Exercise 1 (Regular Languages). (**??** points)

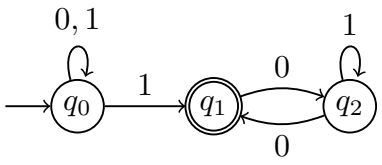
(i) Give a regular expression that describes the language [3]

$$L := \{w \in \{0,1\}^* \mid w \text{ contains at least one } 0 \text{ and at least one } 1\}.$$

(ii) Give a *deterministic* finite automaton that recognises the *complement* of the language L as specified in (i). [2]

(iii) Show that the automaton constructed in (ii) accepts the word $w := 000$. [1]

(iv) Apply the powerset construction to turn the following nondeterministic finite automaton $\mathfrak{A}$ into a deterministic finite automaton $\mathfrak{A}'$. [4]



(v) Is $\mathfrak{A}'$ minimal? Please justify your answer in the following way: [3]

"yes": give a distinguishing word for each pair of states;

"no": give two equivalent states and explain why they are equivalent.

Exercise 2 (Context-Free Languages). (**??** points)

(i) Give a context-free grammar G which generates the language 5

$$L := \{a^{2k}b^{2l}c^{k+l} \mid k, l \geq 0\}.$$

(ii) Give a derivation of the word $aabbbbccc \in L$ from the start symbol of G. 2

(iii) Let G' be the following context-free grammar: 5

$$\begin{aligned} S &\to AB \mid BC \\ A &\to BA \mid a \\ B &\to CC \mid b \\ C &\to AB \mid a \end{aligned}$$

and let $w := baaba$. Employ the CYK-Algorithm to show that $w \in L(G')$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \leq i \leq j \leq 5)$$

where $w[i,j] := a_i \dots a_j$ for $w = a_1a_2a_3a_4a_5$.

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	