Exercise 1 (Regular Languages). (**??** points)

(i) Give a regular expression that describes the following language: 3

$$L := \{w \in \{a, b, c\}^* \mid \text{each occurrence of } a \text{ in } w \text{ is directly followed by } b\}.$$

(ii) Give a deterministic finite automaton (DFA) that recognises the following language: 5

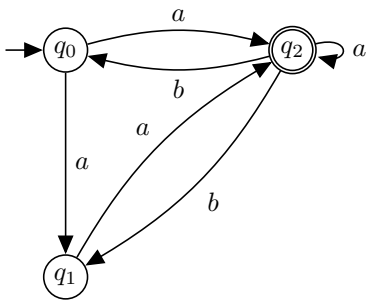
$$L := \{w \in \{a, b\}^* \mid |w|_a \geq 2, |w|_b \text{ is odd}\}$$

where $|w|_x$ refers to the number of occurrences of symbol x in word w.

(**Hint:** a possible solution employs states of the form q_{ij} where $i \in \{0, 1, 2\}$ counts a's while $j \in \{0, 1\}$ remembers even/odd numbers of b's.)

(iii) Show that the automaton constructed in (ii) accepts the word $w := abaa$. 1

(iv) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA) $\mathfrak{A}$ into a deterministic finite automaton (DFA) $\mathfrak{A}'$. 4



Exercise 2 (Context-Free Languages). (**??** points)

(i) Answer the following questions: 3

(a) The language $\{a^n b^n c^n \mid n \in\}$ is context-free.
☐ Yes ☐ No

(b) The class of context-free languages is closed under all of the following operations *except for*
☐ Union ☐ Complementation ☐ Concatenation

(c) If G is a context-free grammar, there is an algorithm for deciding whether $L(G)$ is empty or not.
☐ Yes ☐ No

(ii) Give a context-free grammar G which generates the language 3

$$L := \{a^n b^{2n} c^m \mid m, n \geq 1\}.$$

(iii) Give a derivation of the word $abbccc \in L$ from the start symbol of G. 1

(iv) Let G' be the following context-free grammar: 5

$$\begin{aligned} S &\to AB \mid BC \\ A &\to BA \mid a \\ B &\to CC \mid b \\ C &\to AB \mid a \end{aligned}$$

and let $w := aaaaa$. Employ the CYK-Algorithm to show that $w \in L(G')$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \leq i \leq j \leq 5)$$

where $w[i,j] := a_i \dots a_j$ for $w = a_1 a_2 a_3 a_4 a_5$.

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	