Name: ______________________

Student number: ______________________

Exercise 1 (Regular Languages). (13 points)

(i) Which of the following claims hold? $\boxed{3}$

(a) The language of all words over $\{a, b\}$ that contain an even number of a's and an odd number of b's is regular.

☐ Yes ☐ No

(b) The class of regular languages is closed under concatenation ($L_1 \cdot L_2$) and iteration (L^*).

☐ Yes ☐ No

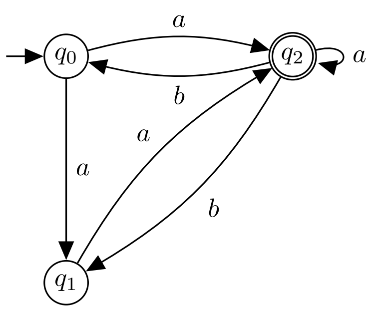
(c) Non-deterministic finite automata with ϵ-transitions (ϵ-NFA) are more expressive that deterministic finite (DFA), that is, they recognise a larger class of languages.

☐ Yes ☐ No

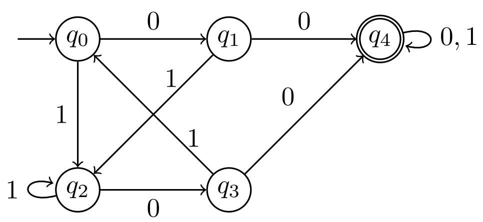
(ii) Give a regular expression that describes the language $\boxed{3}$

$$L := \{w \in \{0,1\}^* \mid w \text{ contains at least one } 0 \text{ and at least one } 1\}.$$

(iii) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA) $\mathfrak{A}$ into a deterministic finite automaton (DFA) $\mathfrak{A}'$. $\boxed{4}$



(iv) Minimise the following deterministic finite automaton (DFA) $\mathfrak{A}$. $\boxed{3}$
(**Reminder:** the minimal DFA is obtained by merging equivalent states. Here, two states are considered equivalent if they accept the same outgoing words.)



Exercise 2 (Context-Free Languages). (12 points)

(i) Which of the following claims hold? $\boxed{3}$

(a) The language $\{a^k b^l c^m \mid 1 \le k \le l \le m\}$ is context-free.

☐ Yes ☐ No

(b) The class of context-free languages is closed under concatenation ($L_1 \cdot L_2$) and iteration (L^*).

☐ Yes ☐ No

(c) The word problem for context-free languages is decidable, that is, given a context-free grammar $G = \langle N, \Sigma, P, S \rangle$ and a word $w \in \Sigma^*$, it is decidable whether $w \in L(G)$ or not.

☐ Yes ☐ No

(ii) Give a context-free grammar G which generates the language $\boxed{3}$

$$L := \{a^k b^l c^{k+l} \mid k, l \ge 1\}.$$

(iii) Give a derivation of the word $aabccc \in L$ from the start symbol of G. $\boxed{1}$

(iv) Let G' be the following context-free grammar: $\boxed{5}$

$$\begin{array}{lcl} S & \to & AB \mid BC \\ A & \to & BA \mid a \\ B & \to & CC \mid b \\ C & \to & AB \mid a \end{array}$$

and let $w := abaab$. Employ the CYK-Algorithm to show that $w \in L(G')$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \le i \le j \le 5)$$

where $w[i,j] := a_i \dots a_j$ for $w = a_1 a_2 a_3 a_4 a_5$.

$i\backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	