

Exercise 1 (Regular Languages).

(?? points)

(i) Which of the following claims hold?

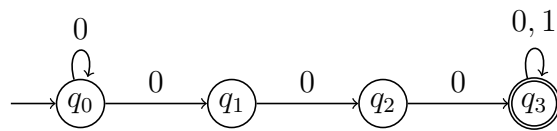
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(a) The language $L = \{a^k b^l \mid 1 \leq k \leq l\}$ is regular.☐ Yes ☐ No(b) The class of regular languages is closed under concatenation and iteration (L^*).☐ Yes ☐ No(c) Given two regular expressions α and β , it is decidable whether $L(\alpha) = L(\beta)$ or not.☐ Yes ☐ No(ii) Give a regular expression that describes the language of all words over $\{a, b\}$ in which each occurrence of a is followed by at least two b s.

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(iii) Apply the powerset construction to turn the following nondeterministic finite automaton (NFA) \mathfrak{A} into a deterministic finite automaton (DFA) \mathfrak{A}' .

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(iv) Is \mathfrak{A}' minimal? Please justify your answer in the following way:

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“yes”: give a distinguishing word for each pair of states;

“no”: give two equivalent states and explain why they are equivalent.

Exercise 2 (Context-Free Languages).

(?? points)

(i) Which of the following claims hold?

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(a) The language $L = \{a^k b^l c^m \mid 1 \leq k \leq l \leq m\}$ is context free.☐ Yes ☐ No(b) The class of context-free languages is closed under concatenation and iteration (L^*).☐ Yes ☐ No(c) Given a context-free grammars G_1 and G_2 , it is decidable whether $L(G_1) = L(G_2)$ or not.☐ Yes ☐ No(ii) Give a context-free grammar G which generates the language

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$$L := \{a^k b^{2k} c^l \mid k, l \geq 1\}.$$

(iii) Give a derivation of the word $abbcc \in L$ from the start symbol of G .

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(iv) Let G' be the following context-free grammar:

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$$S \rightarrow AB \mid BC$$

$$A \rightarrow BA \mid a$$

$$B \rightarrow CC \mid b$$

$$C \rightarrow AB \mid a$$

and let $w := ababa$. Employ the CYK-Algorithm to show that $w \in L(G')$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \quad (1 \leq i \leq j \leq 5)$$

where $w[i,j] := a_i \dots a_j$ for $w = a_1 a_2 a_3 a_4 a_5$.

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	

(v) Give a derivation tree of the word $ababa \in L(G')$.

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