

**Exercise 1** (Regular Languages).

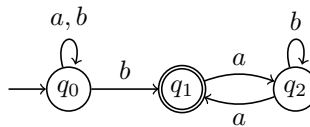
(12 points)

(i) Which of the following claims hold? 3

(a) Deterministic finite automata (DFA) are strictly less expressive (i.e., describe less languages) than regular expressions.

☐ Yes    ☐ No

(b) The class of regular languages is closed under union, intersection and complement.

☐ Yes    ☐ No(c) The language  $\{a^k b^l \mid k, l \in \mathbb{N}\}$  is context-free, but not regular.☐ Yes    ☐ No(ii) Construct a regular expression that describes the language of all words over  $\{0, 1\}$  that contain at least one pair of consecutive 0's and at least one pair of consecutive 1's. 3(iii) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA)  $\mathfrak{A}$  into a deterministic finite automaton (DFA)  $\mathfrak{A}'$ . 4(iv) Is  $\mathfrak{A}'$  minimal? Please justify your answer in the following way: 3

“yes”: give a distinguishing word for each pair of states;

“no”: give two equivalent states and explain why they are equivalent.

**Exercise 2** (Context-Free Languages).

(13 points)

(i) Which of the following claims hold?

3

(a) The class of context-free languages is closed under union, intersection and complement.

☐ Yes    ☐ No(b) The language  $\{a^k b^l c^m \mid k \geq l \geq m \geq 1\}$  is context-free.☐ Yes    ☐ No(c) Given a context-free grammar  $G$ , it is decidable whether  $L(G) = \emptyset$  or not.☐ Yes    ☐ No(ii) Give a context-free grammar  $G$  which generates the language

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$$L := \{a^k b^l c^{k+l} \mid k, l \geq 1\}.$$

(iii) Give a derivation of the word  $aabccc \in L$  from the start symbol of  $G$ .

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(iv) Let  $G'$  be the following context-free grammar:

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$$S \rightarrow AB \mid BC$$

$$A \rightarrow BA \mid a$$

$$B \rightarrow CC \mid b$$

$$C \rightarrow AB \mid a$$

and let  $w := aabab$ . Employ the CYK-Algorithm to show that  $w \in L(G')$ . Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \quad (1 \leq i \leq j \leq 5)$$

where  $w[i,j] := a_i \dots a_j$  for  $w = a_1 a_2 a_3 a_4 a_5$ .

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	