Name: ____________________

Student number: ____________________

Exercise 1 (Regular Languages). (13 points)

(i) Which of the following claims hold? [3]

(a) The language of all words over $\{a, b\}$ that contain exactly as many a's as b's is regular.

☐ Yes ☐ No

(b) The class of regular languages is closed under union, intersection and complement.

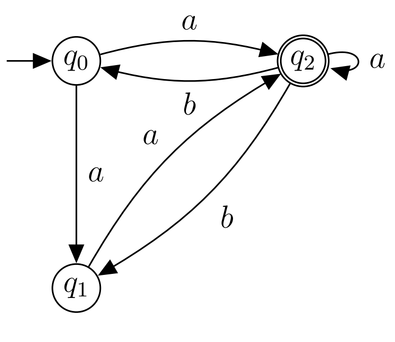
☐ Yes ☐ No

(c) The emptiness problem for regular languages is decidable, that is, given a regular expression α it is decidable whether $L(\alpha) = \emptyset$ or not.

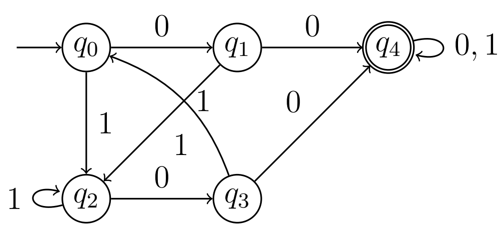
☐ Yes ☐ No

(ii) Construct a regular expression that describes the language of all words over $\{0, 1\}$ that consist only of 0's or that contain at least two 1's. [3]

(iii) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA) $\mathfrak{A}$ into a deterministic finite automaton (DFA) $\mathfrak{A}'$. [4]



(iv) Minimise the following deterministic finite automaton (DFA) $\mathfrak{A}$. [3]
(**Reminder:** the minimal DFA is obtained by merging equivalent states. Here, two states are considered equivalent if they accept the same outgoing words.)



Exercise 2 (Context-Free Languages). (12 points)

(i) Which of the following claims hold? 3

(a) The language $\{a^k b^l c^m \mid k, l, m \geq 0, k \leq l \text{ or } l \leq m\}$ is context-free.

☐ Yes ☐ No

(b) The class of context-free languages is closed under union, intersection and complement.

☐ Yes ☐ No

(c) The emptiness problem for context-free languages is decidable, that is, given a context-free grammar G, it is decidable whether $L(G) = \emptyset$ or not.

☐ Yes ☐ No

(ii) Give a context-free grammar G which generates the language 3

$$L := \{a^n b^m c^{2n} \mid m, n \geq 1\}.$$

(iii) Give a derivation of the word $aabbcccc \in L$ from the start symbol of G. 1

(iv) Let G' be the following context-free grammar: 5

$$\begin{array}{lcl} S & \rightarrow & SA \mid a \\ A & \rightarrow & BS \\ B & \rightarrow & BB \mid BS \mid b \mid c \end{array}$$

and let $w := abcaa$. Employ the CYK-Algorithm to show that $w \in L(G')$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \leq i \leq j \leq 5)$$

where $w[i,j] := a_i \dots a_j$ for $w = a_1 a_2 a_3 a_4 a_5$.

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	