Modelling and Analysing Concurrent Systems

RIO 2023 Summer School of Informatics
Rio Cuarto, Argentina; February 13–17, 2023

Lecture 1: Milner's Calculus of Communicating Systems

Thomas Noll
Software Modelling and Verification Group
RWTH Aachen University

https://moves.rwth-aachen.de/teaching/ws-22-23/rio/

Outline of Lecture 1

Staff

About me

- Associate professor at the Software Modelling and Verification Group (MOVES) in the Department of Computer Science at RWTH Aachen University
- Research interests:
  - Reliability, Safety and Security of Hardware/Software Systems
  - Static Program Analysis for Software Optimisation and Verification
  - Formal Verification of Artificial Neural Networks
- Teaching activities:
  - Courses on *Concurrency Theory*
  - Courses on *Semantics and Verification of Software*
  - Courses on *Compiler Construction*
  - Courses on *Static Program Analysis*
  - Bridging courses on *Foundations of Informatics*
  - Seminars on advanced topics
  - Supervision of Bachelor's and Master's theses

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- Understand the foundations of concurrent systems
- Understand the main semantical underpinnings of concurrency
- Model, reason about, and compare concurrent systems in a rigorous manner

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  - "Programming Concurrent Systems"
  - synchronisation, scheduling, semaphores, ...

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  - "Model Checking"
  - validation of mutual exclusion, fairness, absence of deadlocks, ...

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  - synchronisation, scheduling, semaphores, ...
- Verifying functional correctness properties
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- Comparing expressivity of models of concurrency
  - "interleaving" vs. "true concurrency"
  - equivalence, refinement, abstraction, ...

Organisation of the Course

Organisation

- All material (slides, exercises, ...) made available via
  `https://moves.rwth-aachen.de/teaching/ws-22-23/rio/`
- Schedule: Mon Feb 13 – Thu Feb 16, 10:30 – 13:00
- Exam Fri Feb 17 morning

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Concurrency and Interaction by Example

Observation: concurrency introduces new phenomena

Example 1.1

$$x := 0;$$
$$(x := x + 1 \parallel x := x + 2)$$

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$$x := 0;$$
$$(x := x + 1 \parallel x := x + 2) \qquad \text{value of } x\text{: } 2$$
$$2$$

- At first glance: x is assigned 3
- But: both parallel components could read x before it is written
- Thus: x is assigned 2,

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- At first glance: x is assigned 3
- But: both parallel components could read x before it is written
- Thus: x is assigned 2, 1, or 3
- If exclusive access to shared memory and atomic execution of assignments guaranteed
  $\Rightarrow$ only possible outcome: 3

Concurrency and Interaction

The problem arises due to the combination of

- concurrency and
- interaction (here: via shared memory)

Concurrency and Interaction

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Conclusion

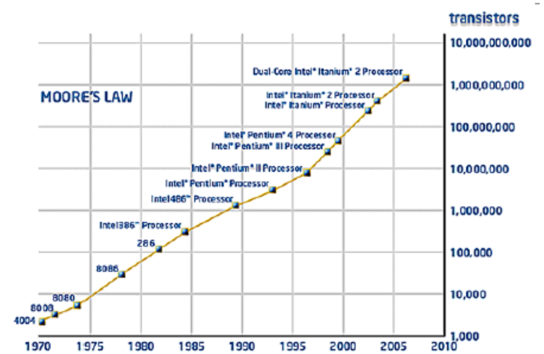
When modelling concurrent systems, the precise description of the mechanisms of both concurrency and interaction is crucially important.

Concurrency Everywhere

Herb Sutter: *The Free Lunch Is Over*, Dr. Dobb's Journal, 30(3), 2005

"The biggest sea change in software development since the OO revolution is knocking at the door, and its name is Concurrency."

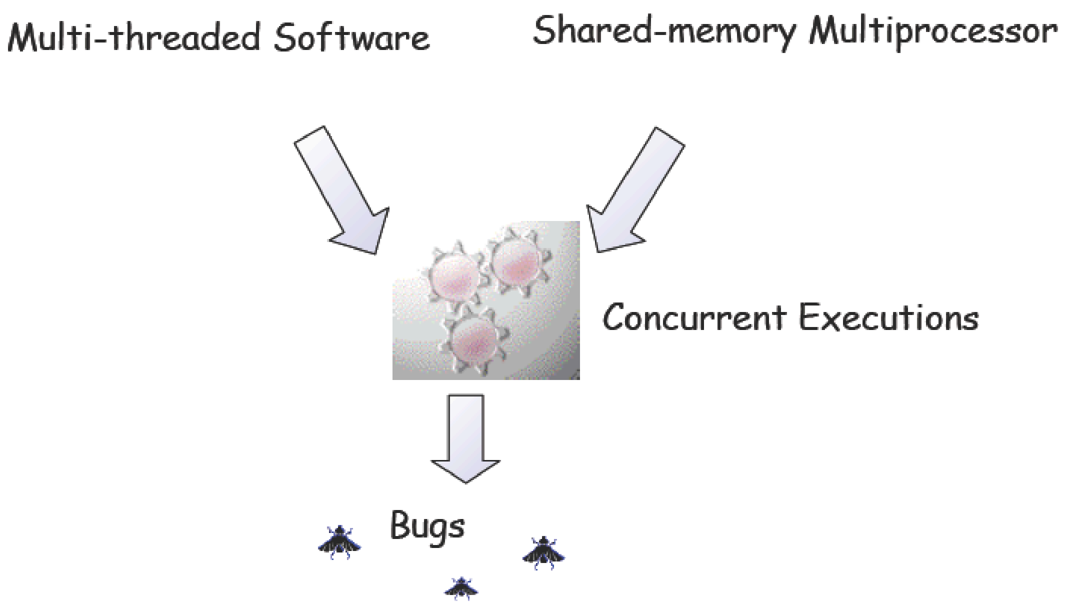
- Operating systems
- Embedded/reactive systems
  - parallelism (at least) between hardware, software, and environment
- High-end parallel hardware infrastructure:
  - high-performance computing
- Low-end parallel hardware infrastructure
  - increasing performance only achievable by parallelism
  - multi-core computers, GPGPUs, FPGAs



Moore's Law: Transistor density doubles every 2 years

Problems Everywhere

- Operating systems:
  - mutual exclusion
  - fairness (no starvation)
  - no deadlocks, ...
- Shared-memory systems:
  - memory models
  - data races
  - inconsistencies ("sequential consistency" vs. relaxed notions)
- Embedded systems:
  - safety
  - liveness, ...



Outline of Lecture 1

Memory Models

An illustrative example

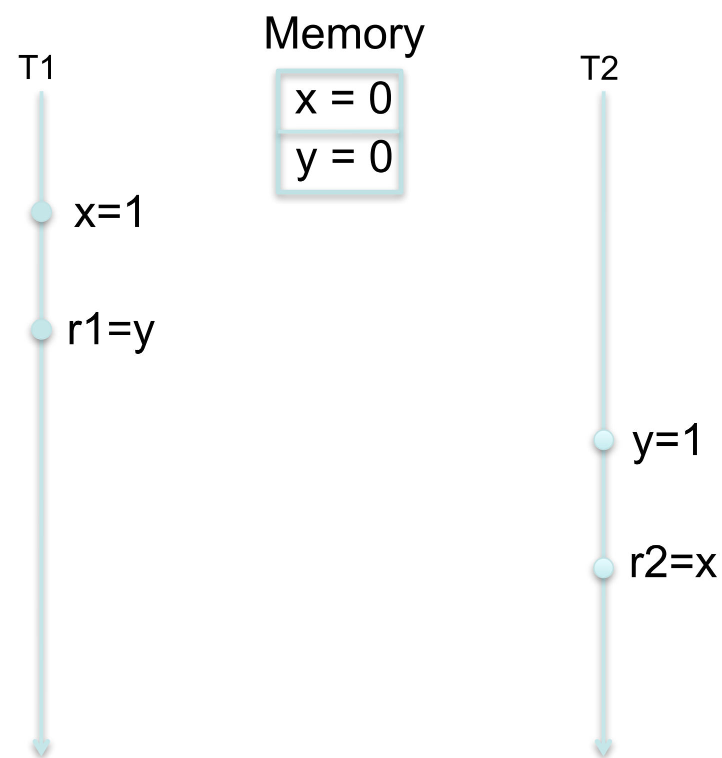
Initially: x = y = 0

thread1:	thread2:
1: x = 1	3: y = 1
2: r1 = y	4: r2 = x

(with global variables x, y and local registers r1, r2)

Memory Models

Sequential Consistency (SC)



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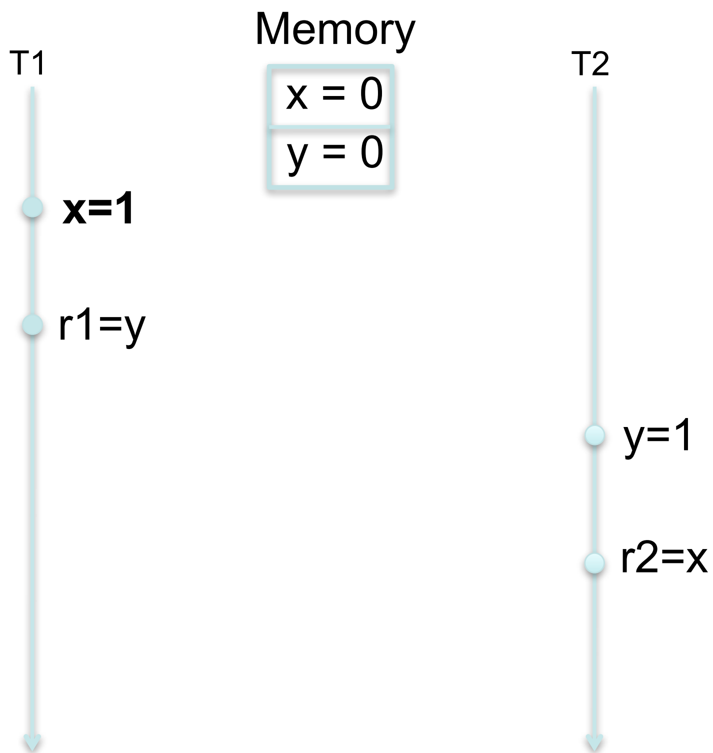

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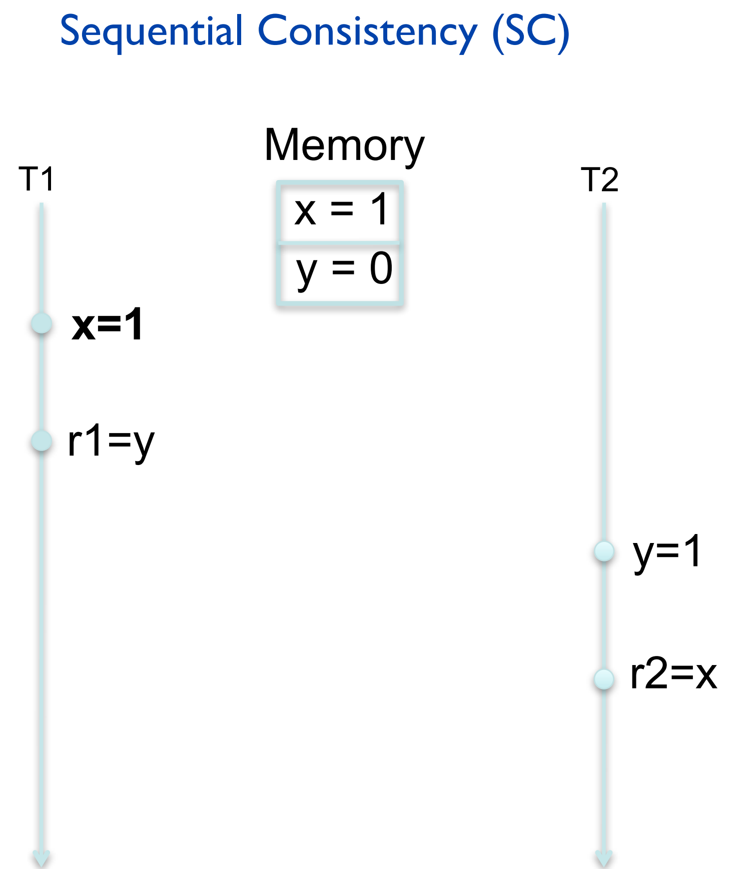

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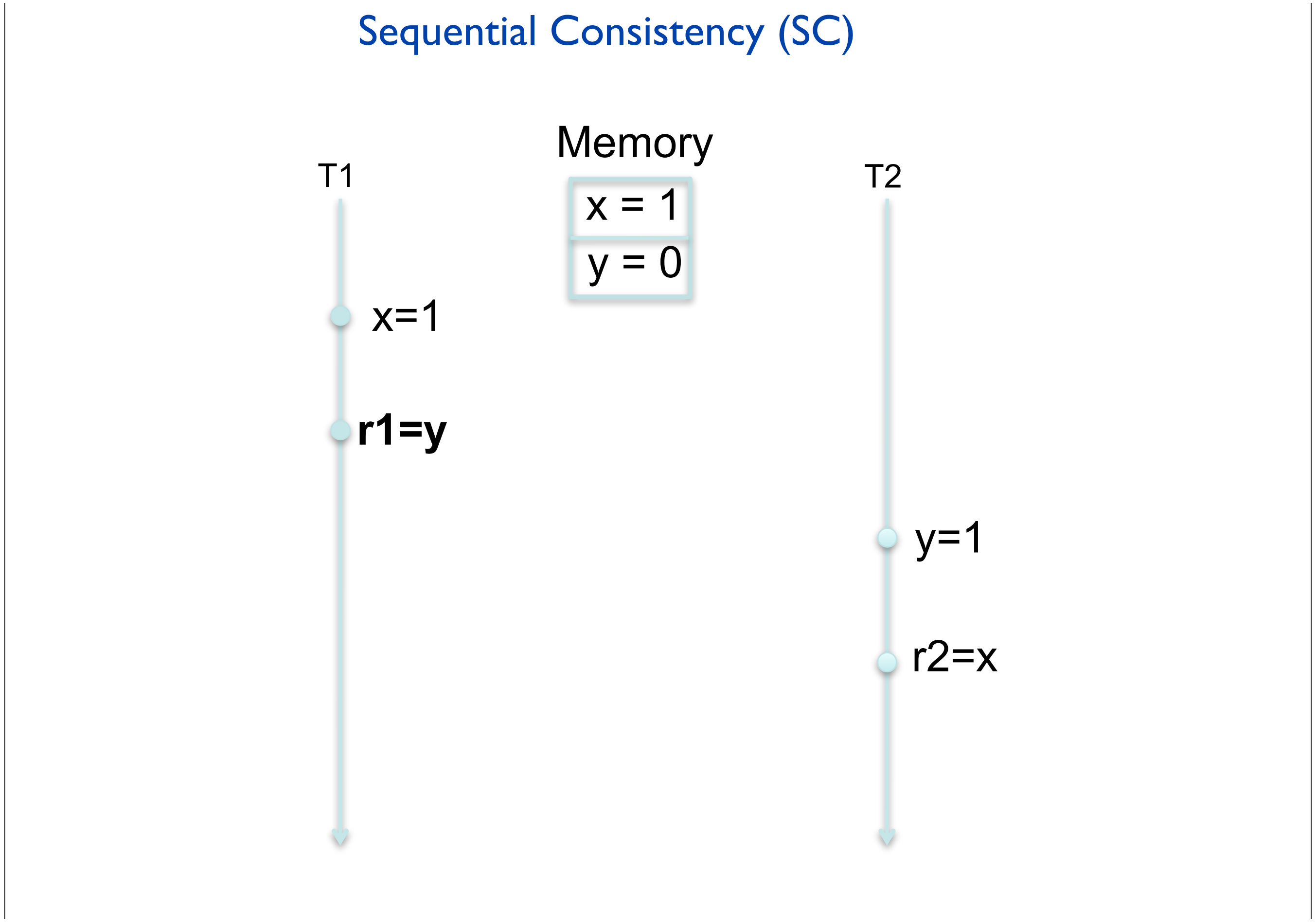

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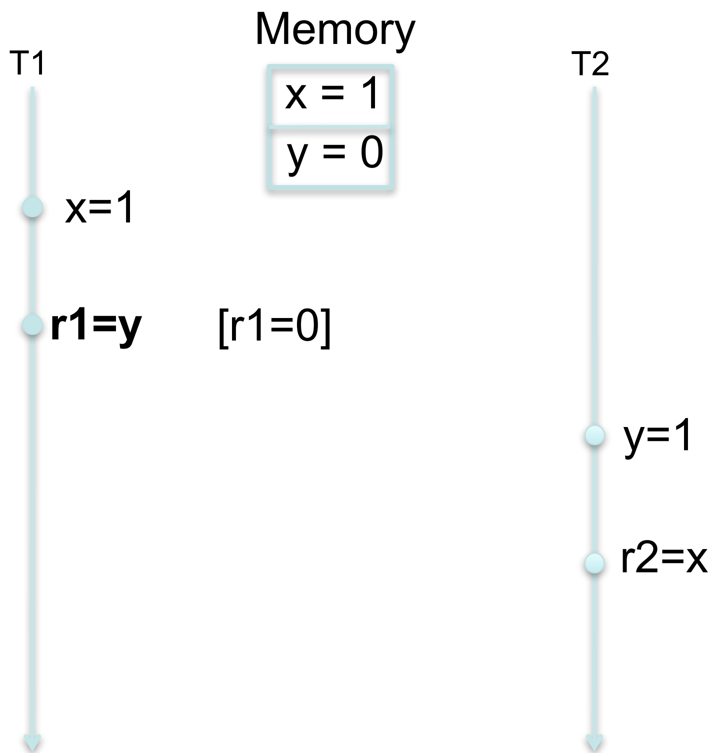

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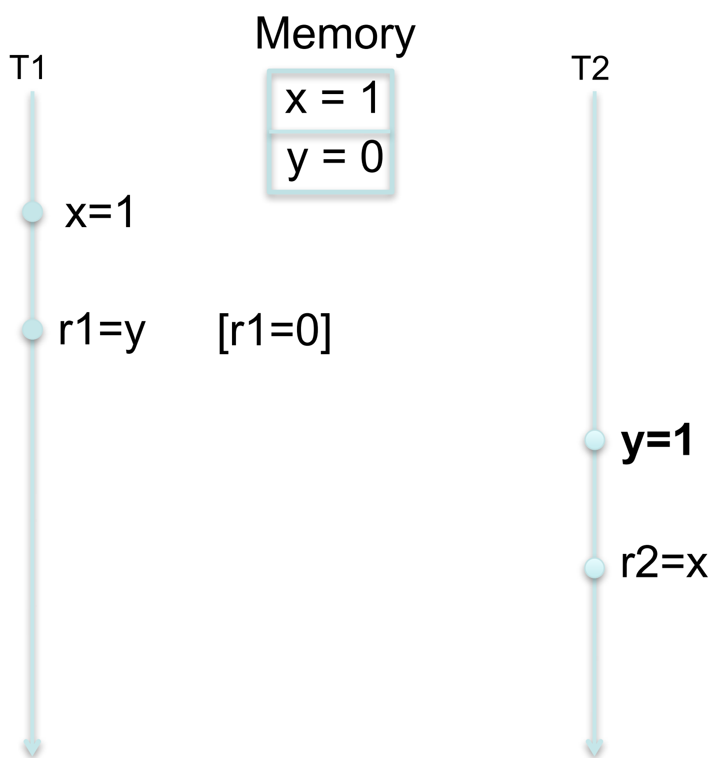

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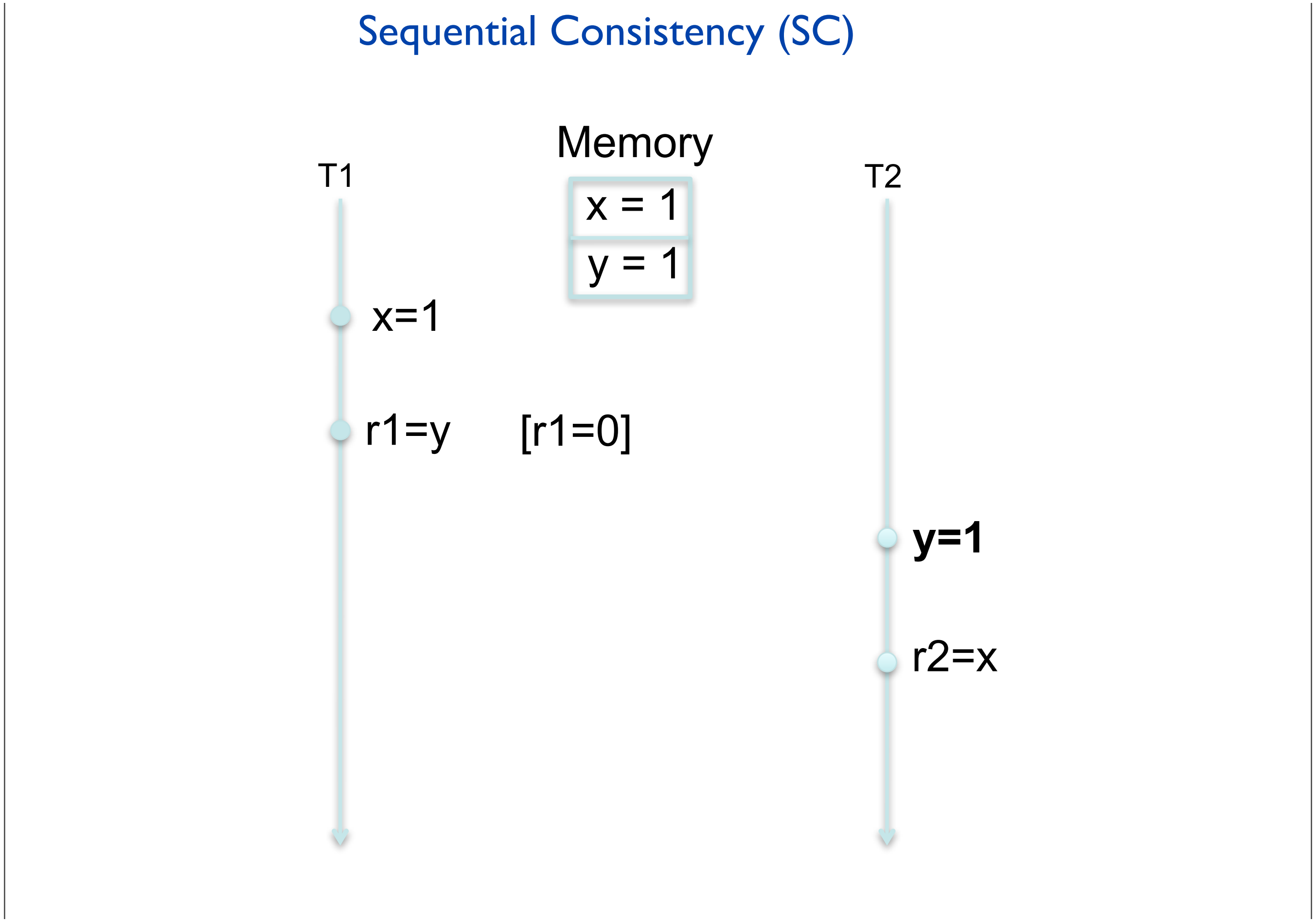

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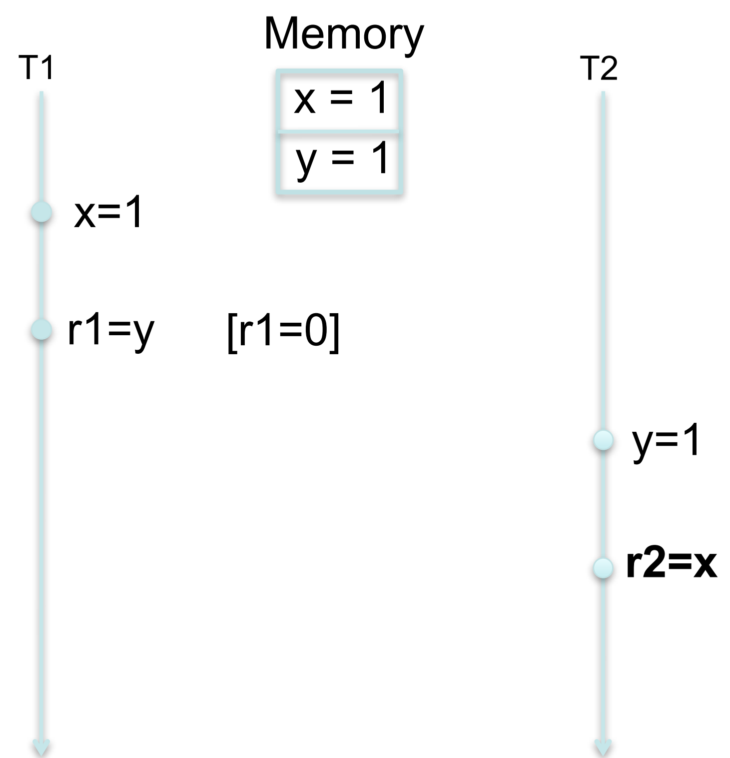

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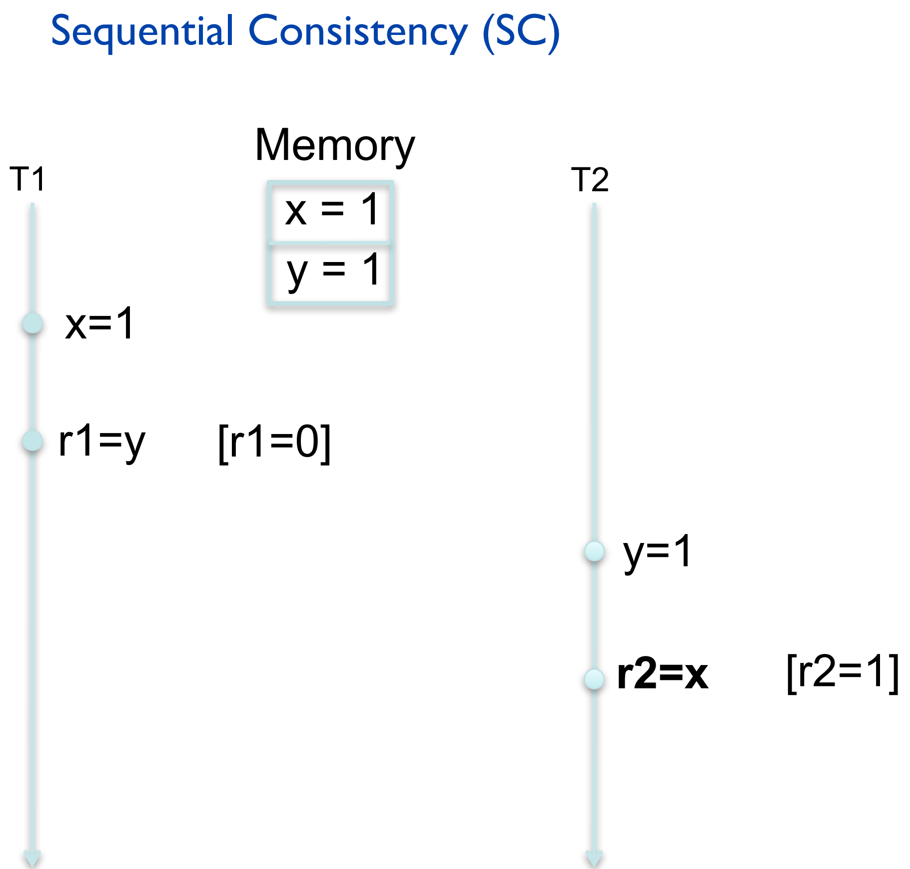
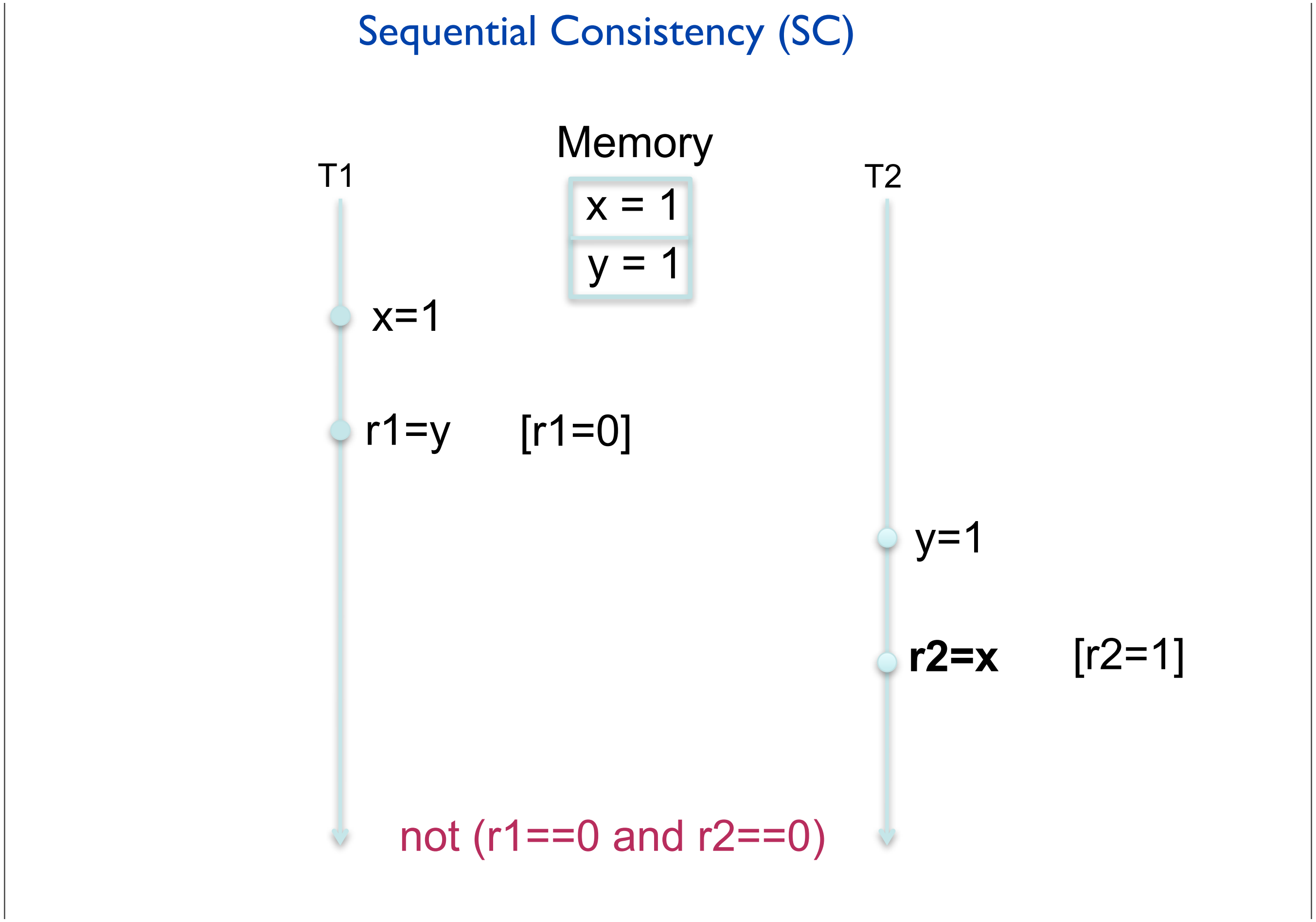
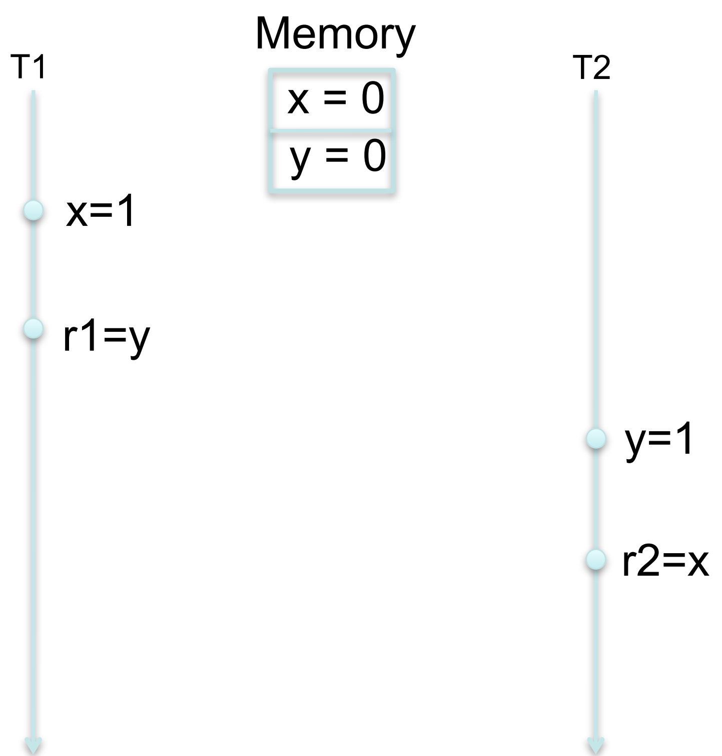



Sequential Consistency (SC)
Memory
x = 1
y = 1
T1
T2
x=1
r1=y
[r1=0]
y=1
r2=x
[r2=1]
not (r1==0 and r2==0)

Memory Models

Total Store Ordering (TSO)



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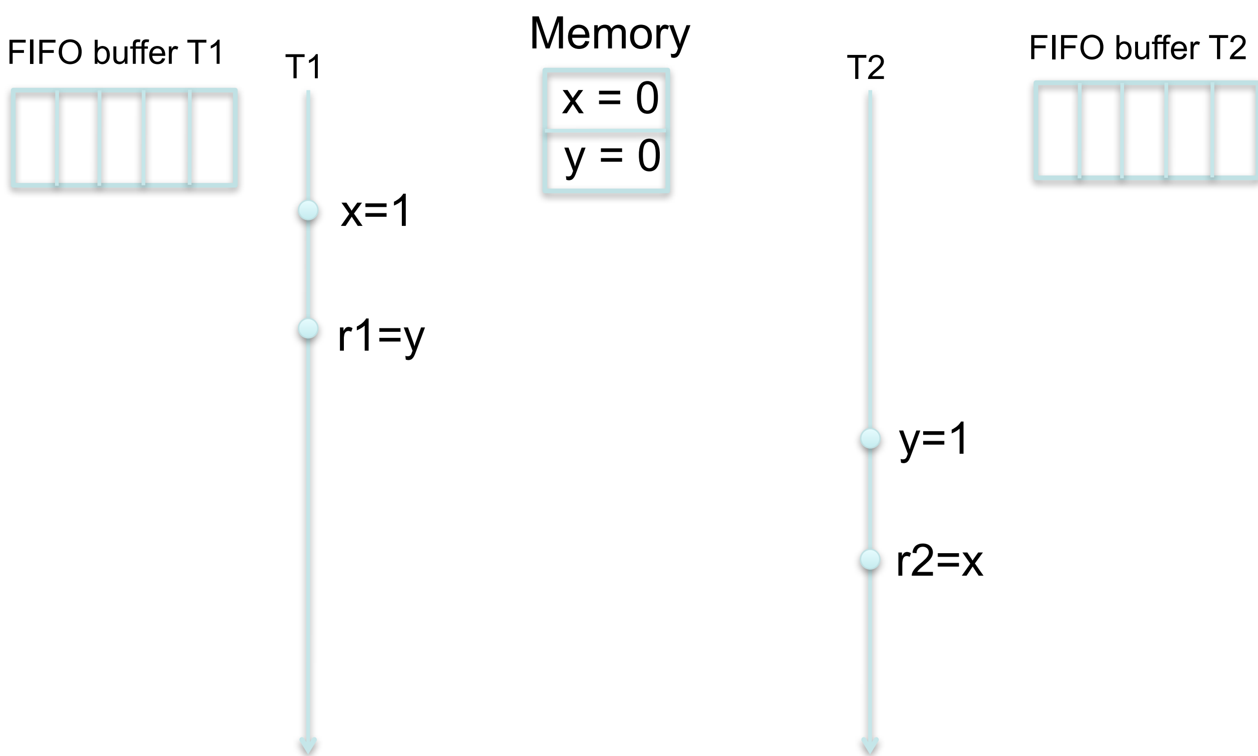

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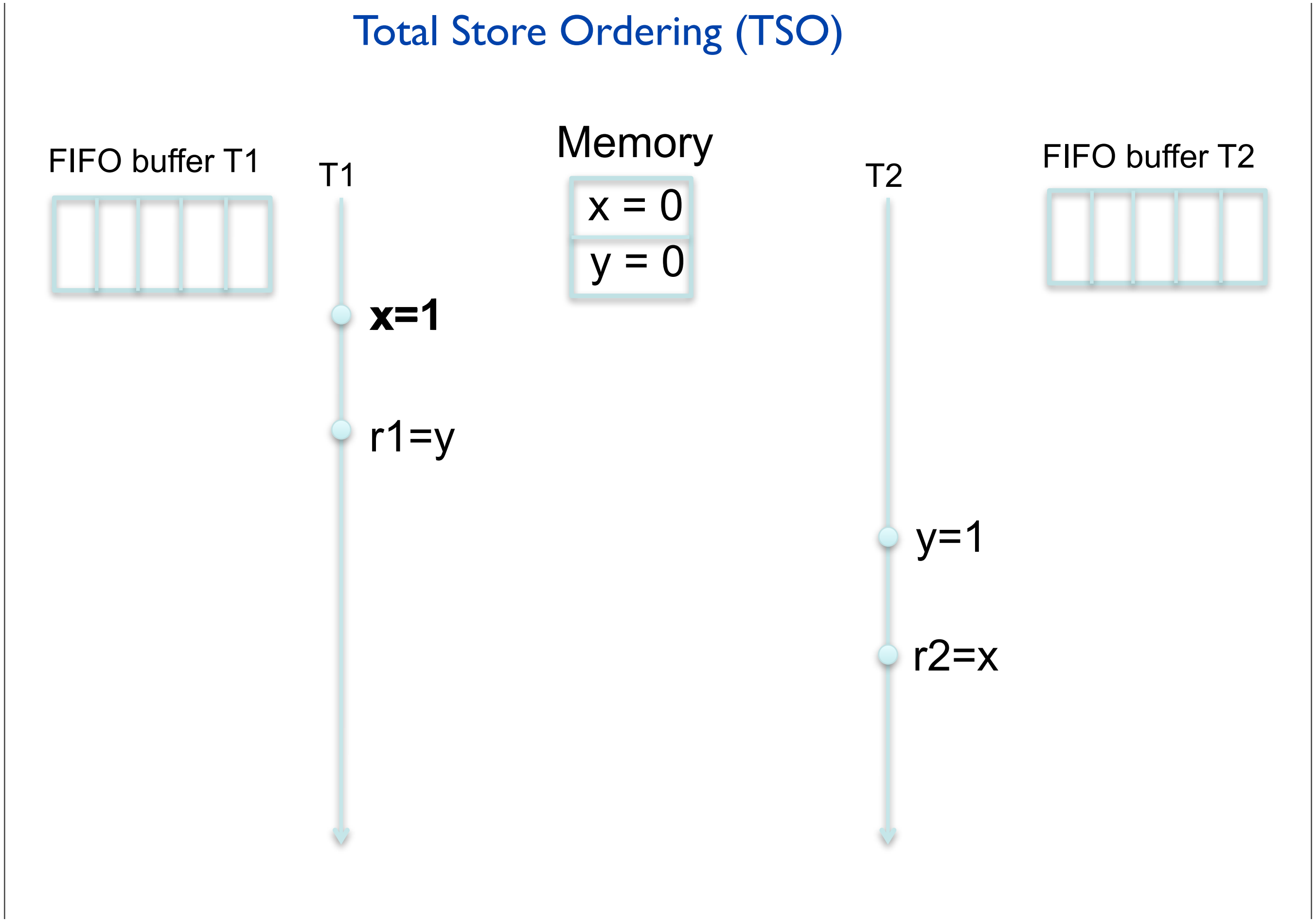

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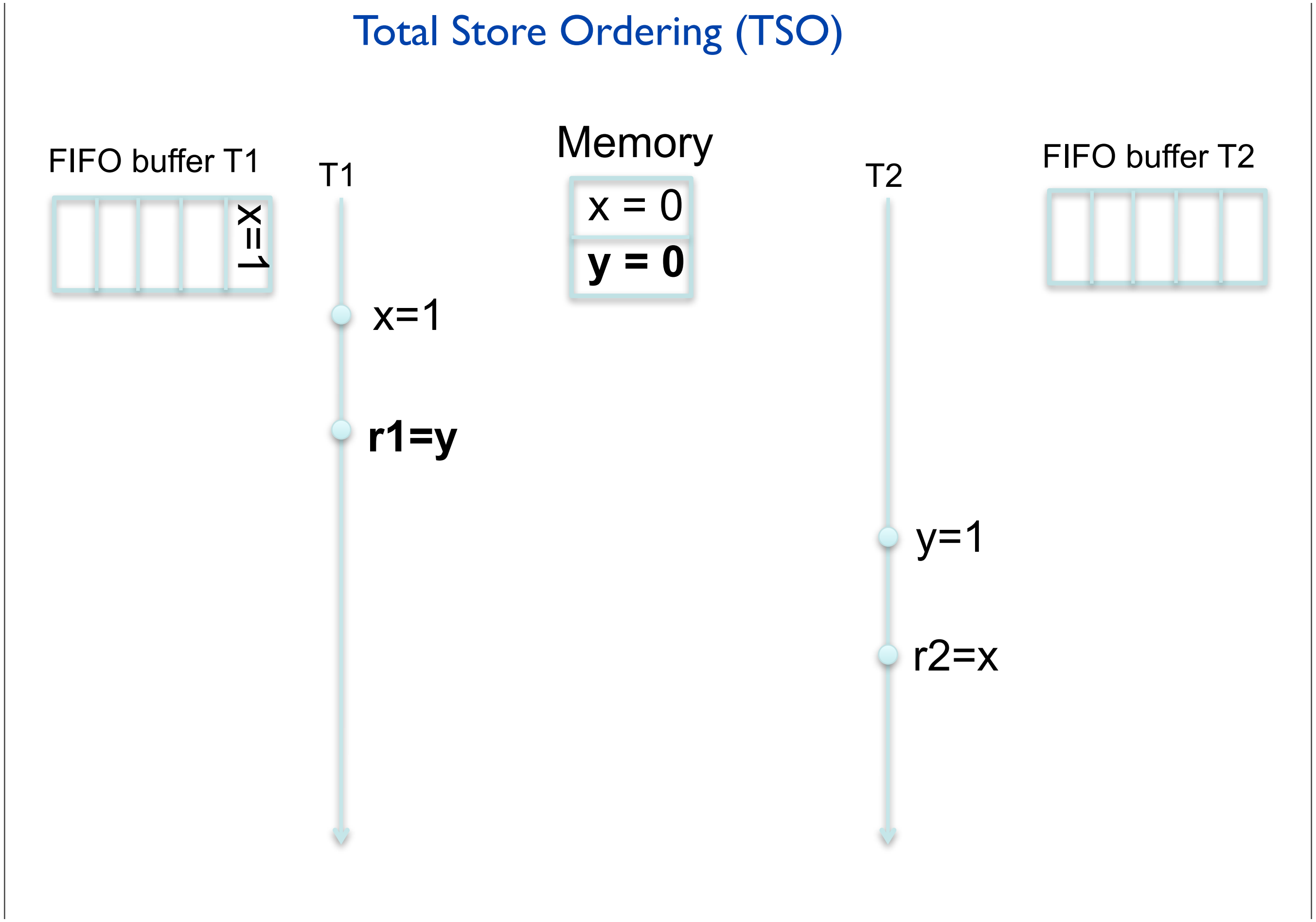

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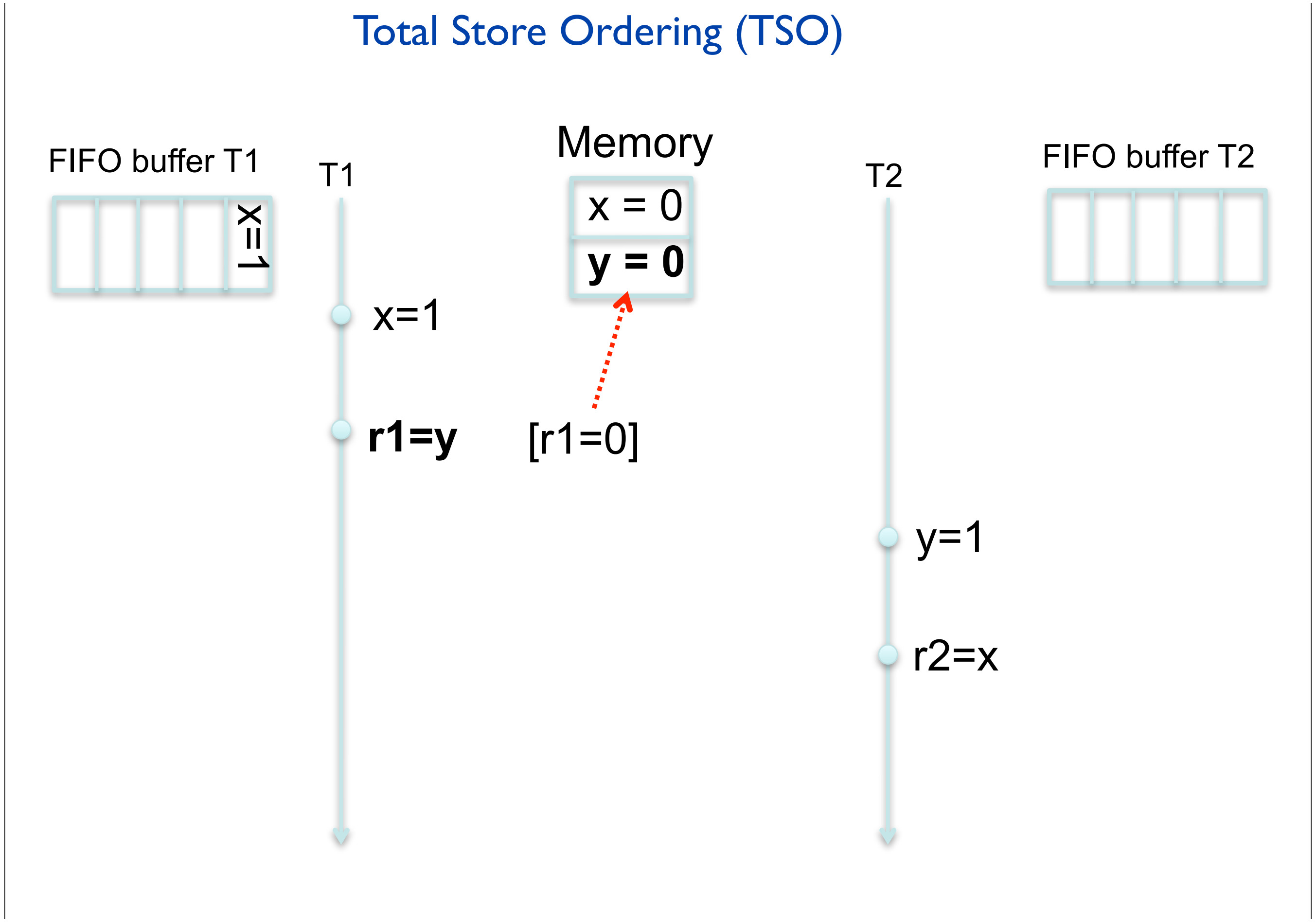

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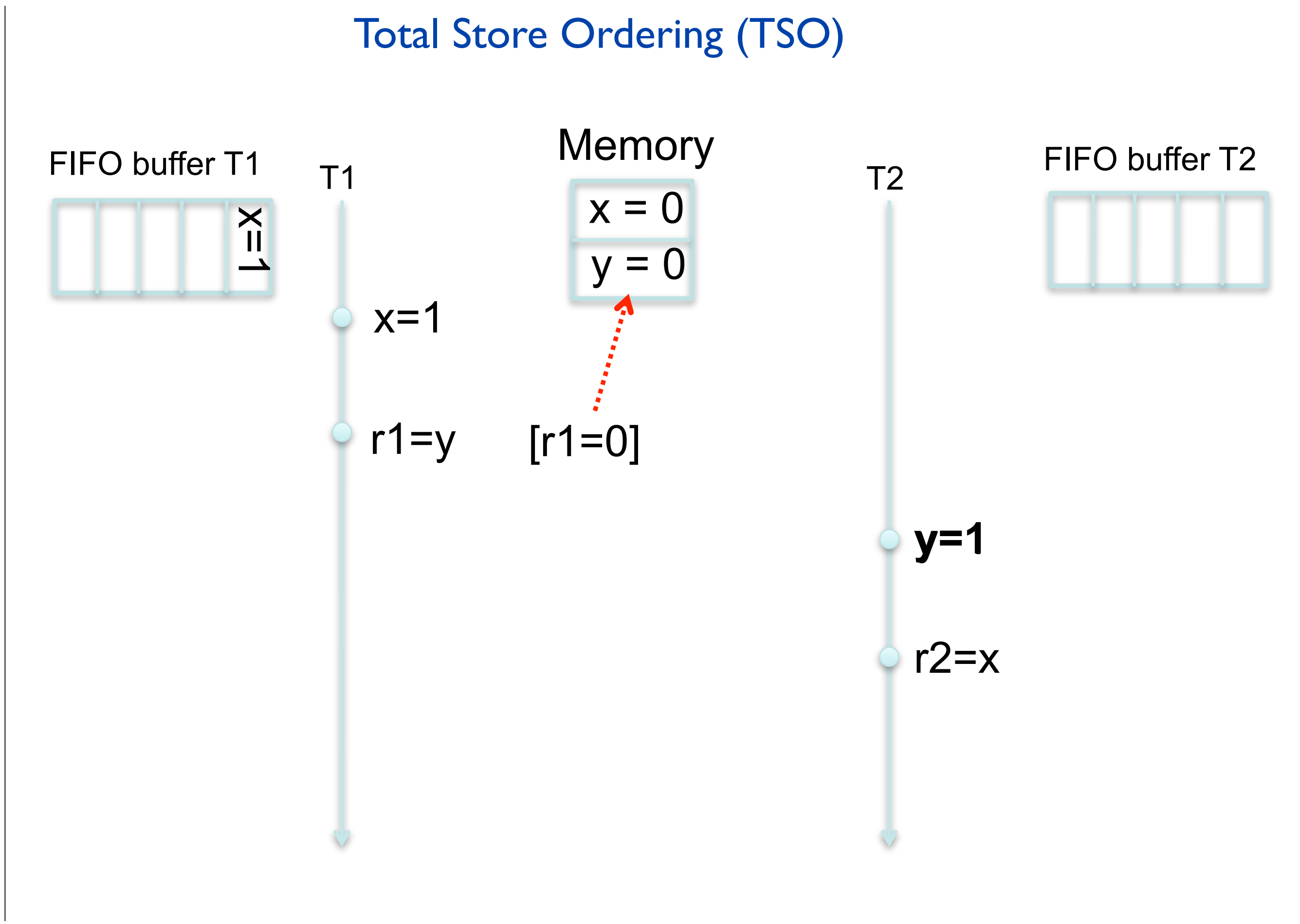

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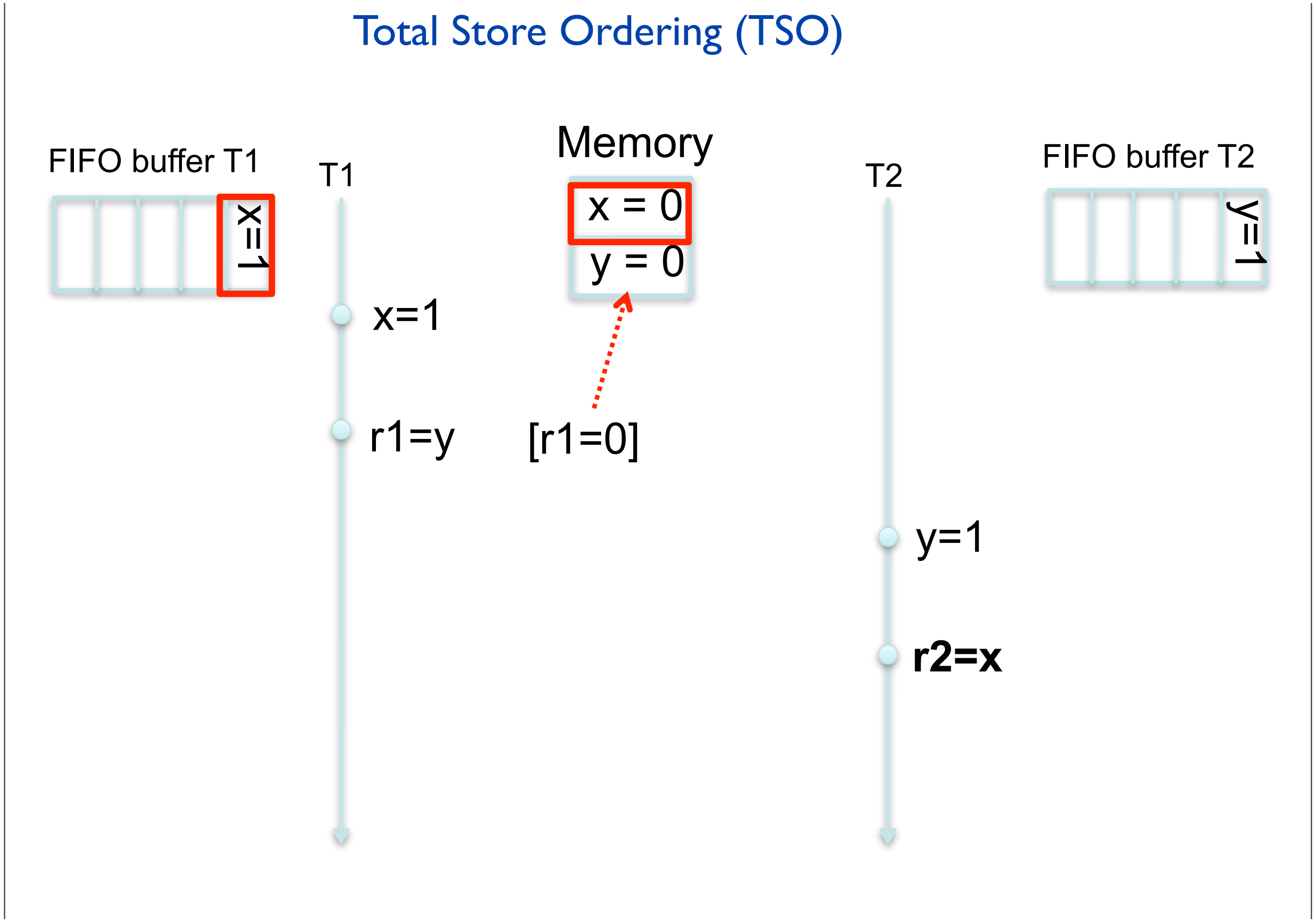

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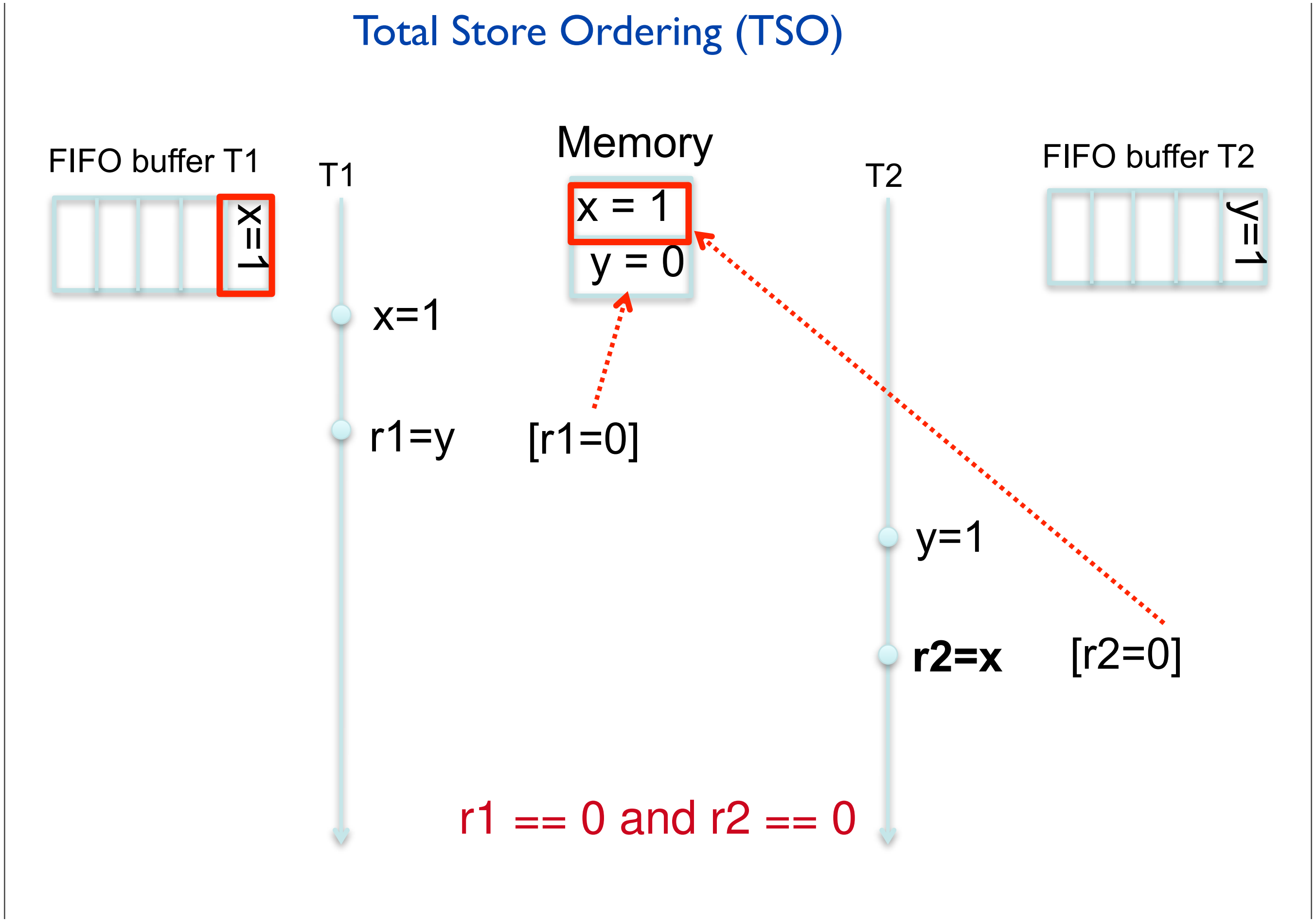

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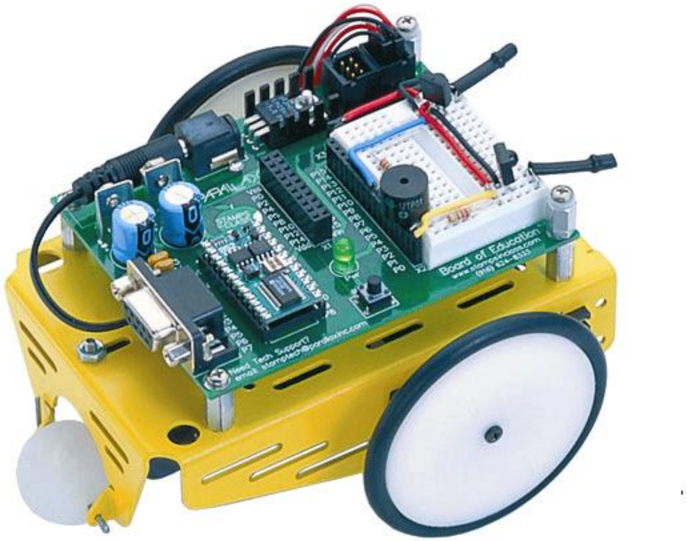
Reactive Systems I

- "Classical" model for sequential systems

$$\mathit{System} : \mathit{Input} \to \mathit{Output}$$

  (transformational systems) is not adequate
- Missing: aspect of interaction



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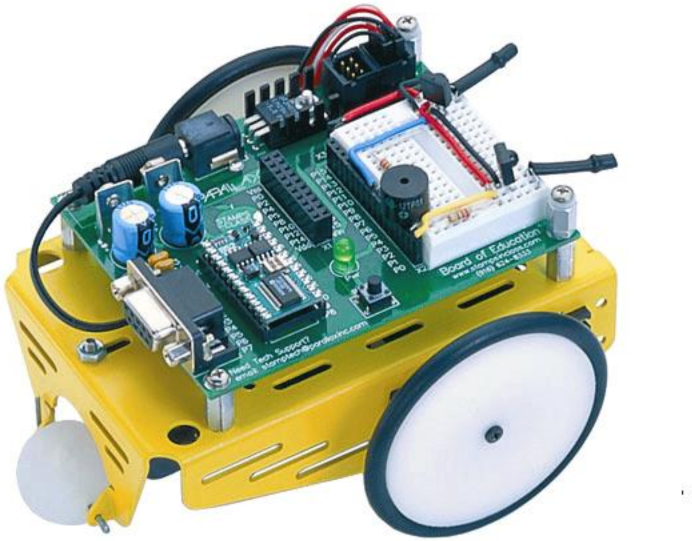
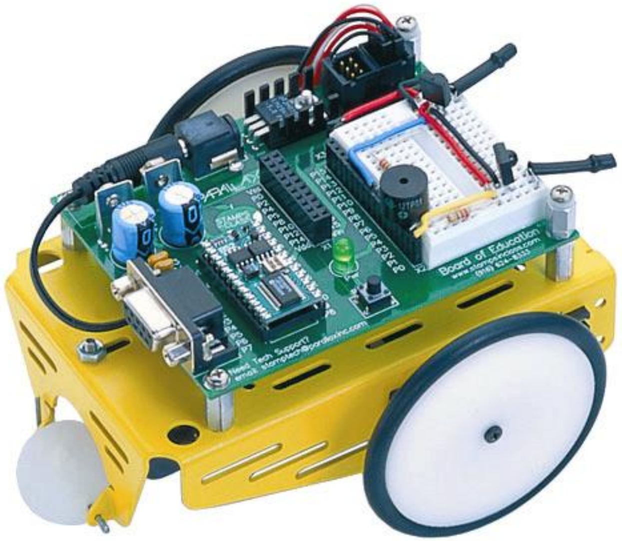


Reactive Systems I

- "Classical" model for sequential systems

$$\mathit{System} : \mathit{Input} \to \mathit{Output}$$

  (transformational systems) is not adequate
- Missing: aspect of interaction
- Rather: reactive systems which interact with environment and among themselves
- Main interest: not terminating computations but infinite behaviour (system maintains ongoing interaction with environment)
- Examples:
  - operating systems
  - embedded systems controlling mechanical or electrical devices (planes, cars, home appliances, ...)
  - power plants, production lines, ...



Reactive Systems II

Observation

Reactive systems are often safety critical, thus trustworthiness has to be ensured.

- Safety properties: “Nothing bad is ever going to happen.”
  - e.g., “at most one process in the critical section”
- Liveness properties: “Eventually something good will happen.”
  - e.g., “every request will finally be answered by the server”
- Fairness properties: “No component will starve to death.”
  - e.g., “any process requiring entry to the critical section will eventually be admitted”
- Reliability, performance, survivability, ...

Outline of Lecture 1

Overview of the Course

Overview

(1) Milner's Calculus of Communicating Systems (CCS)
- introduction and motivation
- syntax of CCS
- semantics of CCS
- the CAAL tool

(2) Behavioural Equivalences
- trace equivalence
- bisimulation
- congruence
- deadlock sensitivity

(3) Logical Specifications
- Hennessy-Milner Logic
- HML and traces
- HML and bisimulation
- adding recursion

(4) Application: Mutual-Exclusion Protocols
- modelling mutex algorithms in CCS
- verification by model checking
- verification by bisimulation checking

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Literature

Luca Aceto, Anna Ingólfsdóttir, Kim Guldstrand Larsen and Jiří Srba: *Reactive Systems: Modelling, Specification and Verification*, Cambridge Univ. Press, 2007

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The Calculus of Communicating Systems

History

- First development:
  Robin Milner: *A Calculus of Communicating Systems*, LNCS 92, Springer, 1980
- Elaboration and larger case studies:
  Robin Milner: *Communication and Concurrency*, Prentice-Hall, 1989
- Extension to mobile systems:
  Robin Milner: *Communicating and Mobile Systems: the π-calculus*, Cambridge University Press, 1999

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Approach

Description of concurrency on a simple and abstract level, using only a few basic primitives

- no explicit storage (variables)
- no explicit representation of values (numbers, Booleans, ...) or data structures

$\Rightarrow$ Concurrent system reduced to communication potential

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Definition 1.2 (Syntax of CCS)

- Let A be a set of (action) names.

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- $Act := A \cup \overline{A} \cup \{\tau\}$ is the set of actions with the silent (or: unobservable) action τ.
- Let Pid be a set of process identifiers.
- The set Prc of process expressions is defined by the following syntax:

$$
\begin{array}{rlll}
P ::= & \mathrm{nil} & & \text{(inaction)} \\
\mid & \alpha.P & & \text{(prefixing)} \\
\mid & P_1 + P_2 & & \text{(choice)} \\
\mid & P_1 \parallel P_2 & & \text{(parallel composition)} \\
\mid & P \setminus L & & \text{(restriction)} \\
\mid & P[f] & & \text{(relabelling)} \\
\mid & C & & \text{(process call)}
\end{array}
$$

where $\alpha \in Act$, $\emptyset \neq L \subseteq A$, $C \in Pid$, and $f : Act \to Act$ such that $f(\tau) = \tau$ and $f(\overline{a}) = \overline{f(a)}$ for each $a \in A$.

Syntax of CCS II

Definition 1.2 (continued)

- A (recursive) process definition is an equation system of the form

$$(C_i = P_i \mid 1 \leq i \leq k)$$

  where $k \geq 1$, $C_i \in Pid$ (pairwise distinct), and $P_i \in Prc$ (with identifiers from $\{C_1, \ldots, C_k\}$).

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Notational Conventions:

- $\overline{\overline{a}}$ means a
- $\sum_{i=1}^{n} P_i$ ($n \in \mathbb{N}$) means $P_1 + \ldots + P_n$ (where $\sum_{i=1}^{0} P_i := \text{nil}$)
- $P \setminus a$ abbreviates $P \setminus \{a\}$
- $[a_1 \mapsto b_1, \ldots, a_n \mapsto b_n]$ stands for $f : Act \to Act$ with $f(a_i) = b_i$ for $i \in [n]$ and $f(\alpha) = \alpha$ otherwise
- Restriction and relabelling bind stronger than prefixing, prefixing stronger than composition, composition stronger than choice:

$$P \setminus a + b.Q \parallel R \quad \text{means} \quad (P \setminus a) + ((b.Q) \parallel R)$$

Outline of Lecture 1

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- The relabelling $P[f]$ allows to adapt the naming of actions.
- The behaviour of a process call C is given by the right-hand side of the corresponding equation.

CCS Examples

Example 1.3

(1) One-place buffer:

$$B = in.\overline{out}.B$$

Software Modeling and Verification Chair

CCS Examples

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$$\begin{aligned} B_\parallel &= (B[out \mapsto com] \parallel B[in \mapsto com]) \setminus com \\ B &= in.\overline{out}.B \end{aligned}$$

"Interaction diagram":

$$\xrightarrow{in} \boxed{\xrightarrow{in} \boxed{\xrightarrow{in} B \xrightarrow{out}} \xrightarrow{com} \boxed{\xrightarrow{in} B \xrightarrow{out}} \xrightarrow{out}} \xrightarrow{out}$$

Outline of Lecture 1

Labelled Transition Systems

Goal: represent system behaviour by (infinite) graph

- nodes = system states
- edges = transitions between states

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Definition 1.4 (Labelled transition system)

A labelled transition system (LTS) is a triple $(S, Act, \longrightarrow)$ consisting of

- a set S of states
- a set Act of (action) labels
- a transition relation $\longrightarrow \subseteq S \times Act \times S$

For $(s, \alpha, s') \in \longrightarrow$ we write $s \xrightarrow{\alpha} s'$. An LTS is called finite if S is so.

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For $(s, \alpha, s') \in \longrightarrow$ we write $s \xrightarrow{\alpha} s'$. An LTS is called finite if S is so.

Remarks:

- Sometimes an initial state $s_0 \in S$ is distinguished ("$LTS(s_0)$").
- (Finite) LTSs correspond to (finite) automata without final states.

Semantics of CCS I

We define the assignment

$$\begin{aligned} \text{syntax} &\longrightarrow \text{semantics} \\ \text{process definition} &\longmapsto \text{LTS} \end{aligned}$$

by induction over the syntactic structure of process expressions. Here we employ derivation rules of the form

$$\text{(rule name)}\,\frac{\text{premise(s)}}{\text{conclusion}}$$

which are composed to form derivation trees (where axioms, i.e., rules without premises, correspond to leaves).

Semantics of CCS II

Reminder: $P ::= \mathrm{nil} \mid \alpha.P \mid P_1 + P_2 \mid P_1 \parallel P_2 \mid P \setminus L \mid P[f] \mid C$

Definition 1.5 (Semantics of CCS)

A process definition $(C_i = P_i \mid 1 \leq i \leq k)$ determines the LTS $(Prc, Act, \longrightarrow)$ whose transitions can be inferred from the following rules ($P, P', Q, Q' \in Prc$, $\alpha \in Act$, $\lambda \in A \cup \overline{A}$, $a \in A$):

$$\text{(Act)}\frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad \text{(Sum}_1\text{)}\frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} \qquad \text{(Sum}_2\text{)}\frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'}$$

$$\text{(Par}_1\text{)}\frac{P \xrightarrow{\alpha} P'}{P \parallel Q \xrightarrow{\alpha} P' \parallel Q} \qquad \text{(Par}_2\text{)}\frac{Q \xrightarrow{\alpha} Q'}{P \parallel Q \xrightarrow{\alpha} P \parallel Q'} \qquad \text{(Com)}\frac{P \xrightarrow{\lambda} P' \quad Q \xrightarrow{\overline{\lambda}} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'}$$

$$\text{(Res)}\frac{P \xrightarrow{\alpha} P' \quad (\alpha, \overline{\alpha} \notin L)}{P \setminus L \xrightarrow{\alpha} P' \setminus L} \qquad \text{(Rel)}\frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]} \qquad \text{(Call)}\frac{P \xrightarrow{\alpha} P' \quad (C = P)}{C \xrightarrow{\alpha} P'}$$

Semantics of CCS III

Example 1.6

(1) One-place buffer: $B = in.\overline{out}.B$

– First step:

$$
\text{(Call)}\;\dfrac{\text{(Act)}\;\dfrac{}{in.\overline{out}.B \xrightarrow{in} \overline{out}.B}}{B \xrightarrow{in} \overline{out}.B}
$$

Semantics of CCS III

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$$\text{(Call)}\ \dfrac{\text{(Act)}\ \dfrac{}{in.\overline{out}.B \xrightarrow{in} \overline{out}.B}}{B \xrightarrow{in} \overline{out}.B}$$

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$$\text{(Act)}\ \dfrac{}{\overline{out}.B \xrightarrow{\overline{out}} B}$$

Semantics of CCS III

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– Second step:

$$\text{(Act)}\ \dfrac{}{\overline{out}.B \xrightarrow{\overline{out}} B}$$

$\Rightarrow$ Complete LTS:

$$\begin{tikzcd} {} \arrow[r] & \boxed{B} \arrow[rr, bend left, "in"] & & \boxed{\overline{out}.B} \arrow[ll, bend left, "\overline{out}"] \end{tikzcd}$$

Semantics of CCS IV

Example 1.6 (continued)

(2) Sequential two-place buffer:
$$\begin{aligned} B_0 &= in.B_1 \\ B_1 &= \overline{out}.B_0 + in.B_2 \\ B_2 &= \overline{out}.B_1 \end{aligned}$$

– First step:

$$\dfrac{\dfrac{}{in.B_1 \xrightarrow{in} B_1}\,(\mathrm{Act})}{B_0 \xrightarrow{in} B_1}\,(\mathrm{Call})$$

Example 1.6 (continued)

(2) Sequential two-place buffer:
$$\begin{aligned} B_0 &= in.B_1 \\ B_1 &= \overline{out}.B_0 + in.B_2 \\ B_2 &= \overline{out}.B_1 \end{aligned}$$

– First step:

$$\text{(Call)}\,\dfrac{\text{(Act)}\,\dfrac{}{in.B_1 \xrightarrow{in} B_1}}{B_0 \xrightarrow{in} B_1}$$

– Second step:

$$\text{(Call)}\,\dfrac{\text{(Sum}_1\text{)}\,\dfrac{\text{(Act)}\,\dfrac{}{\overline{out}.B_0 \xrightarrow{\overline{out}} B_0}}{\overline{out}.B_0 + in.B_2 \xrightarrow{\overline{out}} B_0}}{B_1 \xrightarrow{\overline{out}} B_0}$$

Semantics of CCS IV

Example 1.6 (continued)

(2) Sequential two-place buffer:
$$B_0 = in.B_1$$
$$B_1 = \overline{out}.B_0 + in.B_2$$
$$B_2 = \overline{out}.B_1$$

- First step:

$$\text{(Call)}\dfrac{\text{(Act)}\dfrac{}{in.B_1 \xrightarrow{in} B_1}}{B_0 \xrightarrow{in} B_1}$$

- Second step:

$$\text{(Call)}\dfrac{\text{(Sum}_1\text{)}\dfrac{\text{(Act)}\dfrac{}{\overline{out}.B_0 \xrightarrow{\overline{out}} B_0}}{\overline{out}.B_0 + in.B_2 \xrightarrow{\overline{out}} B_0}}{B_1 \xrightarrow{\overline{out}} B_0}$$

- Like second step (with (Sum_2)): $B_1 \xrightarrow{in} B_2$
- Like first step: $B_2 \xrightarrow{\overline{out}} B_1$

Example 1.6 (continued)

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– Like second step (with (Sum_2)): $B_1 \xrightarrow{in} B_2$

– Like first step: $B_2 \xrightarrow{\overline{out}} B_1$

– Complete LTS:

$$\begin{tikzcd} {} \arrow[r] & \boxed{B_0} \arrow[r, bend left, "in"] & \boxed{B_1} \arrow[l, bend left, "\overline{out}"] \arrow[r, bend left, "in"] & \boxed{B_2} \arrow[l, bend left, "\overline{out}"] \end{tikzcd}$$

empty one entry full

Semantics of CCS V

Example 1.6 (continued)

(3) Parallel two-place buffer:

$$
\begin{aligned}
B_{\|} &= (B[f] \parallel B[g]) \setminus com \\
B &= in.\overline{out}.B
\end{aligned}
$$

$(f := [out \mapsto com], g := [in \mapsto com])$

First step:

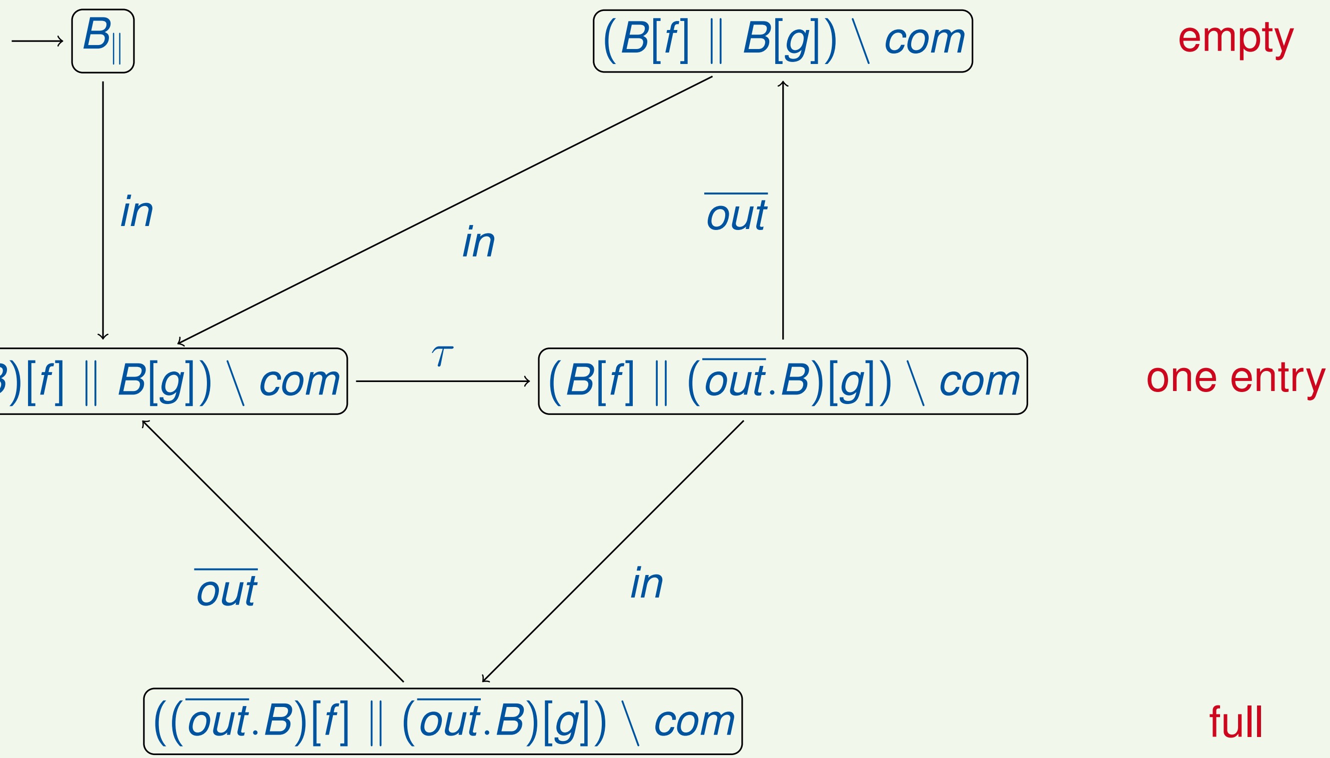
$$
\text{(Call)}\,\dfrac{\text{(Res)}\,\dfrac{\text{(Par}_1\text{)}\,\dfrac{\text{(Rel)}\,\dfrac{\text{(Call)}\,\dfrac{\text{(Act)}\,\dfrac{}{in.\overline{out}.B \xrightarrow{in} \overline{out}.B}}{B \xrightarrow{in} \overline{out}.B}}{B[f] \xrightarrow{in} (\overline{out}.B)[f]}}{B[f] \parallel B[g] \xrightarrow{in} (\overline{out}.B)[f] \parallel B[g]}}{(B[f] \parallel B[g]) \setminus com \xrightarrow{in} ((\overline{out}.B)[f] \parallel B[g]) \setminus com}}{B_{\|} \xrightarrow{in} ((\overline{out}.B)[f] \parallel B[g]) \setminus com}
$$

Semantics of CCS VI

Example 1.6 (continued)

(3) Parallel two-place buffer: $B_{\|} = (B[f] \parallel B[g]) \setminus com \quad (f := [out \mapsto com], g := [in \mapsto com])$
$B = in.\overline{out}.B$

Complete LTS:



Outline of Lecture 1

The Power of Recursive Definitions

So far: only finite state spaces – not necessarily true!

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$$C = up.(C \parallel down.\mathrm{nil})$$

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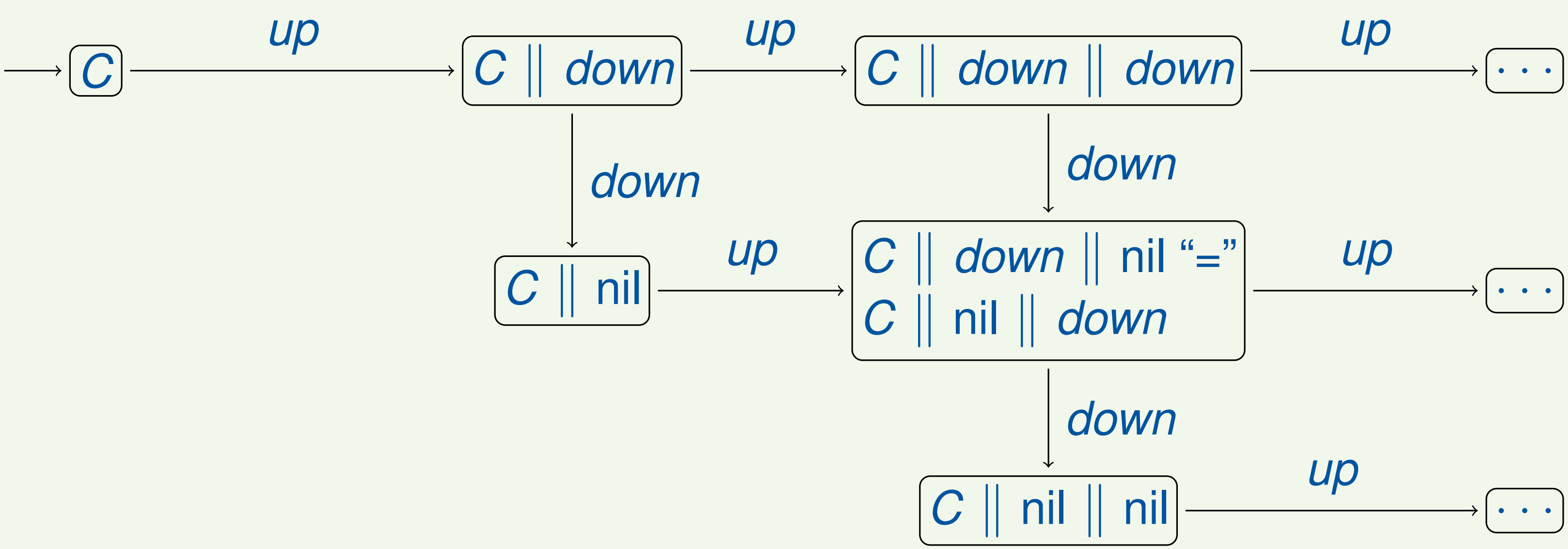
gives rise to infinite LTS (abbreviating $down := down.\mathrm{nil}$):



The Power of Recursive Definitions

So far: only finite state spaces – not necessarily true!

Example 1.7 (Counter)

$$C = up.(C \parallel down.\mathrm{nil})$$

gives rise to infinite LTS (abbreviating $down := down.\mathrm{nil}$):

$$\begin{tikzcd}
{} \arrow[r] & \boxed{C} \arrow[r, "up"] & \boxed{C \parallel down} \arrow[r, "up"] \arrow[d, "down"] & \boxed{C \parallel down \parallel down} \arrow[r, "up"] \arrow[d, "down"] & \boxed{\cdots} \\
& & \boxed{C \parallel \mathrm{nil}} \arrow[r, "up"] & \boxed{\begin{array}{l} C \parallel down \parallel \mathrm{nil} \text{ “=”} \\ C \parallel \mathrm{nil} \parallel down \end{array}} \arrow[r, "up"] \arrow[d, "down"] & \boxed{\cdots} \\
& & & \boxed{C \parallel \mathrm{nil} \parallel \mathrm{nil}} \arrow[r, "up"] & \boxed{\cdots}
\end{tikzcd}$$

Sequential “specification”:

$$C_0 = up.C_1$$
$$C_n = up.C_{n+1} + down.C_{n-1} \qquad (n > 0)$$

Outline of Lecture 1

Preliminaries

Concurrency and Interaction

A Closer Look at Memory Models

A Closer Look at Reactive Systems

Overview of the Course

The Approach

Syntax of CCS

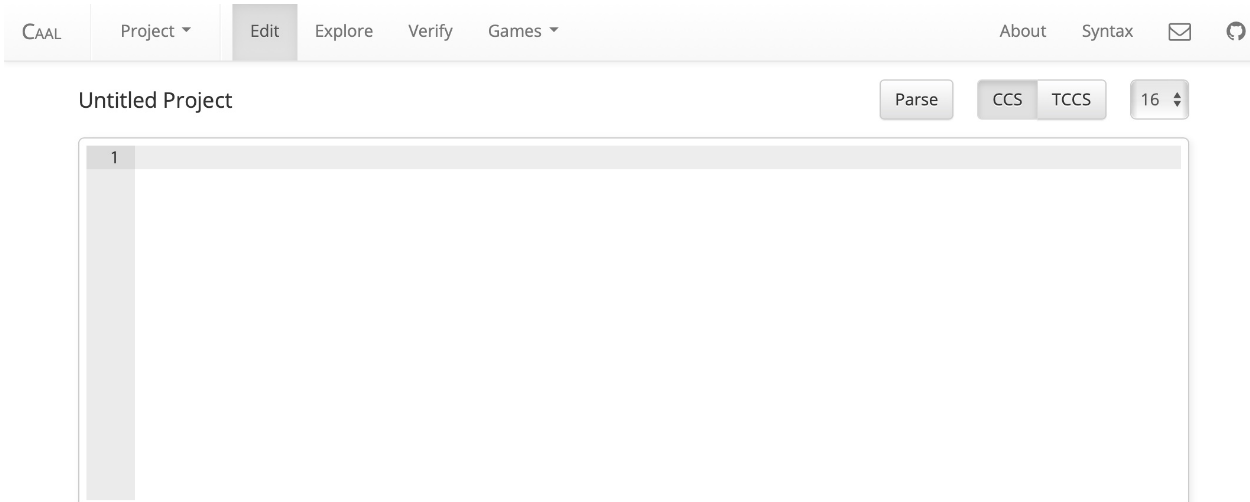
Intuitive Meaning and Examples

Formal Semantics of CCS

Infinite State Spaces

The CAAL Tool

The CAAL Tool



CAAL (Concurrency Workbench, Aalborg Edition; `https://caal.cs.aau.dk/`)

- Smart editor
- Visualisation of generated LTS
- Equivalence checking w.r.t. several bisimulation, simulation and trace equivalences
- Generation of distinguishing formulae for non-equivalent processes
- Model checking of recursive HML formulae
- (Bi)simulation and model checking games.