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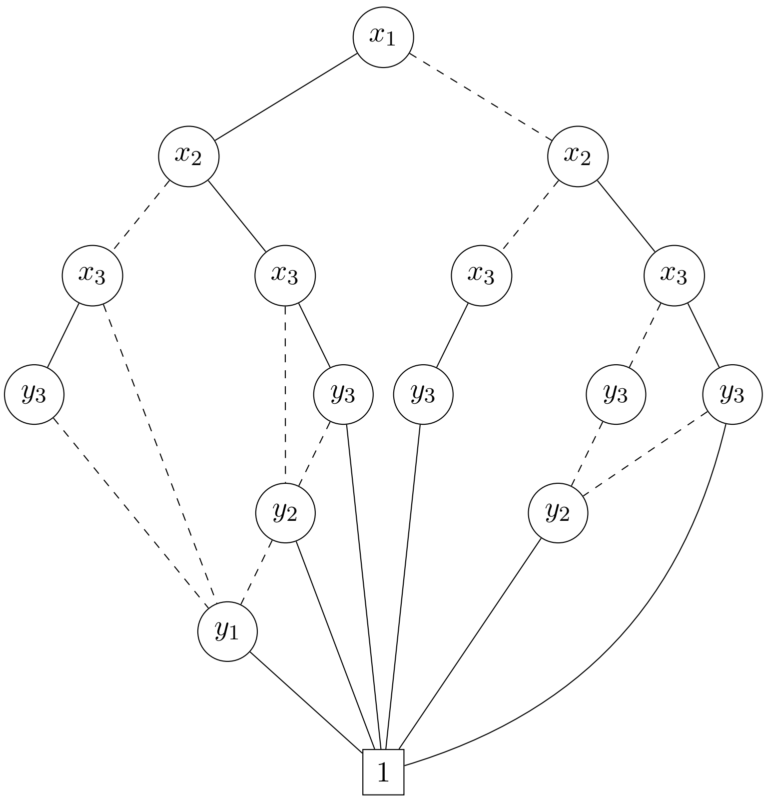
Advanced Model Checking
Summer term 2014

– Series 9 –

Hand in on **25 June** before the exercise class.

Exercise 1 (2 points)

Given the following ROBDD, determine the boolean function $f(x_1, x_2, x_3, y_1, y_2, y_3)$ that it represents.



Exercise 2 (3 points)

Consider the family of functions (for $k > 0$) given by

$$f_k(x_0, \ldots, x_{n-1}, a_0, \ldots, a_{k-1}) = x_{|a|}$$

where $n = 2^k$, $x_i \in \{0,1\}$ for $0 \leq i \leq n$ and $a_j \in \{0,1\}$ for $0 \leq j \leq k$ and $|a| = \sum_{j=0}^{k-1} a_j 2^j$. In other words, the function returns the $|a|$-th bit of $(x_1 \ldots x_n)$.

Provide the ROBDDs that represent f_3 for the following two variable orderings:

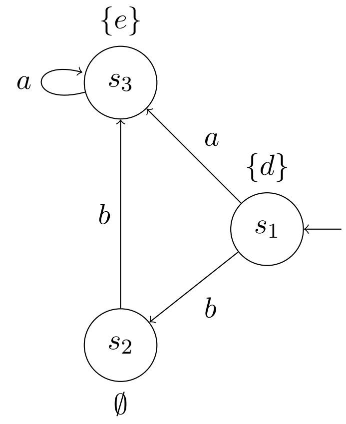
(i) $a_0 < \ldots < a_{k-1} < x_0 < \ldots < x_{n-1}$,

(ii) $a_0 < x_0 < a_1 < x_1 \ldots < a_{k-1} < x_{k-1} < x_k < \ldots < x_{n-1}$.

Hint: it can be useful to find a better variable ordering first.

Exercise 3 (1 points)

Consider the following transition system.



(i) Define the switching functions that represent the transition system (as defined in the lecture/book).

(ii) Encode the switching function for the transition relation using an ROBDD.

Exercise 4 (4 points)

Let $\overline{x}$, $\overline{x}'$ and $\overline{b}$ be three vectors of Boolean variables of size $n > 0$ and let a_i denote the i-th variable in a vector $\overline{a}$. Assume that the transition relation of a transition system TS with state space $S = Eval(\overline{x})$ is given by means of a switching function $\Delta : Eval(\overline{x}, \overline{x}') \longrightarrow \{0, 1\}$ (as seen in the lecture). Let $Q = \{B_0, \ldots, B_{2^n-1}\}$ be a partition of the state space ($S = \bigcup_{i=0}^{2^n-1} B_i$, $B_i \cap B_j = \emptyset$ for $i \neq j$ and $B_i = \emptyset$ is possible) represented as a switching function $f_Q\colon Eval(\overline{x}, \overline{b}) \longrightarrow \{0, 1\}$ defined by

$$f_Q(s, B_i) = 1 \iff s \in B_i.$$

(i) Using the available switching functions and the usual operations on them, define another switching function $f_{\text{sig}^Q}\colon Eval(\overline{x}, \overline{k})$ given by

$$f_{\text{sig}^Q}(s, B) = 1 \iff \exists s' \in Post(s) \; . \; s' \in B.$$

(ii) Let the characteristic functions $\chi_{Sat(a)}$ and $\chi_{Sat(b)}$ for the only two atomic propositions $a, b \in AP$ in TS be given. Explain how the coarsest partition that respects the labeling can be computed in terms of a switching function. In other words, how can a function $f_{Q^*}\colon Eval(\overline{x}, \overline{b}) \longrightarrow \{0, 1\}$ that represents the partition $Q^* = \{\{s \mid L(s) = A\} \mid A \in 2^{AP}\}$ in the way described earlier, be derived? You may use the usual operations on switching functions as well as the "special" switching functions $i|_{\overline{b}}\colon Eval(\overline{b}) \longrightarrow \{0, 1\}$ that evaluate to 1 if and only if the input is the binary encoding of the number i.

(iii) Conceptually explain how the functions $f_Q(s, B_i)$ and $f_{\text{sig}^Q}(s, B_i)$ can be used to compute the coarsest bisimulation equivalence on TS.

(iv) Assuming that all switching functions are stored as ROBDDs using the variable ordering $x_1 < x_1' < x_2 < \ldots < x_n < x_n' < b_1 < \ldots < b_n$, how can the proposed method of (iii) be implemented cleverly? *Hint:* How can the reducedness of the ROBDD(s) be exploited?