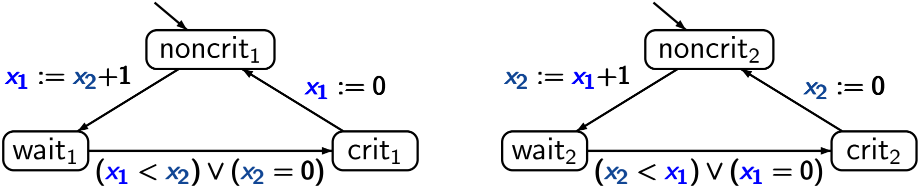
END LOOP
```

initially:

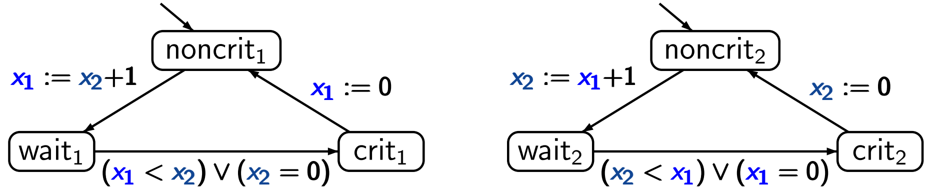
$x_1 = x_2 = 0$

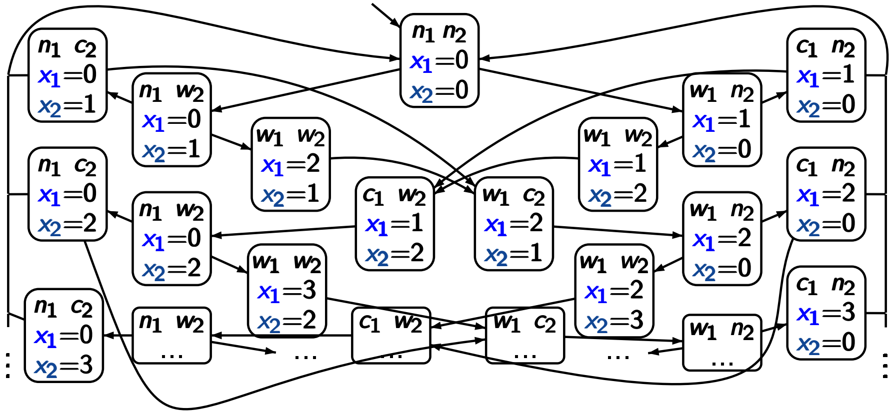
symmetric protocol for P_2

Program graphs for the Bakery algorithm



Transition system for the Bakery algorithm





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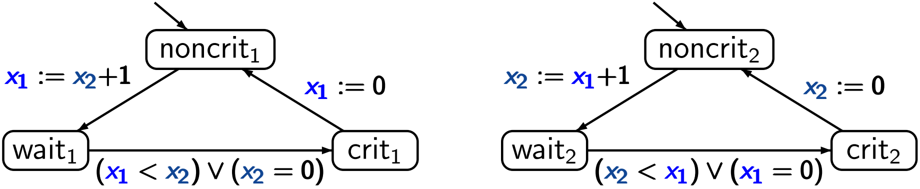

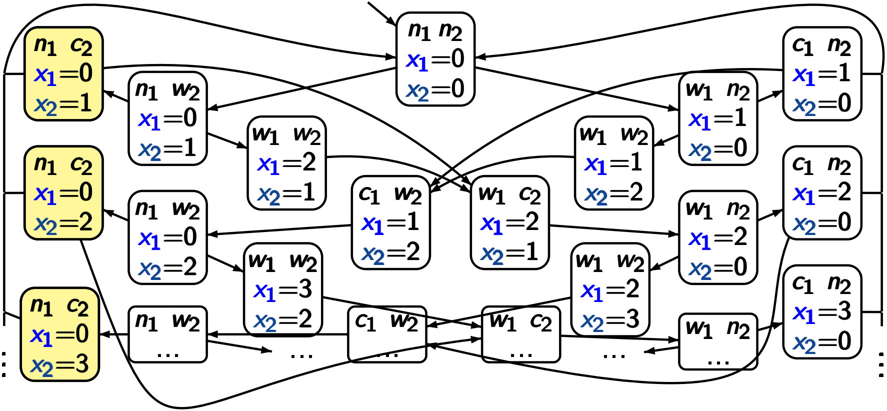

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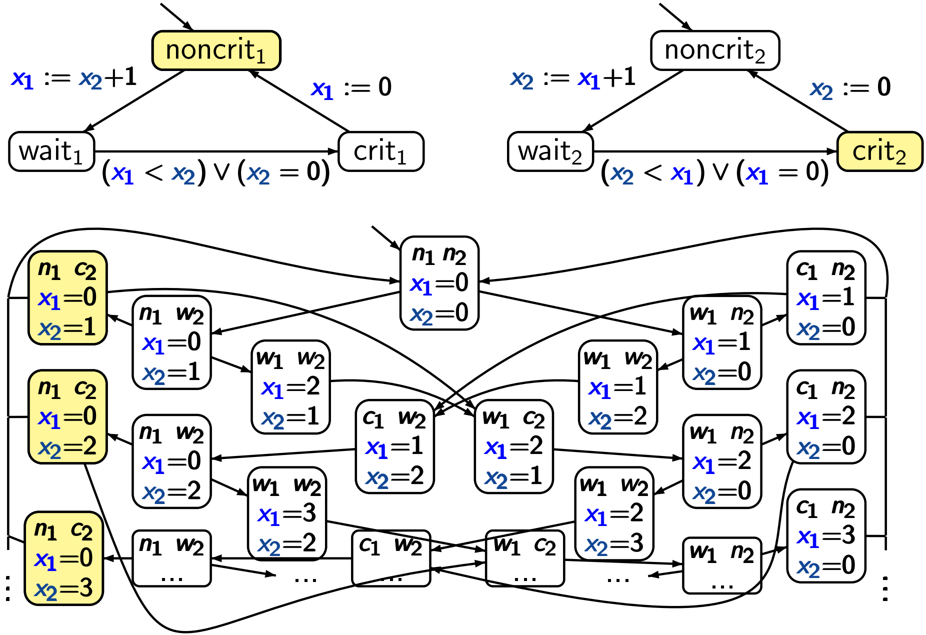

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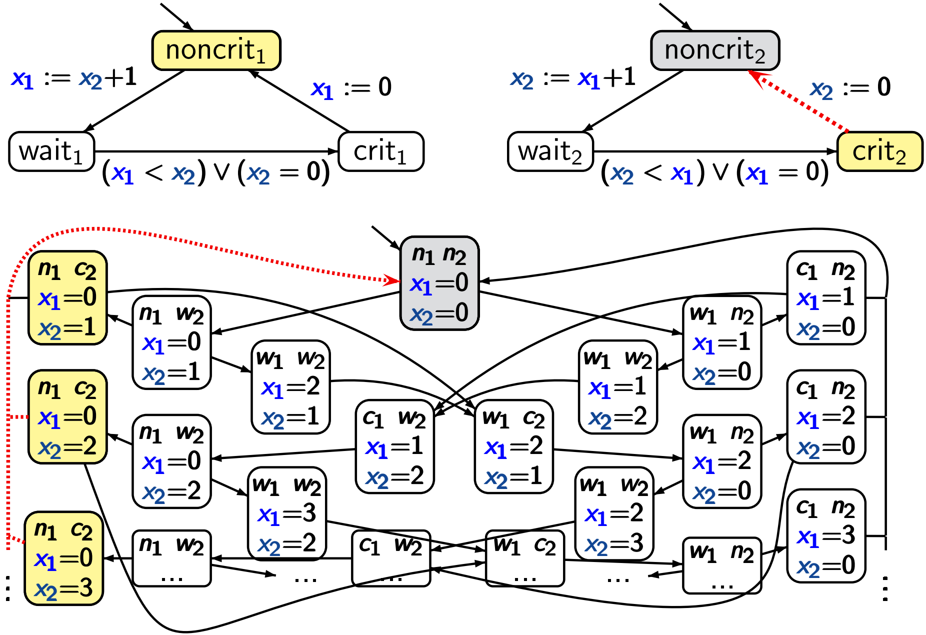

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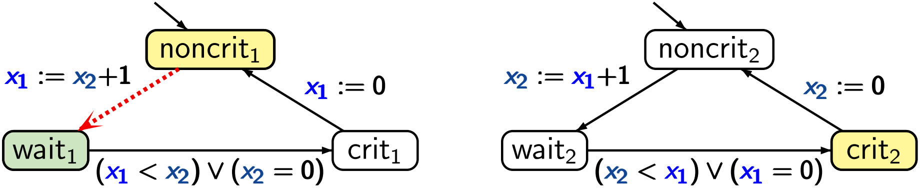

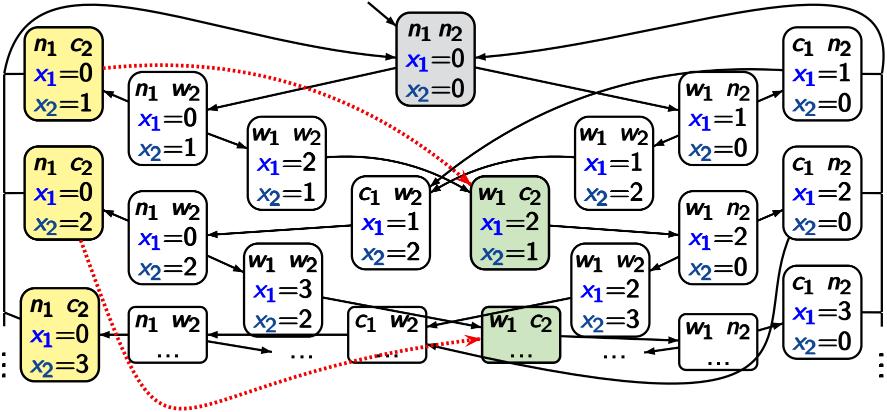

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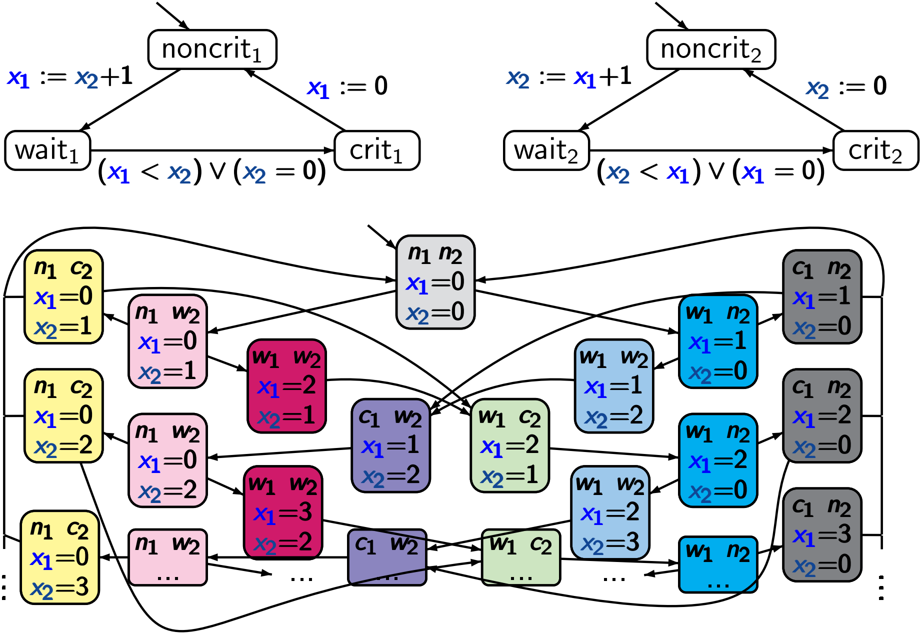
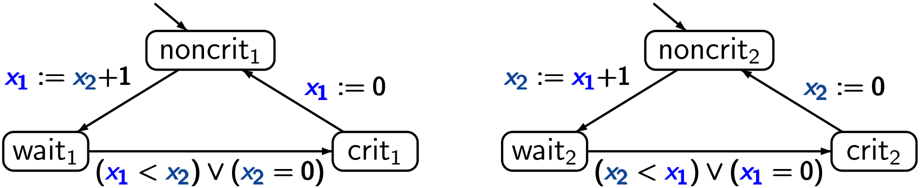
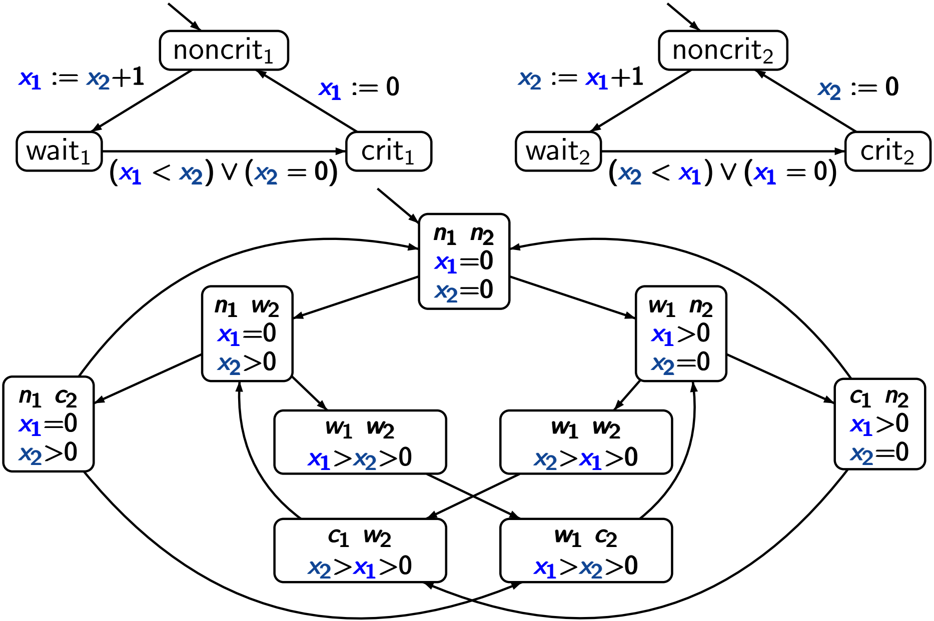


Bakery algorithm: bisimulation quotient



infinite transition system with a
finite bisimulation quotient

Bakery algorithm: bisimulation quotient



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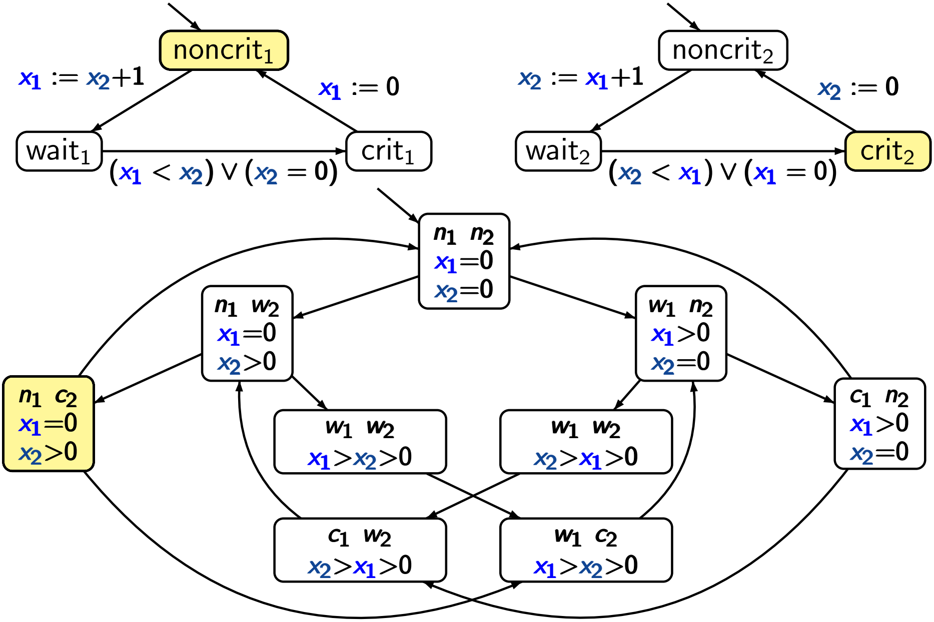

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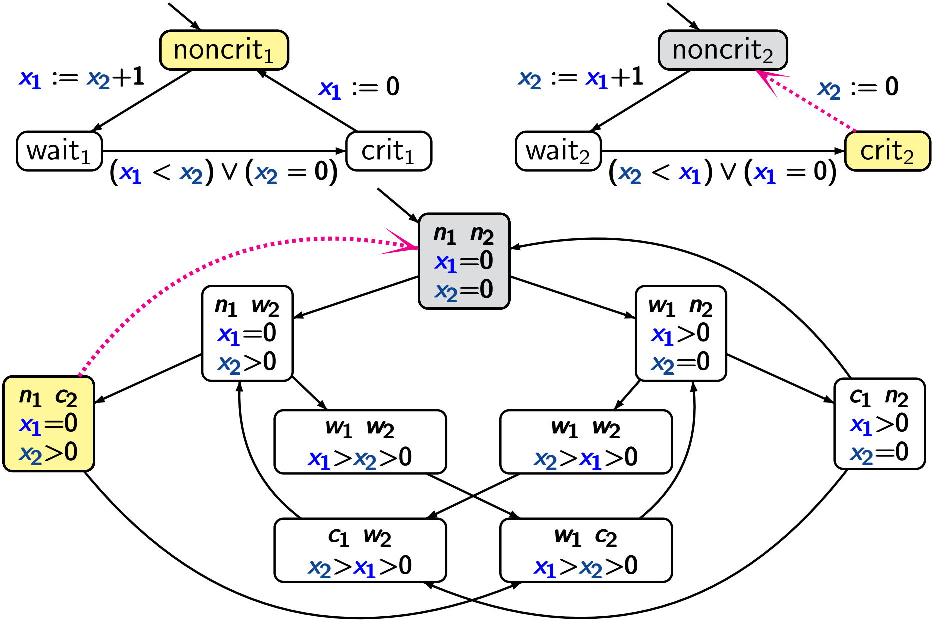

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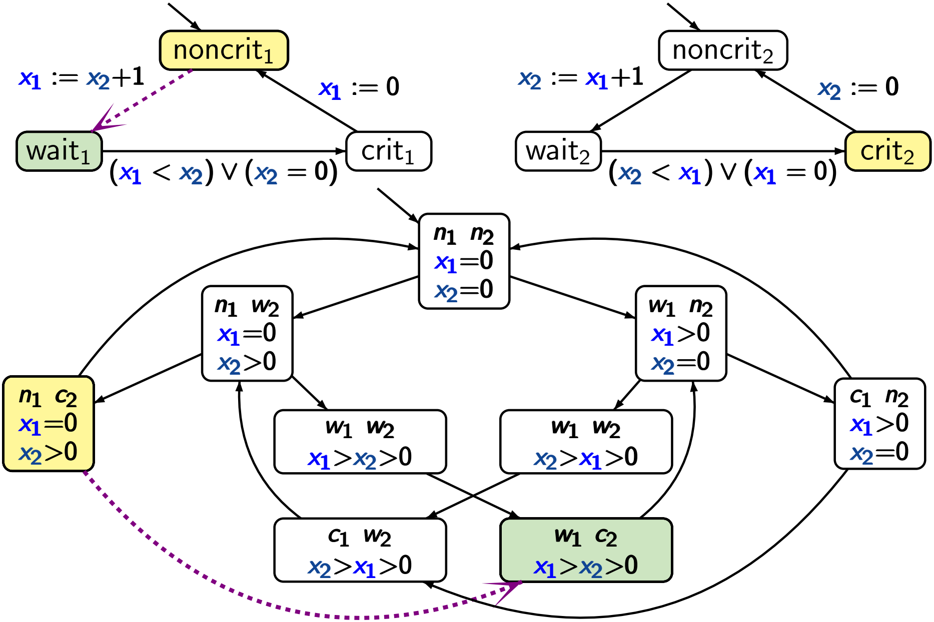

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation
- CTL, CTL*-equivalence ⟵
- computing the bisimulation quotient
- abstraction stutter steps
- simulation relations

Recall: CTL*

CTL* state formulas

$$\Phi \ ::= \ \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

CTL* path formulas

$$\varphi \ ::= \ \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

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derived operators:

- $\Diamond$, $\Box$, . . . as in **LTL**

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derived operators:

- $\Diamond$, $\Box$, ... as in **LTL**
- universal quantification: $\forall\varphi \overset{\text{def}}{=} \neg\exists\neg\varphi$

Recall: CTL* and CTL

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CTL: sublogic of **CTL***

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CTL: sublogic of **CTL***

- with path quantifiers $\exists$ and $\forall$
- restricted syntax of path formulas:
  - *no* boolean combinations of path formulas
  - arguments of temporal operators $\bigcirc$ and U are state formulas

CTL equivalence

CTL equivalence

Let s_1, s_2 be states of a TS $\mathcal{T}$ without terminal states

CTL equivalence

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s_1, s_2 are **CTL** equivalent if for all **CTL** formulas Φ:

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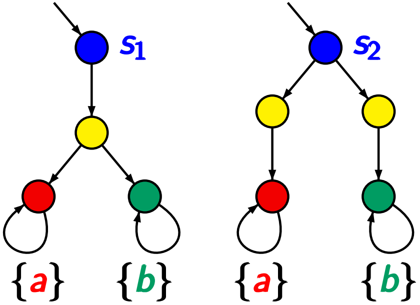

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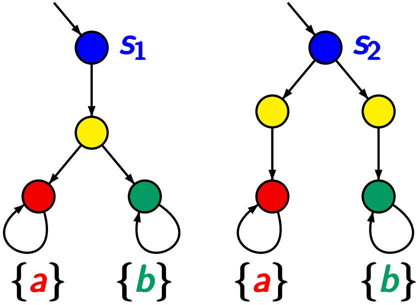

s_1, s_2 are not **CTL** equivalent

$s_1 \models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$

$s_2 \not\models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$

CTL, CTL* and LTL equivalence

Let s_1, s_2 be states of a TS $\mathcal{T}$ without terminal states

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analogous definition for **CTL*** and **LTL**

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s_1, s_2 are **CTL*** equivalent if for all **CTL*** formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

s_1, s_2 are **LTL** equivalent if for all **LTL** formulas φ:

$$s_1 \models \varphi \quad \text{iff} \quad s_2 \models \varphi$$

CTL/CTL* and bisimulation

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bisimulation equivalence
= **CTL** equivalence
= **CTL*** equivalence

CTL/CTL* and bisimulation

	bisimulation equivalence
=	**CTL** equivalence
=	**CTL*** equivalence

$\longleftarrow$ for finite TS

	bisimulation equivalence	
=	CTL equivalence	
=	CTL* equivalence	

$\longleftarrow$ for finite TS

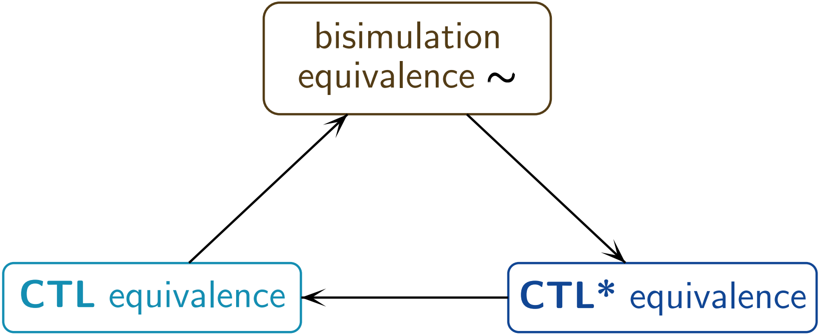
Let $\mathcal{T}$ be a finite TS without terminal states, and s_1, s_2 states in $\mathcal{T}$. Then:

$$s_1 \sim_{\mathcal{T}} s_2$$

iff s_1 and s_2 are CTL equivalent

iff s_1 and s_2 are CTL* equivalent

CTL/CTL* and bisimulation



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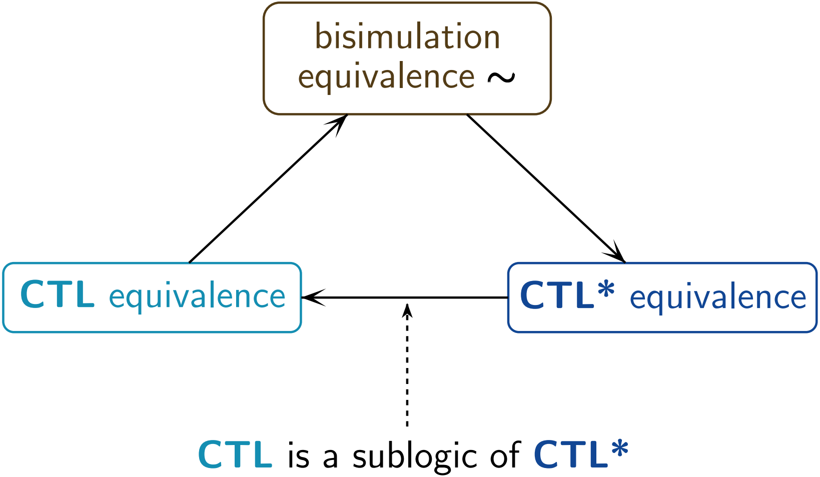

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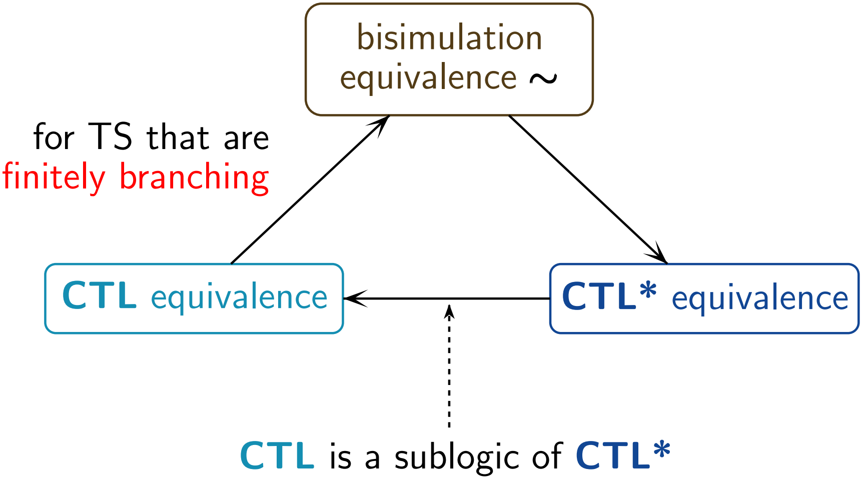
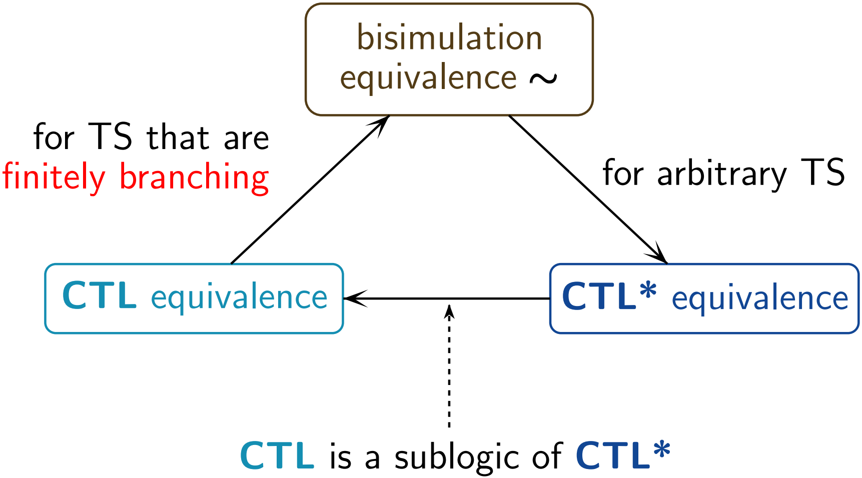


CTL/CTL* and bisimulation



Bisimulation equivalence ⇒ CTL* equivalence

For arbitrary (possibly infinite) transition systems without terminal states:

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For arbitrary (possibly infinite) transition systems without terminal states:

If s_1, s_2 are states with $s_1 \sim_{\mathcal{T}} s_2$ then for all **CTL*** formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

show by structural induction on **CTL*** formulas:

(a) if s_1, s_2 are states with $s_1 \sim_{\mathcal{T}} s_2$ then for all **CTL*** state formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

(b) if π_1, π_2 are paths with $\pi_1 \sim_{\mathcal{T}} \pi_2$ then for all **CTL*** path formulas φ:

$$\pi_1 \models \varphi \quad \text{iff} \quad \pi_2 \models \varphi$$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

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$$\pi_1 \models \varphi \quad \text{iff} \quad \pi_2 \models \varphi$$

$\pi_1 \sim_{\mathcal{T}} \pi_2 \overset{\text{def}}{\iff}$ π_1 and π_2 are statewise bisimulation equivalent

Bisimulation equivalence ⇒ CTL* equivalence

statewise bisimulation equivalent paths:

$$
\begin{array}{ccc}
s_1 & \sim_{\mathcal{T}} & s_2 \\
\downarrow & & \downarrow \\
s_{11} & \sim_{\mathcal{T}} & s_{12} \\
\downarrow & & \downarrow \\
s_{21} & \sim_{\mathcal{T}} & s_{22} \\
\downarrow & & \downarrow \\
s_{31} & \sim_{\mathcal{T}} & s_{32} \\
\downarrow & & \downarrow \\
\ldots & & \ldots \\
\uparrow & & \uparrow
\end{array}
$$

path π_1 path π_2

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

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Proof by structural induction

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Proof by structural induction

base of induction:

(a) $\Phi = \mathit{true}$ or $\Phi = a \in AP$

(b) $\varphi = \Phi$ for some state formula Φ
s.t. statement (a) holds for Φ

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Proof by structural induction

step of induction:

(a) consider $\Phi = \Phi_1 \wedge \Phi_2$, $\neg\Psi$ or $\exists\varphi$ s.t.

(a) holds for Φ_1, Φ_2, Ψ

(b) holds for φ

(b) consider $\varphi = \varphi_1 \wedge \varphi_2$, $\neg\varphi'$, $\bigcirc\varphi'$, $\varphi_1 \mathsf{U} \varphi_2$ s.t.

(b) holds for $\varphi_1, \varphi_2, \varphi'$

Path lifting for $\sim_{\mathcal{T}}$

$$
\begin{array}{ccc}
s_1 & \sim_{\mathcal{T}} & s_2 \\
\downarrow & & \\
s_{11} & & \\
\downarrow & & \\
s_{21} & & \\
\downarrow & & \\
s_{31} & & \\
\downarrow & & \\
\vdots & &
\end{array}
\quad \text{can be completed to} \quad
\begin{array}{ccc}
s_1 & \sim_{\mathcal{T}} & s_2 \\
\downarrow & & \downarrow \\
s_{11} & \sim_{\mathcal{T}} & s_{12} \\
\downarrow & & \downarrow \\
s_{21} & \sim_{\mathcal{T}} & s_{22} \\
\downarrow & & \downarrow \\
s_{31} & \sim_{\mathcal{T}} & s_{32} \\
\downarrow & & \downarrow \\
\vdots & & \vdots
\end{array}
$$

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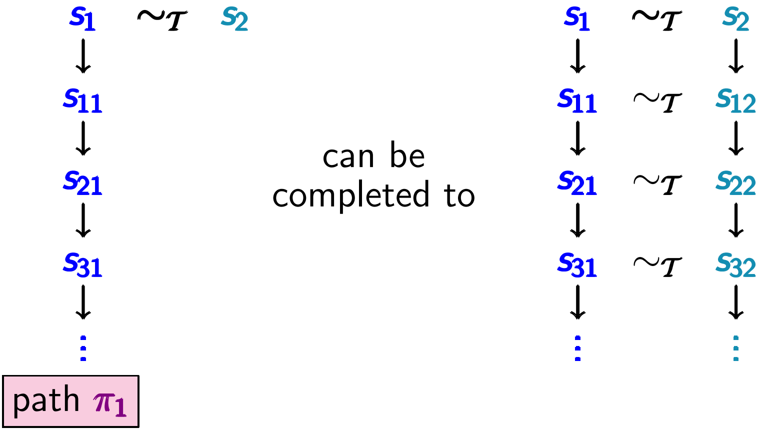
If $s_1 \sim_{\mathcal{T}} s_2$ then for all $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ with $\pi_1 \sim_{\mathcal{T}} \pi_2$

Path lifting for $\sim_{\mathcal{T}}$



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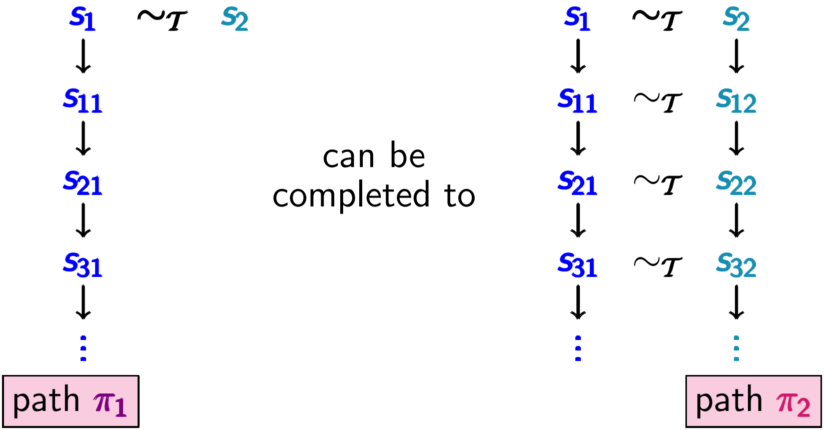
Path lifting for $\sim_{\mathcal{T}}$



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Correct or wrong?

If s_1, s_2 are <u>not</u> **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

Correct or wrong?

If s_1, s_2 are not **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

correct.

Correct or wrong?

If s_1, s_2 are not **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

correct.

If s_1, s_2 not **CTL** equivalent then there exists a **CTL** formula Φ with

$$s_1 \models \Phi \wedge s_2 \not\models \Phi$$

or $$s_1 \not\models \Phi \wedge s_2 \models \Phi$$

Correct or wrong?

If s_1, s_2 are <u>not</u> **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

correct.

If s_1, s_2 not **CTL** equivalent then there exists a **CTL** formula Φ with

$$s_1 \models \Phi \wedge s_2 \not\models \Phi$$

$$\text{or} \quad s_1 \not\models \Phi \wedge s_2 \models \Phi \quad \Longrightarrow \quad s_1 \models \neg\Phi \wedge s_2 \not\models \neg\Phi$$

If s_1, s_2 are <u>not</u> **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

correct.

If s_1, s_2 are <u>not</u> **LTL** equivalent then there exists a **LTL** formula φ with $s_1 \models \varphi$ and $s_2 \not\models \varphi$

If s_1, s_2 are not **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

correct.

If s_1, s_2 are not **LTL** equivalent then there exists a **LTL** formula φ with $s_1 \models \varphi$ and $s_2 \not\models \varphi$

wrong.

Correct or wrong?

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correct.

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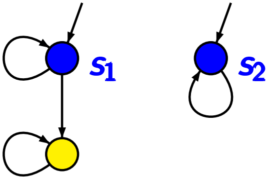
wrong.



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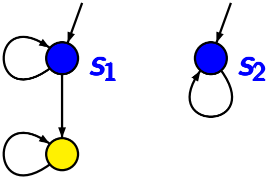
wrong.

$Traces(s_2) \subset Traces(s_1)$



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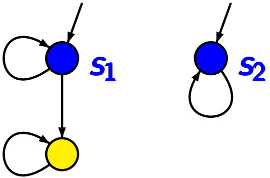
wrong.

$Traces(s_2) \subset Traces(s_1)$

hence: $s_1 \models \varphi$ implies $s_2 \models \varphi$



CTL equivalence $\Longrightarrow$ bisimulation equivalence

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If $\mathcal{T}$ is a finite TS then, for all states s_1, s_2 in $\mathcal{T}$:

if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

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$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_2) : s_1,\ s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation

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Proof: show that

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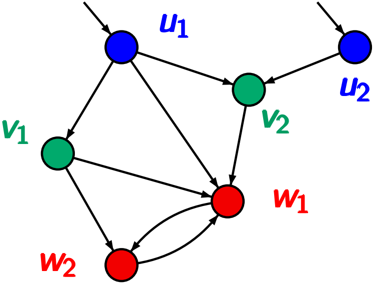
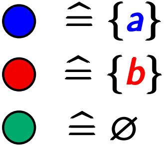
is a bisimulation, i.e., for all $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

(2) if $s_1 \longrightarrow t_1$ then there exists a transition $s_2 \longrightarrow t_2$ s.t. $(t_1, t_2) \in \mathcal{R}$

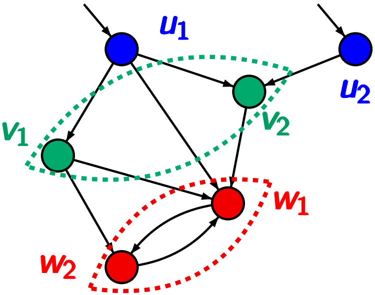


u_1
u_2
v_1
v_2
w_1
w_2

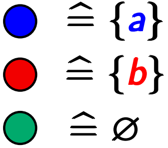


$\widehat{=} \{a\}$
$\widehat{=} \{b\}$
$\widehat{=} \varnothing$

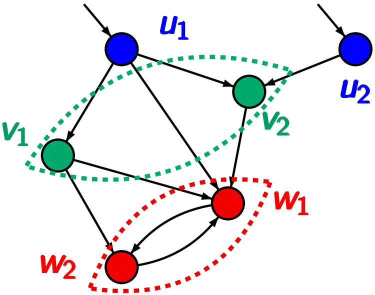
Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$



Example: CTL master formulas



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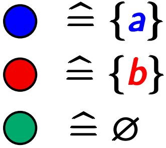
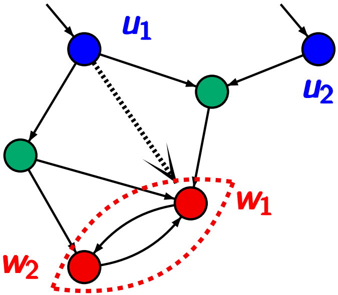
but $u_1 \not\sim_{\mathcal{T}} u_2$



Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
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but $u_1 \not\sim_{\mathcal{T}} u_2$

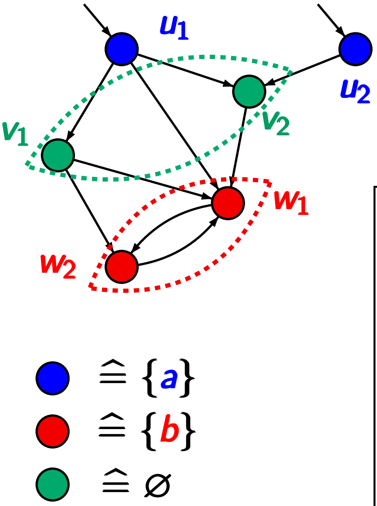
as $u_1 \longrightarrow \{w_1, w_2\}$

$u_2 \not\longrightarrow \{w_1, w_2\}$

$\widehat{=} \{a\}$

$\widehat{=} \{b\}$

$\widehat{=} \varnothing$



bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

CTL master formulas:

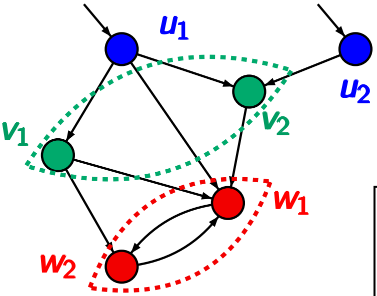
$w_1, w_2 \models$?

$v_1, v_2 \models$?

$u_1 \models$?

$u_2 \models$?

Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
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$\bullet \;\widehat{=}\; \{a\}$
$\bullet \;\widehat{=}\; \{b\}$
$\bullet \;\widehat{=}\; \varnothing$

CTL master formulas:

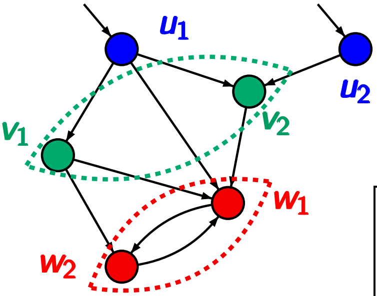
$w_1, w_2 \models b$

$v_1, v_2 \models$?

$u_1 \models$?

$u_2 \models$?

Example: CTL master formulas



● $\widehat{=} \{a\}$ (blue)
● $\widehat{=} \{b\}$ (red)
● $\widehat{=} \varnothing$ (green)

bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

CTL master formulas:

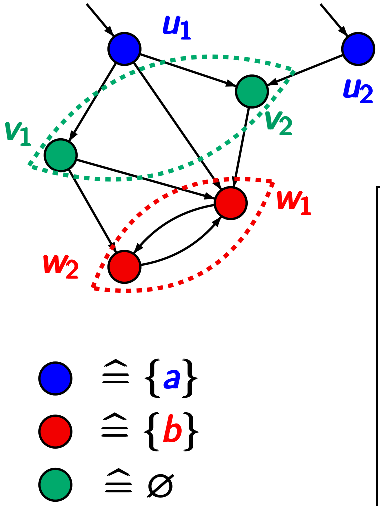
$w_1, w_2 \models b$

$v_1, v_2 \models \neg a \wedge \neg b$

$u_1 \models$?

$u_2 \models$?

Example: CTL master formulas



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CTL master formulas:

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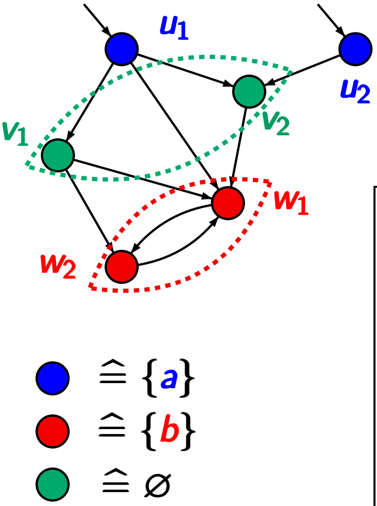
$u_1 \models (\exists \bigcirc b) \wedge a$

$u_2 \models$?

Example: CTL master formulas



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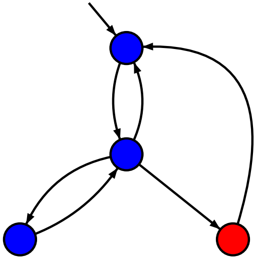
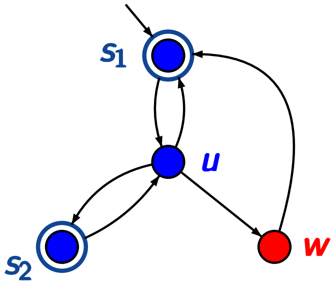
...master formulas for $\sim_{\mathcal{T}}$-classes?



$AP = \{blue, red\}$

...master formulas for $\sim_{\mathcal{T}}$-classes?



$AP = \{blue, red\}$

$s_1 \sim_{\mathcal{T}} s_2 \not\sim_{\mathcal{T}} u$

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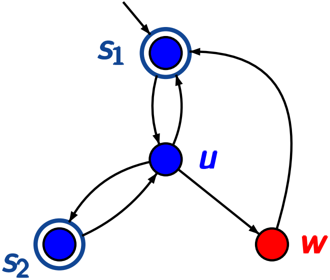


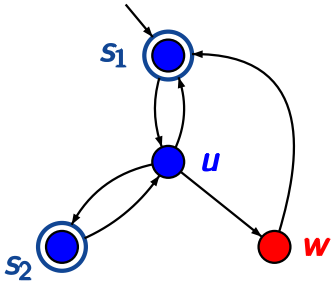
$AP = \{blue, red\}$

$s_1 \sim_{\mathcal{T}} s_2 \not\sim_{\mathcal{T}} u$

$\Phi_w = ?$

$\Phi_C = ?$ where $C = \{s_1, s_2\}$

$\Phi_u = ?$



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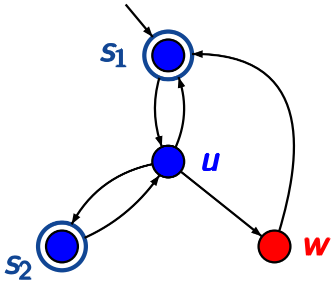

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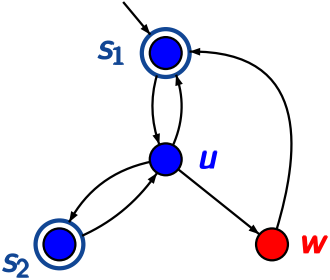

$AP = \{blue, red\}$

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$$\Phi_u = \exists \bigcirc red$$

CTL equivalence $\Longrightarrow$ bisimulation equivalence

If $\mathcal{T}$ is a finite TS then, for all states s_1, s_2 in $\mathcal{T}$:

if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

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- wrong for infinite TS

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- wrong for infinite TS
- but also holds for finitely branching TS

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- wrong for infinite TS
- but also holds for finitely branching TS

↑

possibly infinite-state TS such that

- ∗ the number of initial states is finite
- ∗ for each state the number of successors is finite

CTL equivalence $\Longrightarrow$ bisimulation equivalence

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be finitely branching.

CTL equivalence $\Longrightarrow$ bisimulation equivalence

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be finitely branching.

- S_0 is finite
- $Post(s)$ is finite for all $s \in S$

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Proof: as for finite TS.

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Proof: as for finite TS. Amounts showing that

$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation.

CTL equivalence $\Longrightarrow$ bisimulation equivalence

If $\mathcal{T}$ is a finitely branching TS then for all states s_1, s_2:

if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

Proof: show that

$$\mathcal{R} \overset{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation, i.e., for $(s_1, s_2) \in \mathcal{R}$:

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Summary: CTL/CTL* and bisimulation

CTLEQ5.2-2-SUM

Summary: CTL/CTL* and bisimulation

Let $\mathcal{T}$ be a finite TS without terminal states, and s_1, s_2 states in $\mathcal{T}$. Then:

	$s_1 \sim_{\mathcal{T}} s_2$
iff	s_1 and s_2 are **CTL** equivalent
iff	s_1 and s_2 are **CTL*** equivalent

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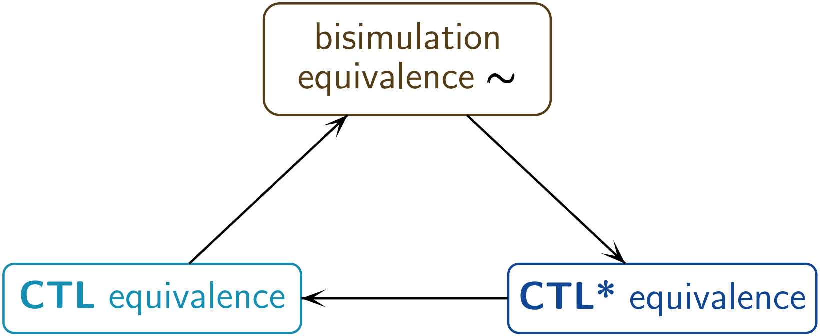
$s_1 \sim_{\mathcal{T}} s_2$

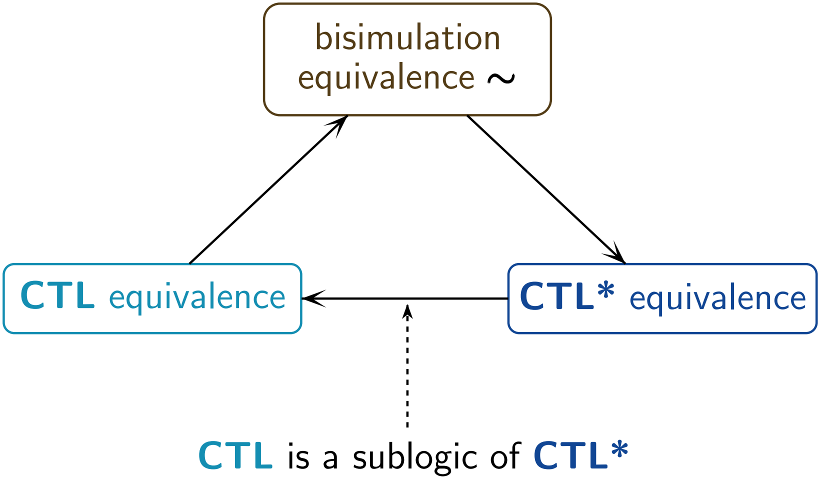
iff s_1 and s_2 are **CTL** equivalent

iff s_1 and s_2 are **CTL*** equivalent

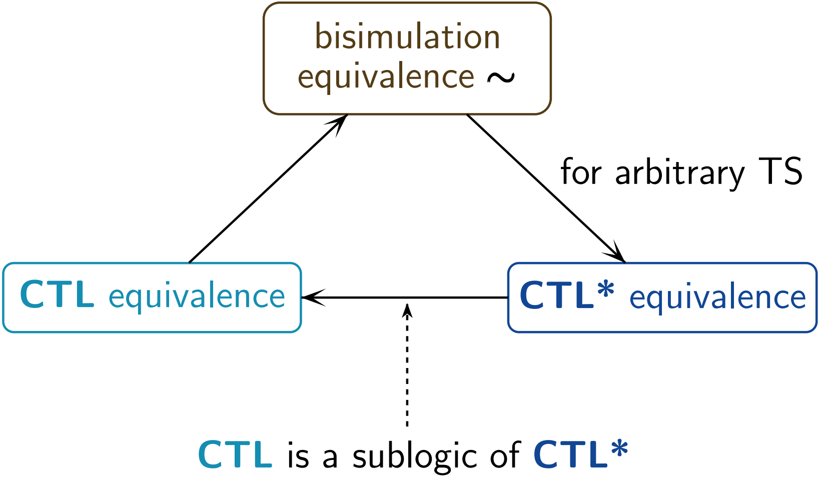
Summary: CTL/CTL* and bisimulation

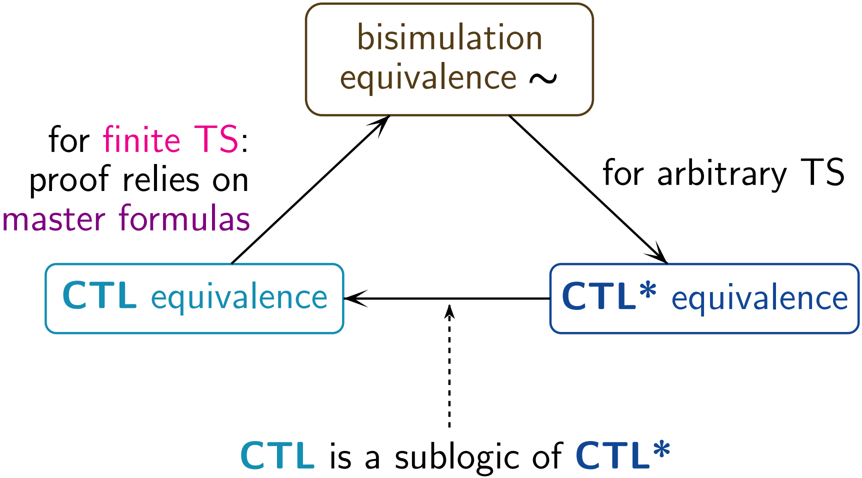


Summary: CTL/CTL* and bisimulation

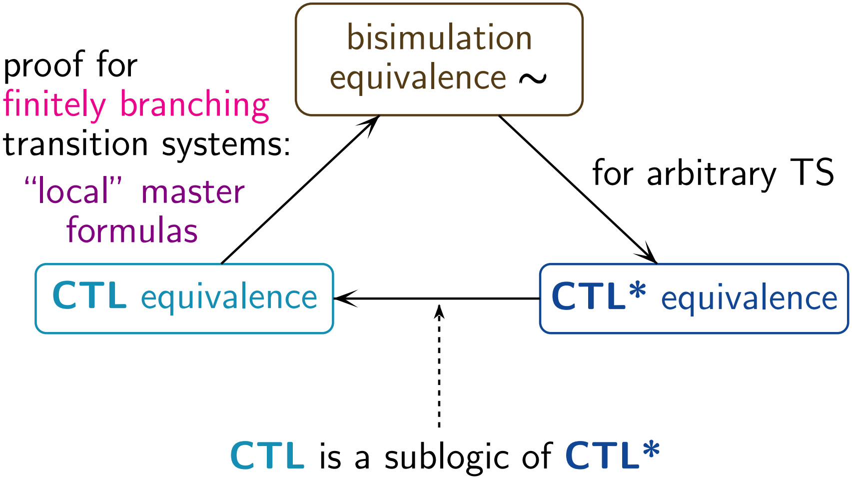


Summary: CTL/CTL* and bisimulation




bisimulation
equivalence ∼
for finite TS:
proof relies on
master formulas
for arbitrary TS
CTL equivalence
CTL* equivalence
CTL is a sublogic of CTL*

Summary: CTL/CTL* and bisimulation



CTL/CTL* and bisimulation for TS

CTL/CTL* and bisimulation for TS

so far: we considered

- **CTL**/**CTL*** equivalence
- bisimulation equivalence $\sim_{\mathcal{T}}$

for the states of a single transition system $\mathcal{T}$

CTL/CTL* and bisimulation for TS

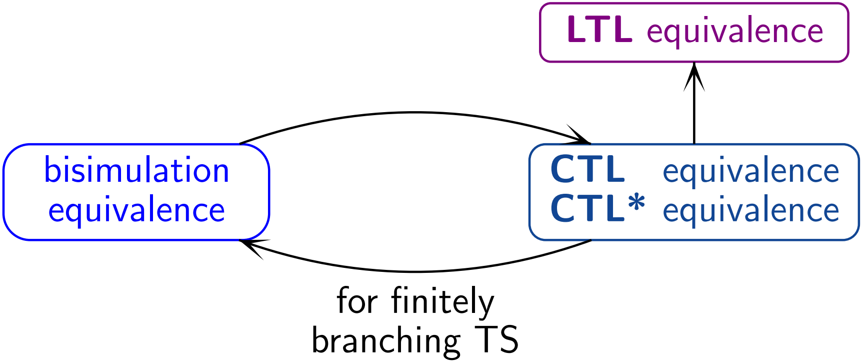
If $\mathcal{T}_1$, $\mathcal{T}_2$ are finitely branching TS over AP without terminal states then:

$\mathcal{T}_1 \sim \mathcal{T}_2$

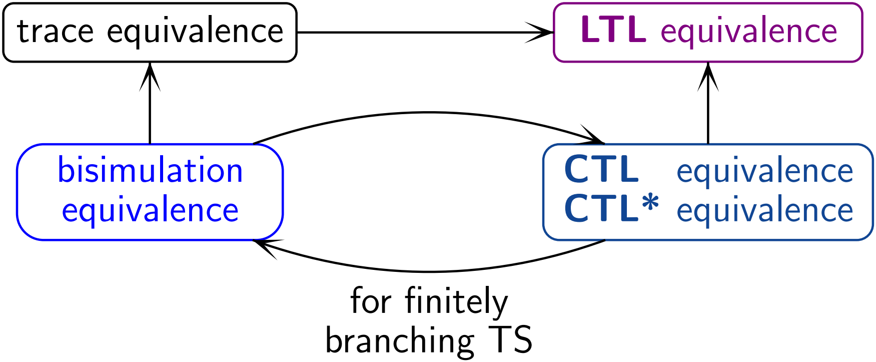
iff $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL** formulas

iff $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL*** formulas

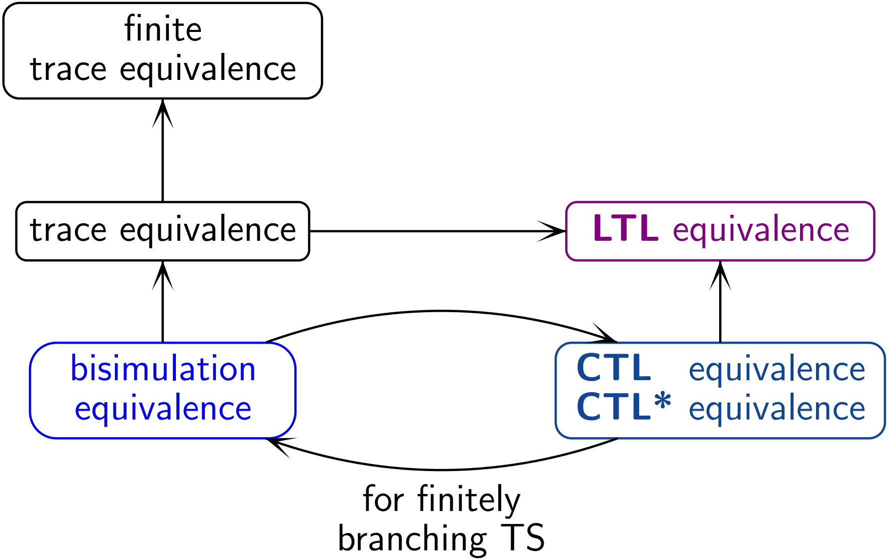
Summary: equivalences



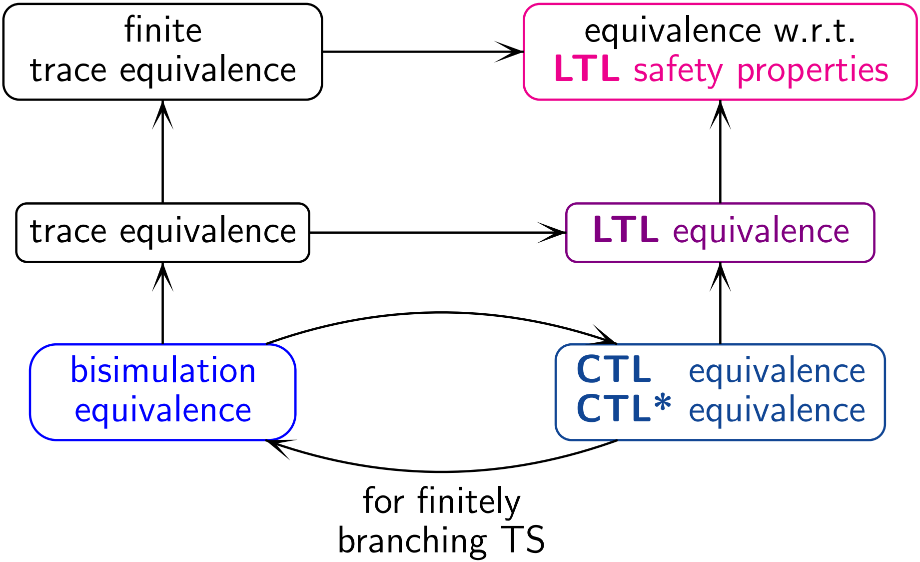
Summary: equivalences



Summary: equivalences



Summary: equivalences



Correct or wrong?

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $\mathrm{CTL}_{\setminus \mathrm{U}}$ formulas then $s_1 \sim_{\mathcal{T}} s_2$.

Correct or wrong?

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $\mathrm{CTL}_{\setminus \mathsf{U}}$ formulas then
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where $\mathrm{CTL}_{\setminus \mathsf{U}} \mathrel{\widehat{=}}$ CTL without until operator $\mathbf{U}$

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where $\text{CTL}_{\setminus \mathbf{U}} \mathrel{\widehat{=}} \text{CTL}$ without until operator $\mathbf{U}$

correct.

Correct or wrong?

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $\mathrm{CTL}_{\setminus \mathbf{U}}$ formulas then
> $s_1 \sim_{\mathcal{T}} s_2$.

where $\mathrm{CTL}_{\setminus \mathbf{U}} \mathrel{\widehat{=}} \mathrm{CTL}$ without until operator $\mathbf{U}$

correct. see the proof

"**CTL** equivalence $\Longrightarrow$ bisimulation equivalence"

$\mathrm{CTL}_{\setminus U}$-equivalence $\Rightarrow$ bisimulation equivalence

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $\mathrm{CTL}_{\setminus U}$ formulas then
> $$s_1 \sim_{\mathcal{T}} s_2.$$

Proof. Show that $\mathrm{CTL}_{\setminus U}$ equivalence is a bisimulation

$\text{CTL}_{\setminus U}$-equivalence $\Rightarrow$ bisimulation equivalence

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

> If s_1, s_2 satisfy the same $\text{CTL}_{\setminus U}$ formulas then
>
> $s_1 \sim_{\mathcal{T}} s_2$.

Proof. Show that $\text{CTL}_{\setminus U}$ equivalence is a bisimulation

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$$\text{and} \quad Sat(\Phi_{C,D}) \subseteq C \setminus D$$

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> $\mathcal{T}$ and its bisimulation quotient $\mathcal{T}/\!\sim$ satisfy the same **CTL*** formulas.

Correct or wrong?

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correct. Recall that $\mathcal{T} \sim \mathcal{T}/\!\sim$ as

$$\mathcal{R} = \{(s, [s]) : s \in S\}$$

is a bisimulation for $(\mathcal{T}, \mathcal{T}/\!\sim)$

here: $[s] = \sim_{\mathcal{T}}$-equivalence class of state s