Modelling and Analysing Concurrent Systems

RIO 2023 Summer School of Informatics
Rio Cuarto, Argentina; February 13–17, 2023

Lecture 4: Application to Mutual-Exclusion Protocols

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https://moves.rwth-aachen.de/teaching/ws-22-23/rio/

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Outline of Lecture 4

Peterson's Mutual Exclusion Algorithm

- **Goal:** ensuring exclusive access to non-shared resources
- Here: two competing processes P_1, P_2 and shared variables
  - b_1, b_2 (Boolean, both initially false) – b_i indicates that P_i wants to enter critical section
  - k (in $\{1, 2\}$, arbitrary initial value) – index of prioritised process
- P_i uses local variable $j := 2 - i$ (index of other process)

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Algorithm 4.1 (Peterson's algorithm for P_i)

while true do
    "non-critical section";
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    while $b_j \wedge k = j$ do skip end;
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Representing Shared Variables in CCS

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- Idea: consider variables as processes that communicate with environment by processing read/write requests

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- Encoding of b_1 with two (process) states B_{1t} (value tt) and B_{1f} (value ff)
- Read access along ports $b1rt$ (in state B_{1t}) and $b1rf$ (in state B_{1f})
- Write access along ports $b1wt$ and $b1wf$ (in both states)

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- Possible behaviours:

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$$B_{1t} = \overline{b1rt}.B_{1t} + b1wf.B_{1f} + b1wt.B_{1t}$$

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- Similarly for b_2 and k:

$$B_{2f} = \overline{b2rf}.B_{2f} + b2wf.B_{2f} + b2wt.B_{2t}$$
$$B_{2t} = \overline{b2rt}.B_{2t} + b2wf.B_{2f} + b2wt.B_{2t}$$

$$K_1 = \overline{kr1}.K_1 + kw1.K_1 + kw2.K_2$$
$$K_2 = \overline{kr2}.K_2 + kw1.K_1 + kw2.K_2$$

Modelling the Processes in CCS

Assumption: P_i cannot fail or terminate within critical section

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CCS representation

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P_2 &= \overline{b2wt}.\overline{kw1}.P_{21}\\
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$$Peterson = (P_1 \parallel P_2 \parallel B_{1f} \parallel B_{2f} \parallel K_1) \setminus L$$

for $L = \{b1rf, b1rt, b1wf, b1wt, b2rf, b2rt, b2wf, b2wt, kr1, kr2, kw1, kw2\}$

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Outline of Lecture 4

Modelling Mutual Exclusion Algorithms

Evaluating the CCS Model

Verifying Properties by Model Checking

Verifying Mutual Exclusion by Bisimulation Checking

The End

Obtaining the LTS using CAAL I

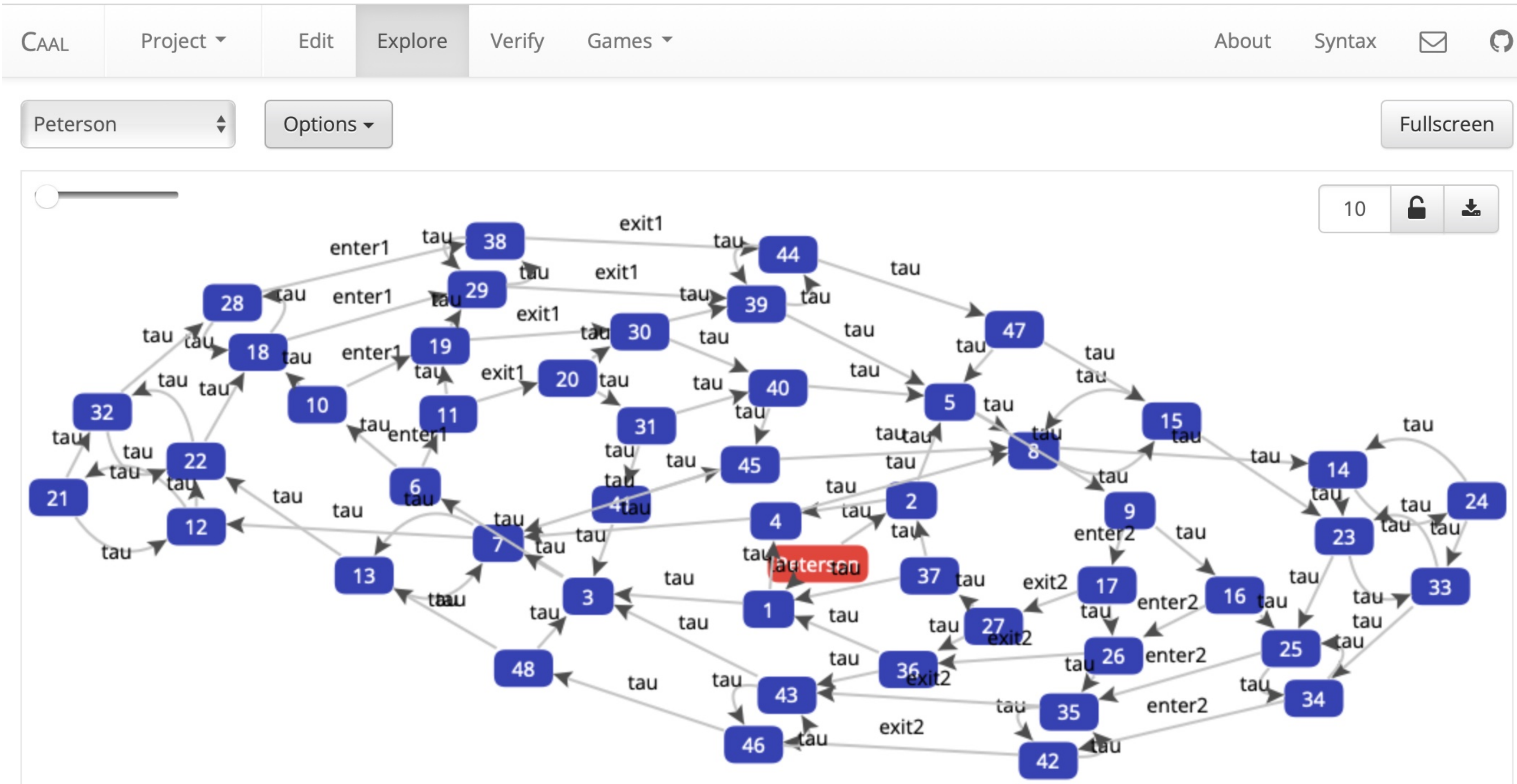
CAAL | Project | Edit | Explore | Verify | Games | About | Syntax

MutEx

Parse | CCS | TCCS | 16

```
* Peterson's algorithm for mutual exclusion.
* See Chapter 7 of "Reactive Systems" for a full description.

B1f = 'b1rf.B1f + b1wf.B1f + b1wt.B1t;
B1t = 'b1rt.B1t + b1wf.B1f + b1wt.B1t;

B2f = 'b2rf.B2f + b2wf.B2f + b2wt.B2t;
B2t = 'b2rt.B2t + b2wf.B2f + b2wt.B2t;

K1 = 'kr1.K1 + kw1.K1 + kw2.K2;
K2 = 'kr2.K2 + kw1.K1 + kw2.K2;

P1 = 'b1wt.'kw2.P11;
P11 = b2rf.P12 + b2rt.(kr2.P11 + kr1.P12);
P12 = enter1.exit1.'b1wf.P1;

P2 = 'b2wt.'kw1.P21;
P21 = b1rf.P22 + b1rt.(kr1.P21 + kr2.P22);
P22 = enter2.exit2.'b2wf.P2;

set L = {b1rf, b2rf, b1rt, b2rt, b1wf, b2wf, b1wt, b2wt, kr1, kr2, kw1, kw2};
Peterson = (P1 | P2 | B1f | B2f | K1) \ L;

MutExCCS = enter1.exit1.MutExCCS + enter2.exit2.MutExCCS;
```

Obtaining the LTS using CAAL II



Source	Action	Target
Peterson	tau	1
Peterson	tau	2

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The Mutual Exclusion Property

- **Done:** formal description of Peterson's algorithm
- **To do:** analysing its behaviour
- **Question:** what does "ensuring mutual exclusion" formally mean?

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Mutual exclusion

At no point in the execution of the algorithm, processes P_1 and P_2 will both be in their critical section at the same time.

Equivalently:
It is always the case that either P_1 or P_2 or both are not in their critical section.

Specifying Mutual Exclusion in HML

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Observations:

- Mutual exclusion is an invariance property ("always")
- P_i is in its critical section iff action *exit i* is enabled

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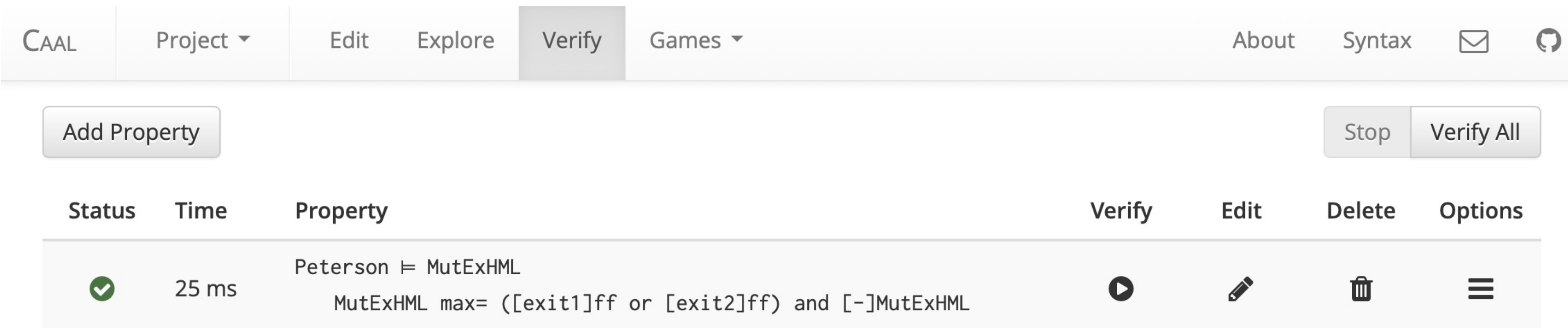
Mutual exclusion in HML

$$
\begin{aligned}
MutExHML &:= Inv(NotBoth) \\
Inv(F) &\stackrel{max}{=} F \wedge [Act]Inv(F) \qquad \text{(cf. Example 3.11)} \\
NotBoth &:= [exit1]\mathrm{ff} \vee [exit2]\mathrm{ff}
\end{aligned}
$$

Model Checking Mutual Exclusion

- Using CAAL Tool
- Supports property specifications by recursive HML formulae:

```
MutExHML max= ([exit1]ff or [exit2]ff) and [-]MutExHML;
```



Absence of Deadlocks

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Absence of deadlocks in HML

$$\begin{aligned} \mathit{NoDeadlocks} &:= \mathit{Inv}(\mathit{CanProgress}) \\ \mathit{Inv}(F) &\stackrel{\mathit{max}}{=} F \land [\mathit{Act}]\mathit{Inv}(F) \\ \mathit{CanProgress} &:= \langle \mathit{Act} \rangle \mathrm{tt} \end{aligned}$$

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Fairness in HML (only first process)

$$
\begin{aligned}
Fair_1 &:= Inv(EventuallyLeaves_1) \\
Inv(F) &\overset{max}{=} F \wedge [Act]Inv(F) \\
EventuallyLeaves_1 &:= Evt(Exits_1) \\
Evt(G) &\overset{min}{=} G \vee [Act]Evt(G) \\
Exits_1 &:= \langle exit1 \rangle \mathrm{tt}
\end{aligned}
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- However, *Peterson* and *MutExSpec* share the same weak traces (i.e., action sequences ignoring τ-transitions).

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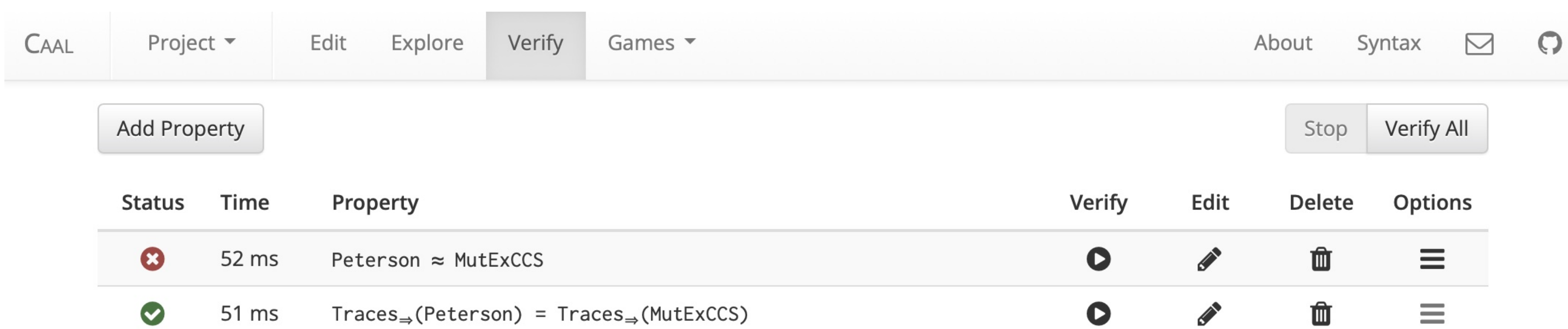

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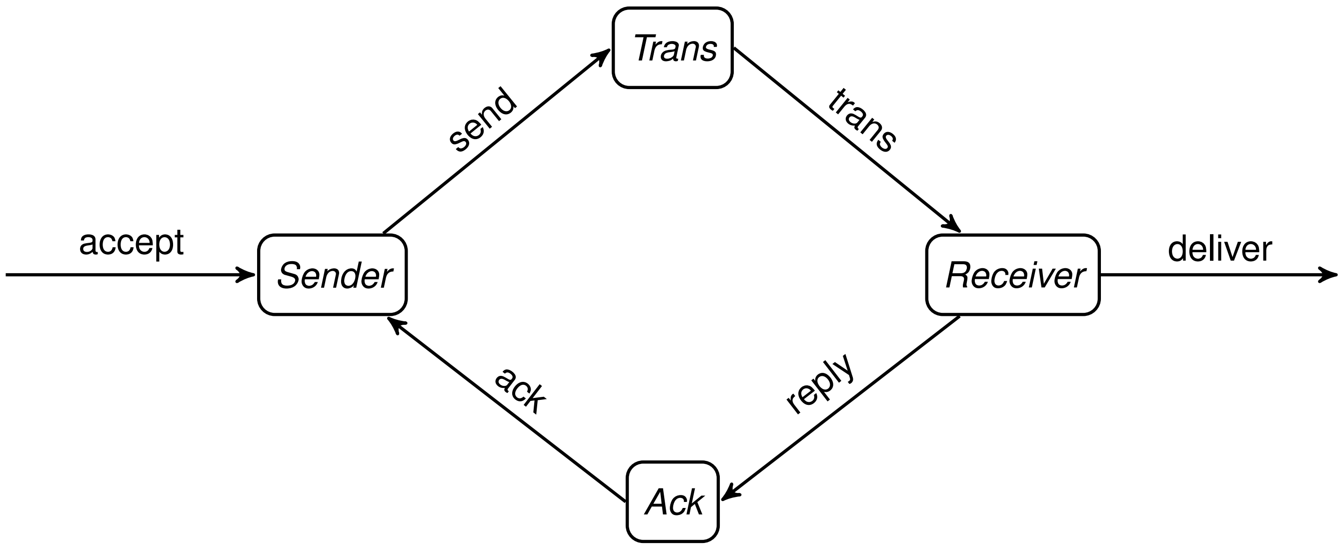
Homework: The Alternating-Bit Protocol



To do (using CAAL):

- Implementation as CCS process definition
- Abstract specification in CCS and bisimilarity checking
- Model checking for deadlocks and livelocks
- Deliverable: short experience report with description of outcomes
- Deadline: March 31, 2023
- More details in `https://moves.rwth-aachen.de/wp-content/uploads/abp.pdf`