Exercise 1 (Regular Languages). (13 points)

(i) Give a finite automaton (DFA, NFA or ϵ-NFA) that recognises the following language: 4

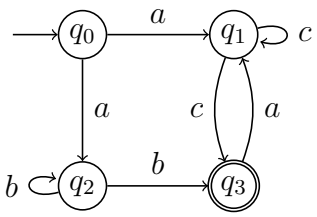
$L := \{w \in \{0, \ldots, 9, .\}^* \mid w$ is a decimal number with a non-empty integer part without leading zeros, possibly with a non-empty fractional part$\}$

Examples: 0, 42, 42.0, 42.00, 42.012

Counterexamples: 00, 01.2, 1., .1

(ii) Show that the automaton constructed in (i) accepts the word $w := 42.0$. 1

(iii) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA) $\mathfrak{A}$ into a deterministic finite automaton (DFA) $\mathfrak{A}'$. 5



(iv) Is $\mathfrak{A}'$ minimal? Please justify your answer in the following way: 3

"yes": give a distinguishing word for each pair of states;

"no": give two equivalent states and explain why they are equivalent.

Exercise 2 (Context-Free Languages). (12 points)

(i) Give a context-free grammar G which generates the language 5

$$L := \{a^k b^l c^m \mid k, l, m \geq 1 \text{ and } (k = l \text{ or } l = m)\}.$$

(ii) Give a derivation of the word $aabbcc \in L$ from the start symbol of G. 1

(iii) Let G' be the following context-free grammar: 5

$$\begin{aligned} S &\to SA \mid a \\ A &\to BS \\ B &\to BB \mid BS \mid b \mid c \end{aligned}$$

and let $w := abcaa$. Employ the CYK-Algorithm to show that $w \in L(G')$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \leq i \leq j \leq 5)$$

where $w[i,j] := a_i \dots a_j$ for $w = a_1 a_2 a_3 a_4 a_5$.

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	