On ∞-Safety of Probabilistic Programs

What is it all about?

Probabilistic programs extend sequential programs with random assignments and conditioning. The interest in probabilistic programs has been rapidly mounting in the last decade due to prominent applications in biology, machine learning, quantum computing, security, to name just a few. À la Michael Hicks's interpretation, the crux of probabilistic programming is to consider normal-looking programs as if they were probability distributions.

Given a probabilistic-program model over the state vector X, a domain set $\mathcal{X}$, an initial set $\mathcal{X}_0 \subset \mathcal{X}$, and an unsafe set $\mathcal{X}_u \subset \mathcal{X}$, the ∞-safety problem asks to bound the failure probability

$$P\left(\exists k \in \mathbb{N} \colon X_k \in \mathcal{X}_u\right),$$

for any initial state X_0 whose support lies within $\mathcal{X}_0$. Accordingly, the K-safety problem, for a fixed $K \in \mathbb{N}$, refers to the problem where one aims to bound the failure probability within finitely many steps up to K, as in the context of bounded model checking.

Our recent work in [FCX+20] approached the problem in a different lens. There we presented a solution to the ∞-safety problem of a continuous-time probabilistic model termed Stochastic Differential Equations (SDEs). Our method computes an exponentially decreasing upper bound, if existent, on the tail probability that an SDE system violates a given safety specification. Such an upper bound facilitates a reduction of the verification problem over the infinite-time horizon to that over a finite one.

The main interest of this thesis topic is to investigate to what extent we can carry the aforementioned results to unbounded verification of probabilistic programs in the absence of conditioning, which can be viewed as discrete-time stochastic dynamics. The key challenge there may lie in the lack of stability or even continuity characterizations of program traces in the discrete-time setting.

What is to be done?

1. A continuity characterization for certain functions over discrete transitions among states (probabilistic distributions) along a program run. That is, we seek for a discrete analogy of the Lie derivative known in the realm of differential geometry.
2. An adaption of Dynkin's formula (stochastic generalization of the Newton-Leibniz axiom) to compute the expected value of an adequately smooth (in the sense of the continuity characterization) function along a program run at a stopping time.
3. Tractable sufficient conditions (in the form of supermartingales) that witness the convergence (or stability) of the tail failure probability.
4. The extension to probabilistic programs over continuous stochastic variables, possibly by combining the quantitative-analysis techniques developed in [San20].

This list is of course non-exhaustive.

What do we expect?

- Solid background in theoretical computer science. Ideally you have taken the lectures Semantics and Verification and Probabilistic Programming.
- Adequate mathematical skills in Algebra, Probability Theory and Convex Optimization. Ideally maths as your minor/elective.
- Passion and endurance for solving difficult (mathematical) problems.

What you can expect?

- Get a chance to work on relevant open problems of theoretical nature.
- Work closely together with us – you can work in our student's room at the chair whenever you like.
- We have a very good coffee machine.

Contact

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References

[FCX+20] Shenghua Feng, Mingshuai Chen, Bai Xue, Sriram Sankaranarayanan, and Naijun Zhan, Unbounded-time safety verification of stochastic differential dynamics, 32nd International Conference on Computer-Aided Verification, CAV 2020, Los Angeles, California, USA, July 19-24, 2020, [Arfifact Evaluated]. Slides. Video-cast.

[San20] Sriram Sankaranarayanan, Quantitative analysis of programs with probabilities and concentration of measure inequalities, Foundations of Probabilistic Programming (Gilles Barthe, Joost-Pieter Katoen, and Alexandra Silva, eds.), Cambridge University Press, 2020, to appear.