

## What's to Come is Still Unsure <sup>\*</sup>

### Synthesizing Controllers Resilient to Delayed Interaction

Mingshuai Chen<sup>1</sup>, Martin Fränzle<sup>2</sup>, Yangjia Li<sup>3,1</sup>, Peter N. Mosaad<sup>2</sup>, Naijun Zhan<sup>1</sup>

✉ chenms@ios.ac.cn   🌐 lcs.ios.ac.cn/~chenms/

<sup>1</sup> State Key Lab. of Computer Science, Institute of Software, Chinese Academy of Sciences, China

<sup>2</sup> Dpt. of Computing Science, Carl v. Ossietzky Universität Oldenburg, Germany

<sup>3</sup> University of Tartu, Estonia

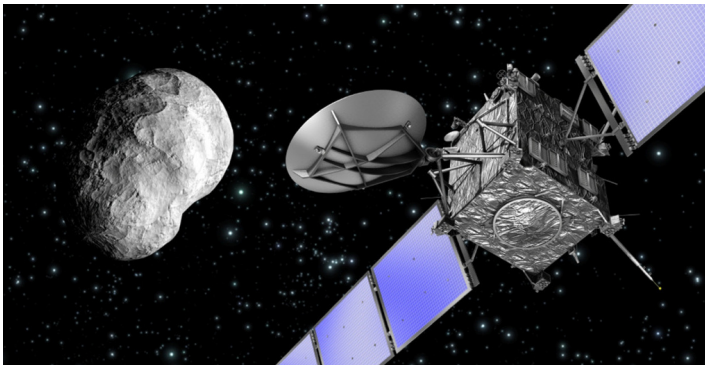
Los Angeles, October 2018

---

\*. William Shakespeare, Twelfth Night/What You Will, Act 2, Scene 3.

# Staying Safe

When Observation & Actuation Suffer from Serious Delays



©ESA

- You could move slowly. (Well, can you ?)
- You could trust autonomy.
- Or you have to anticipate and issue actions early.

## A Pearl of Wisdom



Indecision and delays are the parents of failure.  
(George Canning)

©izQuotes

## A Pearl of Wisdom



Indecision and delays are the parents of failure.  
(George Canning)

©izQuotes

- Only relevant to ordinary people's life?
- Or to scientists, in particular comp. sci. and control folks, too?

## A Pearl of Wisdom



Indecision and delays are the parents of failure.  
(George Canning)

©izQuotes

- Only relevant to ordinary people's life?
- Or to scientists, in particular comp. sci. and control folks, too?

Remember that Canning briefly controlled Great Britain !

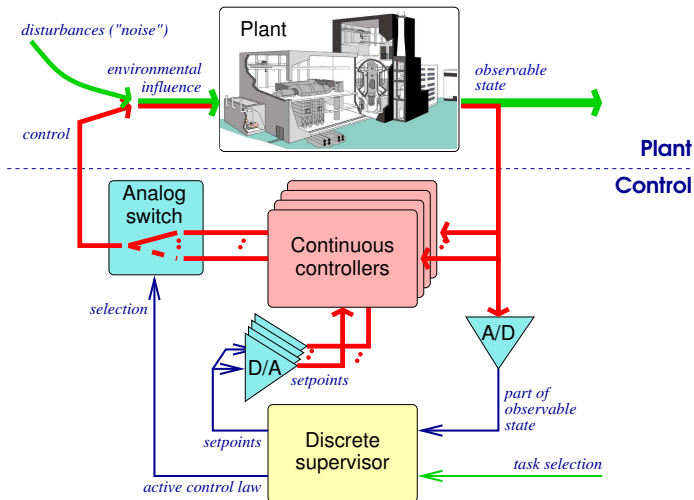
# Outline

- 1 Why Time Delays
- 2 Safety Games under Delay
- 3 Synthesizing Controllers Resilient to Delayed Interaction
- 4 Experimental Evaluation
- 5 Concluding Remarks

# Outline

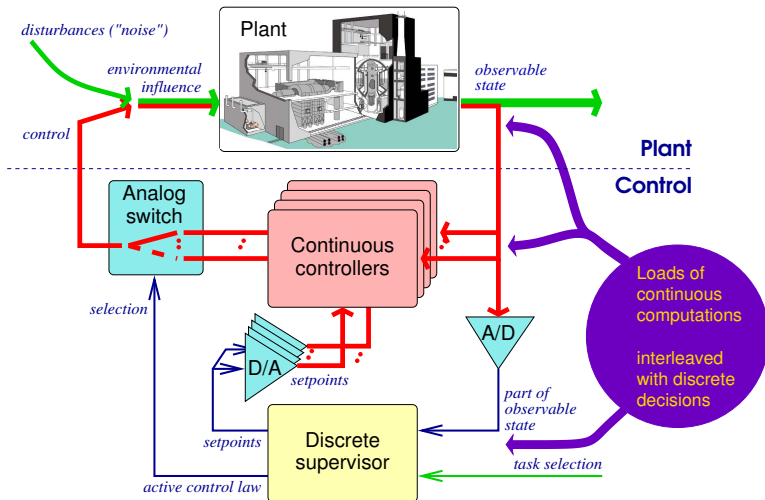
- 1 Why Time Delays
  - Motivation
- 2 Safety Games under Delay
  - Delayed observation and actuation
  - Reducibility to standard safety games
- 3 Synthesizing Controllers Resilient to Delayed Interaction
  - Incremental handling of order-preserving delays
  - Out-of-order delivery
- 4 Experimental Evaluation
  - Performance
- 5 Concluding Remarks
  - Summary

# Hybrid Systems

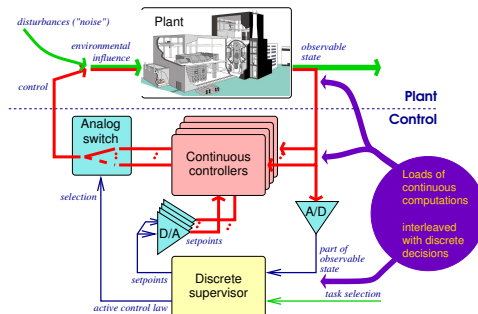




# Hybrid Systems



# Hybrid Systems



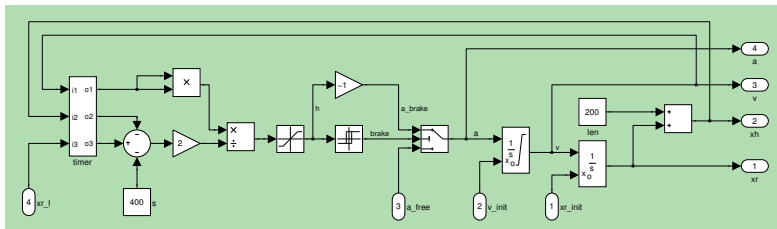
## Crucial question :

- How do the controller and the plant interact ?

## Traditional answer :

- Coupling assumed to be (or at least modeled as) delay-free.
- ⇒ **Mode dynamics** is covered by the **conjunction of the individual ODEs**.
- ⇒ **Switching** btw. modes is an **immediate reaction to environmental conditions**.

## Instantaneous Coupling



©FTCS-3

Following the tradition, above (rather typical) Simulink model assumes

- delay-free coupling between all components,
- instantaneous feed-through within all functional blocks.

### Central questions :

- 1 Is this **realistic**?
- 2 If not, does it have **observable effect on control performance**?
- 3 May that effect be **detrimental or even harmful**?

# Q1 : Is Instantaneous Coupling Realistic?



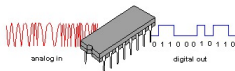
# Q1 : Is Instantaneous Coupling Realistic?



**We are no better :**

As soon as computer scientists enter the scene, serious delays are ahead...

# Q1 : Is Instantaneous Coupling Realistic ?



Digital control needs **A/D and D/A conversion**, which induces latency in signal forwarding.



Digital signal processing, especially in complex sensors like CV, needs **processing time**, adding signal delays.

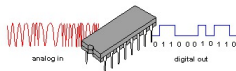


**Networked control** introduces communication latency into the feedback control loop.



Harvesting, fusing, and forwarding data through **sensor networks** enlarge the latter by orders of magnitude.

# Q1 : Is Instantaneous Coupling Realistic? -- No.



Digital control needs **A/D and D/A conversion**, which induces latency in signal forwarding.



Digital signal processing, especially in complex sensors like CV, adds **processing time**, adding signal de-



communication la-  
loop.

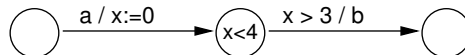


Harvesting, fusing, and forwarding data through **sen-  
sor networks** enlarge the latter by orders of magni-  
tude.

## Q1a : Resultant Forms of Delay

**Delayed reaction :** Reaction to a stimulus is not immediate.

- Easy to model in timed automata, hybrid automata, ... :



- Thus amenable to the pertinent analysis tools.

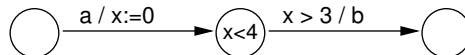
⇒ **Not of interest today.**



# Q1a : Resultant Forms of Delay

**Delayed reaction :** Reaction to a stimulus is not immediate.

- Easy to model in timed automata, hybrid automata, ... :



- Thus amenable to the pertinent analysis tools.

⇒ **Not of interest today.**

**Network delay :** Information of different age coexists and is queuing in the network when piped towards target.

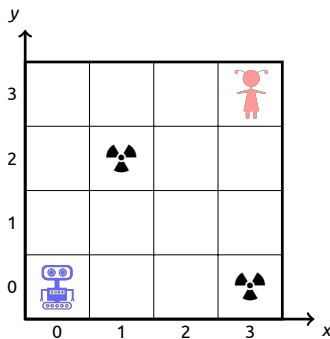
- End-to-end latency may exceed sampling intervals etc. by orders of magnitude
- Not (continuous-time pipelined delay) or not efficiently (discrete-time pipelined delay) expressible in our std. models.

⇒ **Our theme today : discrete-time pipelined delay.**

[M. Chen, M. Fränzle *et al.*. ATVA'18],

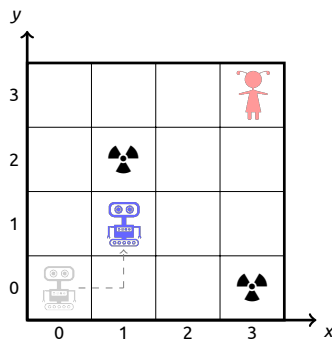
[M. Zimmermann. LICS'18, GandALF'17], [F. Klein & M. Zimmermann. ICALP'15, CSL'15].

## Q2 : Do Delays Have Observable Effect ?



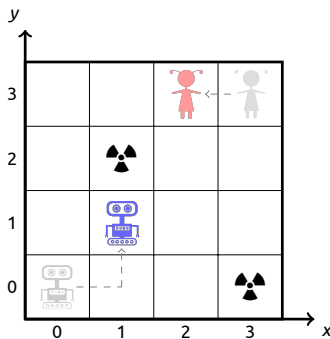
**Figure :** A robot escape game in a  $4 \times 4$  room, with  
 $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  
 $\Sigma_k = \{R, L, U, D\}$ .

## Q2 : Do Delays Have Observable Effect ?



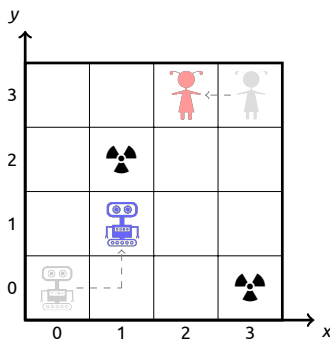
**Figure :** A robot escape game in a  $4 \times 4$  room, with  
 $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  
 $\Sigma_k = \{R, L, U, D\}$ .

## Q2 : Do Delays Have Observable Effect ?



**Figure :** A robot escape game in a  $4 \times 4$  room, with  
 $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  
 $\Sigma_k = \{R, L, U, D\}$ .

## Q2 : Do Delays Have Observable Effect ?



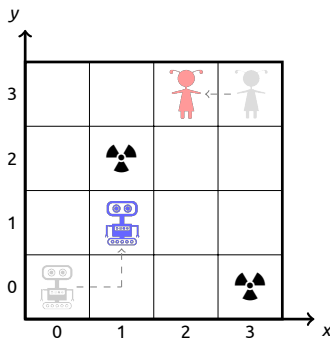
No delay :

Figure : A robot escape game in a  $4 \times 4$  room, with

$$\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\},$$

$$\Sigma_k = \{R, L, U, D\}.$$

## Q2 : Do Delays Have Observable Effect ?

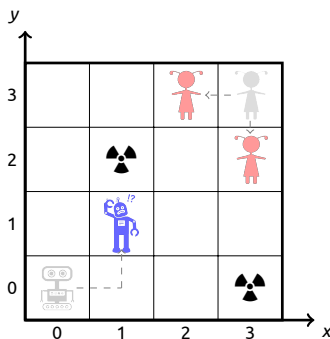


No delay :

Robot always wins by circling around the obstacle at (1,2).

Figure : A robot escape game in a  $4 \times 4$  room, with  $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  $\Sigma_k = \{R, L, U, D\}$ .

## Q2 : Do Delays Have Observable Effect ?



No delay :

Robot always wins by circling around the obstacle at (1,2).

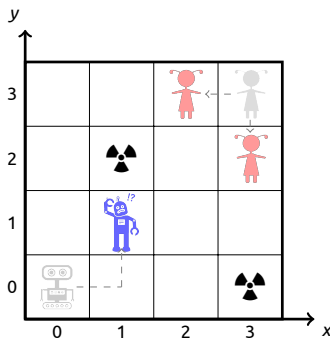
1 step delay :

Figure : A robot escape game in a  $4 \times 4$  room, with

$$\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\},$$

$$\Sigma_k = \{R, L, U, D\}.$$

## Q2 : Do Delays Have Observable Effect ?



No delay :

Robot always wins by circling around the obstacle at (1,2).

1 step delay :

Robot wins by 1-step pre-decision.

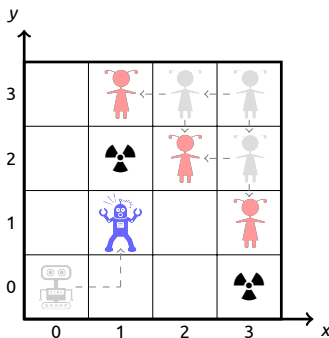
Figure : A robot escape game in a  $4 \times 4$  room, with

$\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,

$\Sigma_k = \{R, L, U, D\}$ .



## Q2 : Do Delays Have Observable Effect ?



No delay :

Robot always wins by circling around the obstacle at (1,2).

1 step delay :

Robot wins by 1-step pre-decision.

2 steps delay :

Figure : A robot escape game in a  $4 \times 4$  room, with  $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  $\Sigma_k = \{R, L, U, D\}$ .

## Q2 : Do Delays Have Observable Effect ?

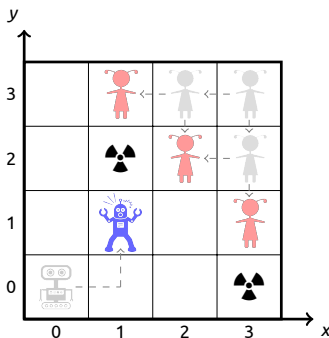


Figure : A robot escape game in a  $4 \times 4$  room, with  
 $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  
 $\Sigma_k = \{R, L, U, D\}$ .

### No delay :

Robot always wins by circling around the obstacle at (1,2).

### 1 step delay :

Robot wins by 1-step pre-decision.

### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

## Q2 : Do Delays Have Observable Effect ?

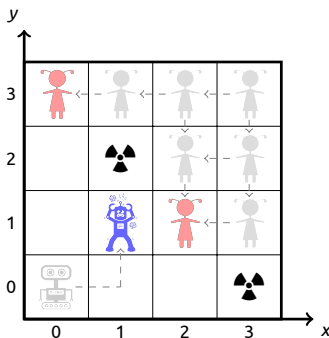


Figure : A robot escape game in a  $4 \times 4$  room, with  
 $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  
 $\Sigma_k = \{R, L, U, D\}$ .

### No delay :

Robot always wins by circling around the obstacle at (1,2).

### 1 step delay :

Robot wins by 1-step pre-decision.

### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

### 3 steps delay :

## Q2 : Do Delays Have Observable Effect ?

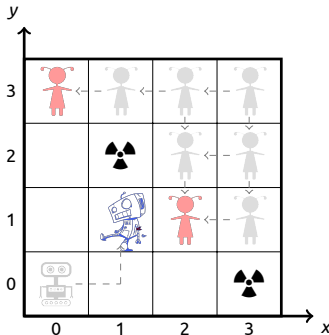


Figure : A robot escape game in a  $4 \times 4$  room, with  $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  $\Sigma_k = \{R, L, U, D\}$ .

### No delay :

Robot always wins by circling around the obstacle at (1,2).

### 1 step delay :

Robot wins by 1-step pre-decision.

### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

### 3 steps delay :

Robot is unwinnable (**uncontrollable**) anymore.

## Q2 : Do Delays Have Observable Effect? -- Yes, they have.

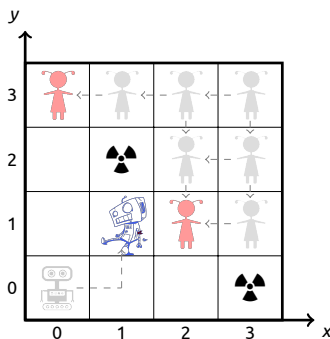


Figure : A robot escape game in a  $4 \times 4$  room, with  $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  $\Sigma_k = \{R, L, U, D\}$ .

### No delay :

Robot always wins by circling around the obstacle at (1,2).

### 1 step delay :

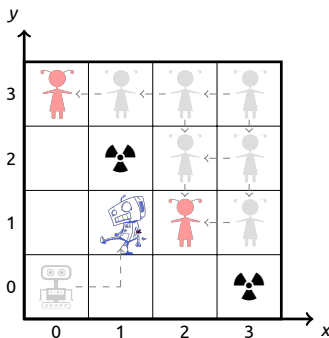
Robot wins by 1-step pre-decision.

### 2 steps delay :

Robot still wins, yet **extra memory** is needed.

### 3 steps delay :

Robot is unwinnable (**uncontrollable**) anymore.



**Figure :** A robot escape game in a  $4 \times 4$  room, with  
 $\Sigma_r = \{RU, UR, LU, UL, RD, DR, LD, DL, \epsilon\}$ ,  
 $\Sigma_k = \{R, L, U, D\}$ .

Robot always wins by circling around the obstacle at (1,2).

Robot wins by 1-step pre-decision.

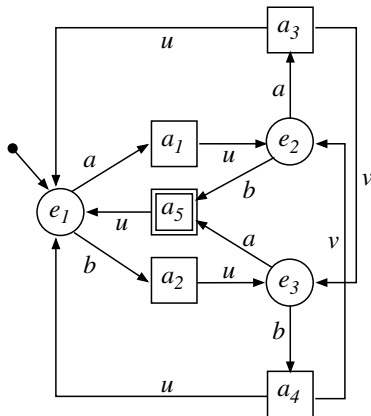
Robot still wins, yet **extra memory** is needed.

Robot is unwinnable (uncontrollable) anymore.

# Outline

- 1 Why Time Delays
  - Motivation
- 2 Safety Games under Delay
  - Delayed observation and actuation
  - Reducibility to standard safety games
- 3 Synthesizing Controllers Resilient to Delayed Interaction
  - Incremental handling of order-preserving delays
  - Out-of-order delivery
- 4 Experimental Evaluation
  - Performance
- 5 Concluding Remarks
  - Summary

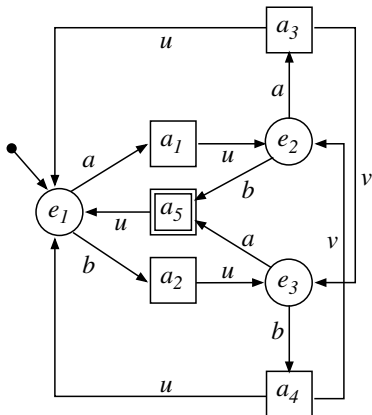
# A Trivial Safety Game



Goal: Avoid  $\boxed{a_5}$  by appropriate actions of player  $e$ .



# A Trivial Safety Game



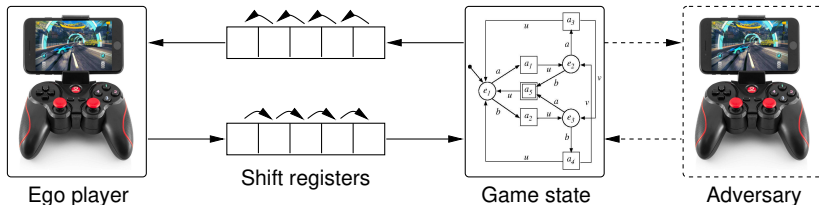
**Goal:** Avoid  $\boxed{a_5}$  by appropriate actions of player  $e$ .

**Strategy:** May always play "a" except in  $e_3$  :

$$e_1, e_2 \mapsto a$$

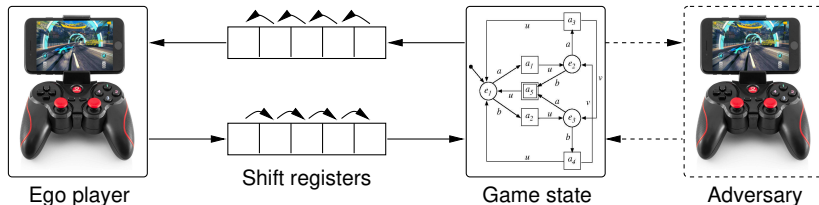
$$e_3 \mapsto b$$

# Playing Safety Game Subject to Discrete Delay



**Observation :** It doesn't make an observable difference for the joint dynamics whether delay occurs in perception, actuation, or both.

## Playing Safety Game Subject to Discrete Delay



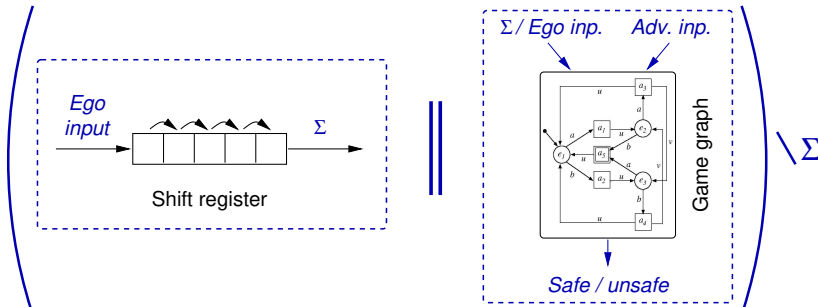
**Observation:** It doesn't make an observable difference for the joint dynamics whether delay occurs in perception, actuation, or both.

**Consequence :** There is an<sup>1</sup> obvious reduction to a safety game of perfect information.

1. In fact, two different ones : To mimic opacity of the shift registers, delay has to be moved to actuation/sensing for ego/adversary, resp. *The two thus play different games!*

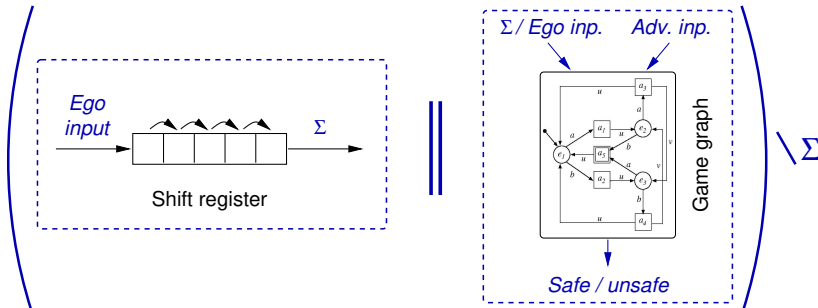
# Reduction to Delay-Free Games

from Ego-Player Perspective



# Reduction to Delay-Free Games

from Ego-Player Perspective

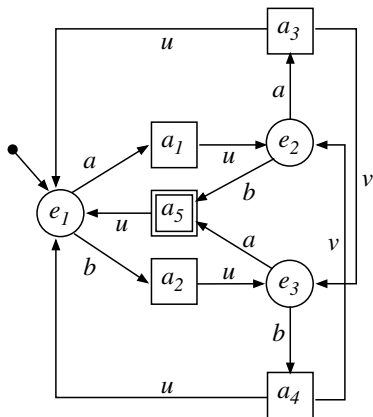


😊 Safety games w. delay **can be solved algorithmically.**

☹ Game graph incurs **blow-up by factor  $|\text{Alphabet}(\text{ego})|^{\text{delay}}$ .**

# The Simple Safety Game

...but with Delay



No delay :

$$e_1, e_2 \mapsto a$$

$$e_3 \mapsto b$$

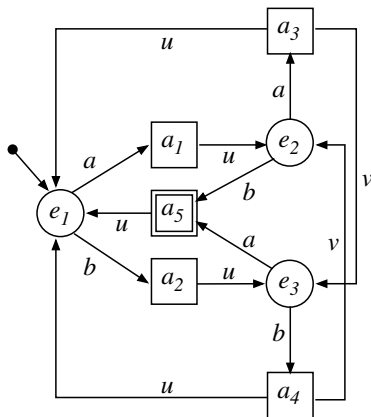
1 step delay : Strategy?

$$a_1, a_4 \mapsto a$$

$$a_2, a_3 \mapsto b$$

# The Simple Safety Game

...but with Delay



No delay :

$$e_1, e_2 \mapsto a$$

$$e_3 \mapsto b$$

1 step delay : Strategy?

$$a_1, a_4 \mapsto a$$

$$a_2, a_3 \mapsto b$$

2 steps delay : Strategy?

$$e_1 \mapsto \begin{cases} a & \text{if 2 steps back} \\ & \text{an "a" was issued,} \\ b & \text{if 2 steps back} \\ & \text{a "b" was issued.} \end{cases}$$

$$e_2 \mapsto b$$

$$e_3 \mapsto a$$

Need memory!

# Outline

- 1 Why Time Delays
  - Motivation
- 2 Safety Games under Delay
  - Delayed observation and actuation
  - Reducibility to standard safety games
- 3 Synthesizing Controllers Resilient to Delayed Interaction
  - Incremental handling of order-preserving delays
  - Out-of-order delivery
- 4 Experimental Evaluation
  - Performance
- 5 Concluding Remarks
  - Summary



# Incremental Synthesis in a Nutshell

**Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .

**Consequence :** That a position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

# Incremental Synthesis in a Nutshell

**Observation :** A winning strategy for delay  $k' > k$  can always be utilized for a safe win under delay  $k$ .

**Consequence :** That a position is winning for delay  $k$  is a necessary condition for it being winning under delay  $k' > k$ .

**Idea :** Incrementally filter out loss states & incrementally synthesize winning strategy for the remaining :

- 1 Synthesize winning strategy for underlying delay-free safety game.
- 2 For each winning state, lift strategy from delay  $k$  to  $k + 1$ .
- 3 Remove states where this does not succeed.
- 4 Repeat from 2 until either delay-resilience suffices (winning) or initial state turns lossy (losing).

- M. Chen, M. Fränzle, Y. Li, P.N. Mosaad, N. Zhan : *What's to come is still unsure : Synthesizing controllers resilient to delayed interaction*. To appear in Proc. of ATVA 2018.

# Incremental Synthesis of Delay-Tolerant Strategies

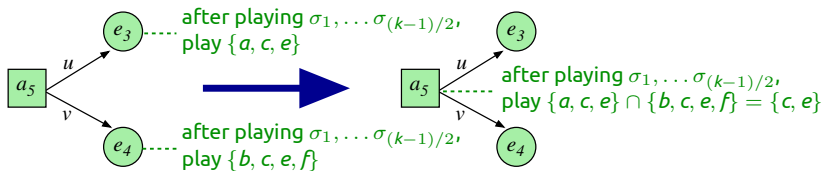
- 1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

# Incremental Synthesis of Delay-Tolerant Strategies

1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

2 Advance to delay  $k + 1$  :

If  $k$  odd: For each (ego-)winning adversarial state define strategy as



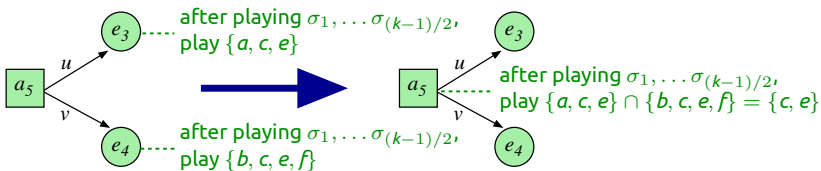
... and eliminate any dead ends by bwd. traversal.

# Incremental Synthesis of Delay-Tolerant Strategies

1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

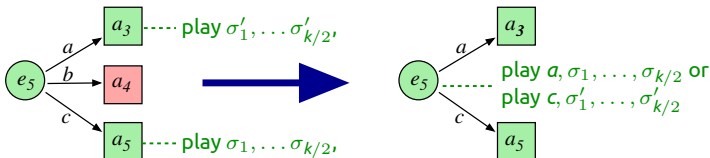
2 Advance to delay  $k + 1$ :

If  $k$  odd: For each (ego-)winning adversarial state define strategy as



... and eliminate any dead ends by bwd. traversal.

If  $k$  even: For each winning ego state define strategy as

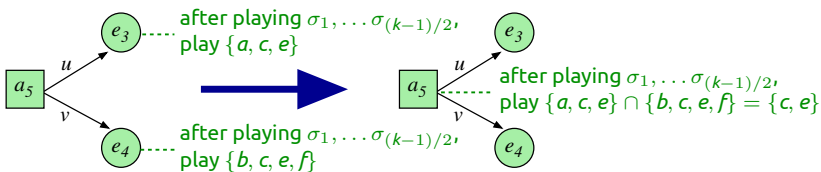


# Incremental Synthesis of Delay-Tolerant Strategies

1 Generate a *maximally permissive* strategy for delay  $k = 0$ .

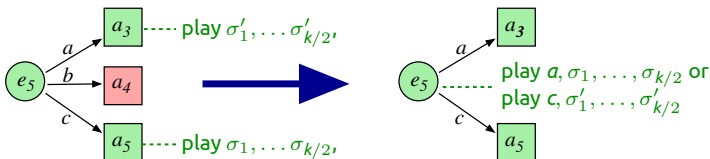
2 Advance to delay  $k + 1$  :

If  $k$  odd: For each (ego-)winning adversarial state define strategy as



... and eliminate any dead ends by bwd. traversal.

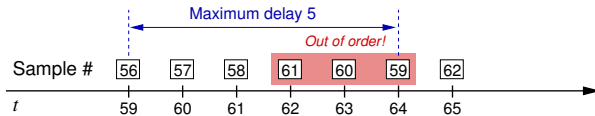
If  $k$  even: For each winning ego state define strategy as



3 Repeat from 2 until either delay-resilience suffices or initial state turns lossy.

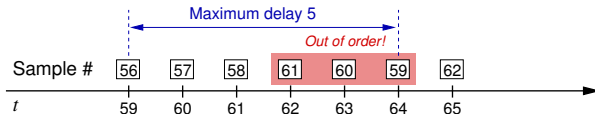
# How About Non-Order-Preserving Delays ?

☹ Observations may arrive out-of-order :

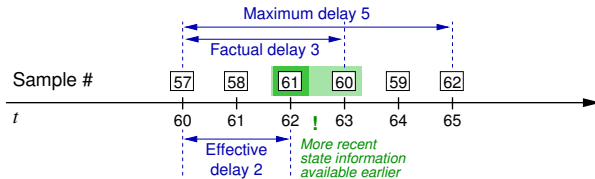


# How About Non-Order-Preserving Delays ?

☹ Observations may arrive out-of-order :



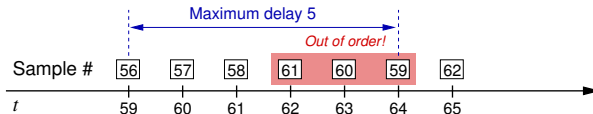
😊 But this may only reduce effective delay, improving controllability :



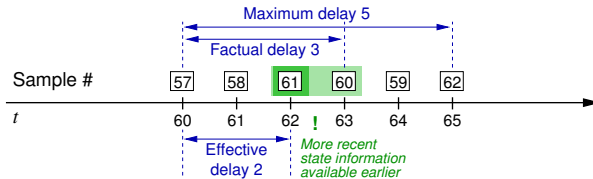


# How About Non-Order-Preserving Delays ?

- ☹ Observations may arrive out-of-order :



- 😊 But this may only reduce effective delay, improving controllability :



- 😊 W.r.t. qualitative controllability, the **worst-case of out-of-order delivery is equivalent to order-preserving delay  $k$ .**
- 😊 Stochastically **expected controllability even better** than for strict delay  $k$ .

# Outline

- 1 Why Time Delays
  - Motivation
- 2 Safety Games under Delay
  - Delayed observation and actuation
  - Reducibility to standard safety games
- 3 Synthesizing Controllers Resilient to Delayed Interaction
  - Incremental handling of order-preserving delays
  - Out-of-order delivery
- 4 Experimental Evaluation
  - Performance
- 5 Concluding Remarks
  - Summary

# Incremental vs. Reduction-Based

Benchmark				Reduction + Explicit-State Synthesis						Incremental Explicit-State Synthesis						
name	S	→	U	$\delta_{\max}$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta_{\max}$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	%
Exmp.trv1	14	20	4	$\geq 22$	0.00	0.00	0.01	0.02	0.02	$\geq 30$	0.00	0.00	<b>0.00</b>	<b>0.01</b>	<b>0.01</b>	–
Exmp.trv2	14	22	4	$= 2$	0.00	0.01	0.01	0.02	–	$= 2$	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.01</b>	–	81.97
Escp.4×4	224	738	16	$= 2$	0.08	11.66	11.73	1059.23	–	$= 2$	0.08	<b>0.13</b>	<b>0.22</b>	<b>0.25</b>	–	99.02
Escp.4×5	360	1326	20	$= 2$	0.18	34.09	33.80	3084.58	–	$= 2$	0.18	<b>0.27</b>	<b>0.46</b>	<b>0.63</b>	–	99.02
Escp.5×5	598	2301	26	$\geq 2$	0.46	96.24	97.10	?	?	$= 2$	0.46	<b>0.68</b>	<b>1.16</b>	<b>1.71</b>	–	98.98
Escp.5×6	840	3516	30	$\geq 2$	1.01	217.63	216.83	?	?	$= 2$	<b>1.00</b>	<b>1.42</b>	<b>2.40</b>	<b>4.30</b>	–	99.00
Escp.6×6	1224	5424	36	$\geq 2$	2.13	516.92	511.41	?	?	$= 2$	<b>2.06</b>	<b>2.90</b>	<b>5.12</b>	<b>10.30</b>	–	98.97
Escp.7×7	2350	11097	50	$\geq 2$	7.81	2167.86	2183.01	?	?	$= 2$	<b>7.71</b>	<b>10.67</b>	<b>19.04</b>	<b>52.47</b>	–	98.99
Escp.7×8	3024	14820	56	$\geq 0$	<b>13.07</b>	?	?	?	?	$= 2$	13.44	<b>18.25</b>	<b>32.69</b>	<b>108.60</b>	–	99.01

Benchmark		Reduction + Yosys + SafetySynth (symbolic)							Incremental Synthesis (explicit-state implementation)								%
name	$\delta_{\max}$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$		
Stub.4×4	$= 2$	1.07	1.24	1.24	1.80	–	–	–	<b>0.04</b>	<b>0.07</b>	<b>0.12</b>	<b>0.18</b>	–	–	–	98.98	
Stub.4×5	$= 2$	1.16	1.49	1.49	2.83	–	–	–	<b>0.08</b>	<b>0.14</b>	<b>0.25</b>	<b>0.44</b>	–	–	–	98.97	
Stub.5×5	$= 2$	1.19	2.61	2.50	13.67	–	–	–	<b>0.21</b>	<b>0.37</b>	<b>0.63</b>	<b>1.17</b>	–	–	–	98.97	
Stub.5×6	$= 2$	1.18	2.60	2.59	23.30	–	–	–	<b>0.42</b>	<b>0.69</b>	<b>1.20</b>	<b>2.49</b>	–	–	–	98.96	
Stub.6×6	$= 4$	1.17	2.76	2.74	19.96	19.69	655.24	–	<b>0.93</b>	<b>1.47</b>	<b>2.60</b>	<b>5.79</b>	<b>7.54</b>	<b>7.60</b>	–	99.89	
Stub.7×7	$= 4$	<b>1.23</b>	<b>2.50</b>	<b>2.48</b>	24.57	<b>23.01</b>	2224.62	–	3.60	5.52	10.08	<b>22.75</b>	31.18	<b>32.98</b>	–	99.88	

**Table :** Benchmark results in relation to reduction-based approaches (time in seconds)

# Incremental vs. Reduction-Based

Benchmark				Reduction + Explicit-State Synthesis						Incremental Explicit-State Synthesis							
name	S	→	U	$\delta_{\max}$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta_{\max}$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	%	
Exmp.trv1	14	20	4	$\geq 22$	0.00	0.00	0.01	0.02	0.02	$\geq 30$	0.00	0.00	<b>0.00</b>	<b>0.01</b>	<b>0.01</b>	–	
Exmp.trv2	14	22	4	$= 2$	0.00	0.01	0.01	0.02	–	$= 2$	0.00	<b>0.00</b>	<b>0.00</b>	<b>0.01</b>	–	81.97	
Escp.4×4	224	738	16	$= 2$	0.08	11.66	11.73	1059.23	–	$= 2$	0.08	<b>0.13</b>	<b>0.22</b>	<b>0.25</b>	–	99.02	
Escp.4×5	360	1326	20	$= 2$	0.18	34.09	33.80	3084.58	–	$= 2$	0.18	<b>0.27</b>	<b>0.46</b>	<b>0.63</b>	–	99.02	
Escp.5×5	598	2301	26	$\geq 2$	0.46	96.24	97.10	?	?	$= 2$	0.46	<b>0.68</b>	<b>1.16</b>	<b>1.71</b>	–	98.98	
Escp.5×6	840	3516	30	$\geq 2$	1.01	217.63	216.83	?	?	$= 2$	<b>1.00</b>	<b>1.42</b>	<b>2.40</b>	<b>4.30</b>	–	99.00	
Escp.6×6	1224	5424	36	$\geq 2$	2.13	516.92	511.41	?	?	$= 2$	<b>2.06</b>	<b>2.90</b>	<b>5.12</b>	<b>10.30</b>	–	98.97	
Escp.7×7	2350	11097	50	$\geq 2$	7.81	2167.86	2183.01	?	?	$= 2$	<b>7.71</b>	<b>10.67</b>	<b>19.04</b>	<b>52.47</b>	–	98.99	
Escp.7×8	3024	14820	56	$\geq 0$	<b>13.07</b>	?	?	?	?	$= 2$	13.44	<b>18.25</b>	<b>32.69</b>	<b>108.60</b>	–	99.01	

Benchmark		Reduction + Yosys + SafetySynth (symbolic)							Incremental Synthesis (explicit-state implementation)							
name	$\delta_{\max}$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$	$\delta = 0$	$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$	%
Stub.4×4 = 2	$= 2$	1.07	1.24	1.24	1.80	–	–	–	<b>0.04</b>	<b>0.07</b>	<b>0.12</b>	<b>0.18</b>	–	–	–	98.98
Stub.4×5 = 2	$= 2$	1.16	1.49	1.49	2.83	–	–	–	<b>0.08</b>	<b>0.14</b>	<b>0.25</b>	<b>0.44</b>	–	–	–	98.97
Stub.5×5 = 2	$= 2$	1.19	2.61	2.50	13.67	–	–	–	<b>0.21</b>	<b>0.37</b>	<b>0.63</b>	<b>1.17</b>	–	–	–	98.97
Stub.5×6 = 2	$= 2$	1.18	2.60	2.59	23.30	–	–	–	<b>0.42</b>	<b>0.69</b>	<b>1.20</b>	<b>2.49</b>	–	–	–	98.96
Stub.6×6 = 4	$= 4$	1.17	2.76	2.74	19.96	19.69	655.24	–	<b>0.93</b>	<b>1.47</b>	<b>2.60</b>	<b>5.79</b>	<b>7.54</b>	<b>7.60</b>	–	99.89
Stub.7×7 = 4	$= 4$	<b>1.23</b>	<b>2.50</b>	<b>2.48</b>	24.57	<b>23.01</b>	2224.62	–	3.60	5.52	10.08	<b>22.75</b>	31.18	<b>32.98</b>	–	99.88

**Table :** Benchmark results in relation to reduction-based approaches (time in seconds)

# Outline

- 1 Why Time Delays
  - Motivation
- 2 Safety Games under Delay
  - Delayed observation and actuation
  - Reducibility to standard safety games
- 3 Synthesizing Controllers Resilient to Delayed Interaction
  - Incremental handling of order-preserving delays
  - Out-of-order delivery
- 4 Experimental Evaluation
  - Performance
- 5 Concluding Remarks
  - Summary

# Concluding Remarks

**Problem :** We face

- increasingly wide-spread use of networked distributed sensing and control,
- **substantial delays thus impacting controllability and control performance,**
- naïve reduction to delay-free settings, yet with an **exponential blow-up.**

**Status :** We present

- insufficiency of memoryless control strategies for discrete safety games under delay,
- **incremental algorithm for efficient delay-tolerant control synthesis,**
- the practically relevant case of non-order-preserving delays.

**Future Work :** We plan to

- integrate **stochastic models** of message delays into safety synthesis processes,
- let synthesis constructively leverage the advantages of (partial) **control on out-of-order delivery,**
- extend to **hybrid setting** combining delayed continuous and delayed discrete reactive behavior.