RHEINISCH-WESTFÄLISCHE TECHNISCHE HOCHSCHULE AACHEN

LEHRSTUHL FÜR INFORMATIK 2
RWTH Aachen · D-52056 Aachen
http://moves.rwth-aachen.de/

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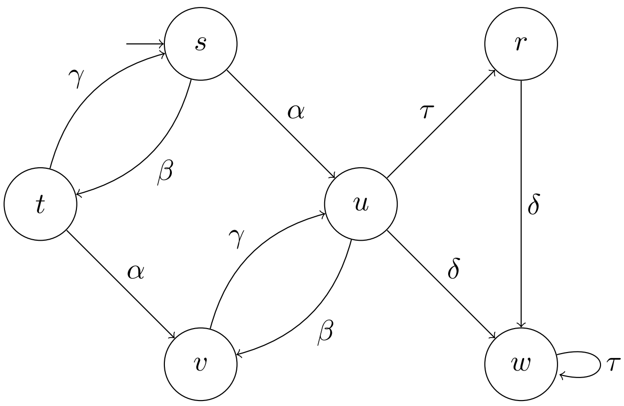
Advanced Model Checking
Summer term 2014

– Series 7 –

Hand in on 4 June before the exercise class.

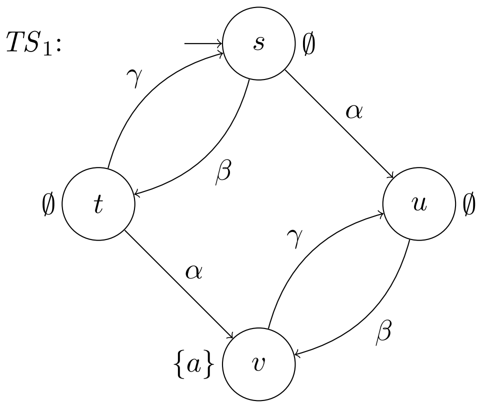
Exercise 1 **(2 points)**

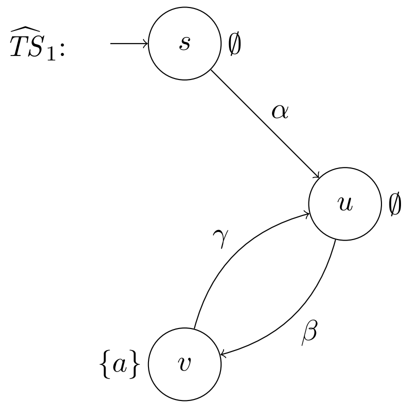
Consider the following transitions system TS with the action set $Act = \{\alpha, \beta, \gamma, \delta, \tau\}$ in which all states are equally labeled. Determine for each pair of actions whether they are independent.



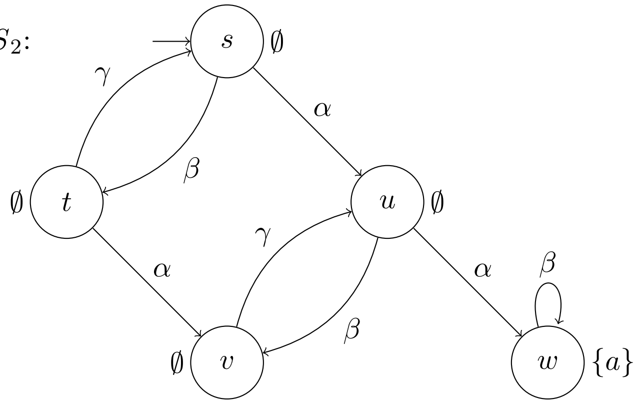
Exercise 2 **(3 points)**

Consider the following four pairs $(TS_i, \widehat{TS}_i)$ of transition systems where $\widehat{TS}_i$ results from reducing TS_i using the appropriate ample sets. Check for each pair $(TS_i, \widehat{TS}_i)$ whether the two transitions systems are stutter trace equivalent and indicate **all** of the ample set conditions (A1)-(A4) that are violated.

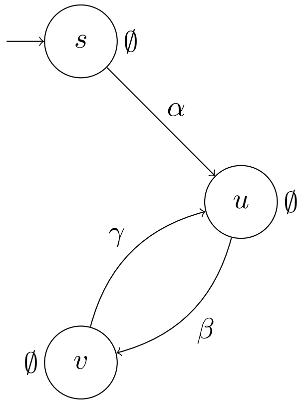




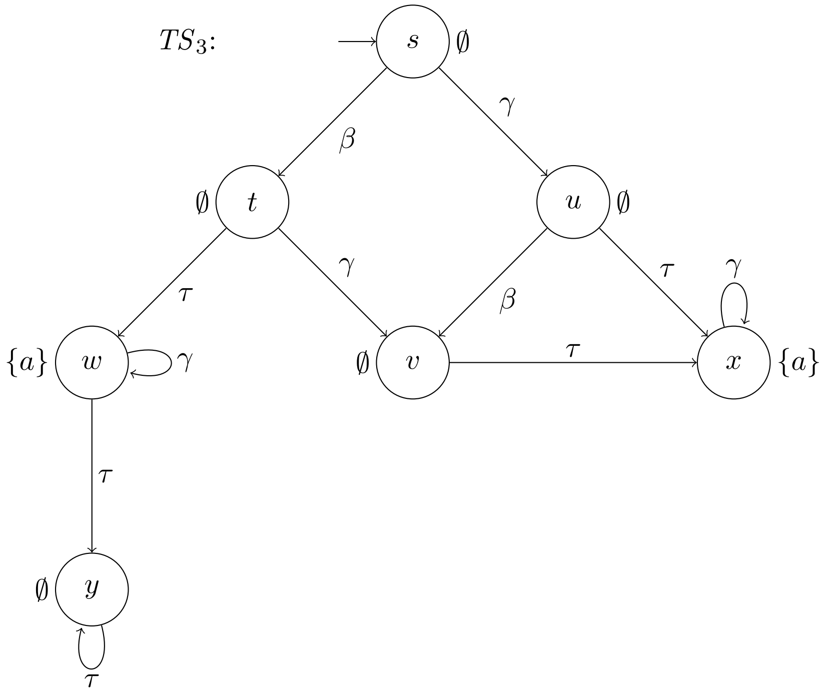
TS_2:



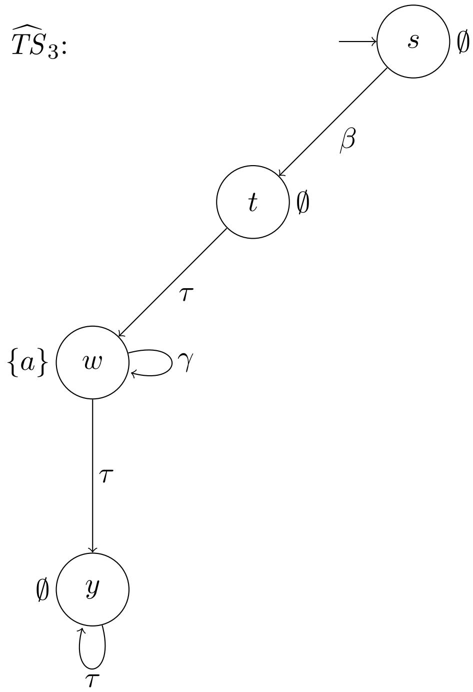
$\widehat{TS}_2$:



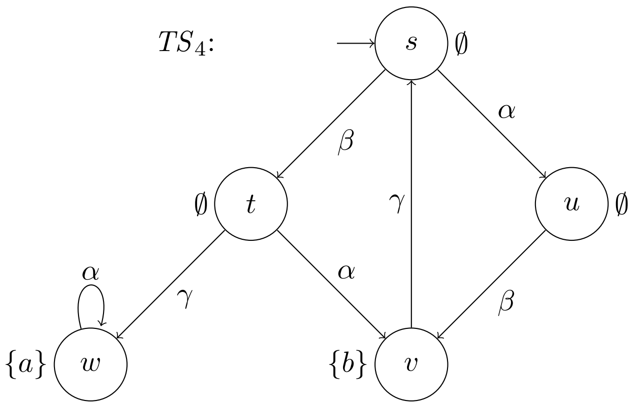
TS_3:



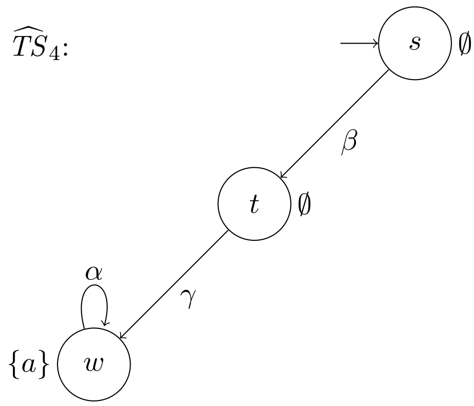
$\widehat{TS}_3$:



TS_4:



$\widehat{TS}_4$:



Exercise 3 **(2 points)**

For $1 \leq i \leq n$, let $TS_i = (S_i, Act_i, \rightarrow_i, I_i, AP, L_i)$ be an action-deterministic transition system such that $Act_i \cap Act_j \cap Act_k = \emptyset$ for $1 \leq i < j < k \leq n$. Consider the parallel composition with synchronization over common actions, i.e., the transition system

$$TS = TS_1 \parallel TS_2 \parallel ... \parallel TS_n.$$

For each state $s = \langle s_1, ..., s_n \rangle$ of TS, let $Act_i(s) = Act_i \cap Act(s)$ be the set of actions of TS_i that are enabled in s. Show that the dependency condition (A2) holds for all sets $ample(\cdot)$ if for each state s of TS the following two conditions hold:

(i) If $ample(s) \neq Act(s)$, then $ample(s) = Act_i(s)$ for some $i \in \{1, ..., n\}$.

(ii) If $ample(s) = Act_i(s) \neq Act(s)$ for some $i \in \{1, \ldots, n\}$, then $ample(s) \cap (\bigcup_{\substack{1 \leq j \leq n \\ j \neq i}} Act_j) = \emptyset$.

Exercise 4 **(3 points)**

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be an action-deterministic transition system and let $\mathcal{I}_{st}$ be the set of all pairs $(\alpha, \beta) \in Act \times Act$ of independent actions α and β where α or β (or both) is (are) a stutter action. Let *stutter permutation equivalence* $\cong_{\text{perm}}$ be the finest equivalence on Act^* such that

$$\overline{\gamma}\alpha\beta\overline{\delta} \cong_{\text{perm}} \overline{\gamma}\beta\alpha\overline{\delta} \quad \text{if} \quad \overline{\gamma}, \overline{\delta} \in Act^* \text{ and } (\alpha, \beta) \in \mathcal{I}_{\text{st}}.$$

The extension of $\cong_{perm}$ to an equivalence for infinite action sequences is defined as follows. If $\widetilde{\alpha} = \alpha_1\alpha_2\alpha_3\ldots$ and $\widetilde{\beta} = \beta_1\beta_2\beta_3\ldots$ are actions sequences in Act^ω, then $\widetilde{\alpha} \sqsubseteq_{\text{perm}} \widetilde{\beta}$ if for all finite prefixes $\alpha_1 \ldots \alpha_n$ of $\widetilde{\alpha}$ there exists a finite prefix $\beta_1 \ldots \beta_m$ of $\widetilde{\beta}$ with $m \geq n$ and a finite word $\overline{\gamma} \in Act^*$ such that

$$\alpha_1 \ldots \alpha_n\overline{\gamma} \cong_{\text{perm}} \beta_1 \ldots \beta_m.$$

We then define the binary relation $\cong^\omega_{\text{perm}}$ on Act^ω by

$$\widetilde{\alpha} \cong^\omega_{\text{perm}} \widetilde{\beta} \quad \text{iff} \quad \widetilde{\alpha} \sqsubseteq_{\text{perm}} \widetilde{\beta} \text{ and } \widetilde{\beta} \sqsubseteq_{\text{perm}} \widetilde{\alpha}.$$

(i) Show that $\cong^\omega_{\text{perm}}$ is an equivalence.