Concurrency Theory

Lecture 16: Interleaving Semantics of Petri Nets

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http://moves.rwth-aachen.de/teaching/ws-19-20/ct

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Overview

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Carl Adam Petri (1926-2010)

The original work[1] does not contain a single (graphical) Petri net!

[1] Petri's PhD dissertation, 1962.

Semantics: executions and traces

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States: current configurations of the machine

- One or more initial states
- Possibly some distinguished final states

Transitions: moves between configurations

Lambda calculus	$(\lambda x.xx)(\lambda y.y)$	$\longrightarrow$	$(\lambda y.y)(\lambda z.z)$
Turing machine	$0010q_1011$	$\longrightarrow$	$001q_201011$
Finite automaton	q_1	$\xrightarrow{a}$	q_2
Pushdown automaton	$(q_1, XYYZ)$	$\xrightarrow{a}$	$(q_2, XYXYYZ)$

Executions: alternating sequences of states and transitions

Petri's question

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C.A. Petri points out a discrepancy between how Theoretical Physics and Theoretical Computer Science described systems in 1962:

Theoretical Physics describes systems as a collection of interacting particles (subsystems), without a notion of global clock or simultaneity

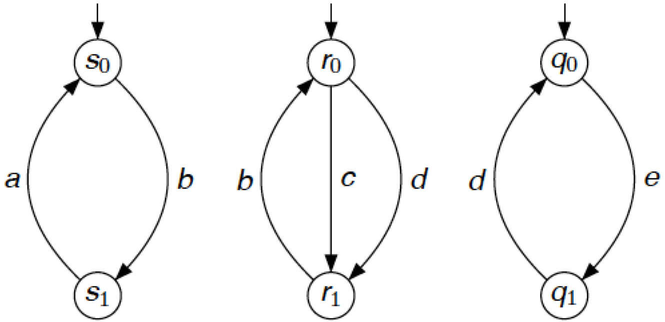
Theoretical Computer Science describes systems as sequential virtual machines going through a temporally ordered sequence of global states

Petri's question:

Which kind of abstract machine should be used to describe the physical implementation of a Turing machine?

Petri net

A graphical representation of interacting finite automata:



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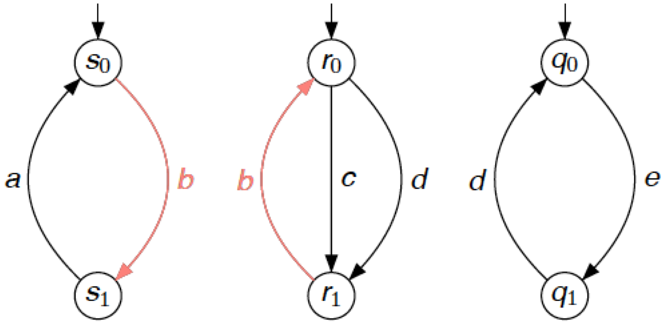

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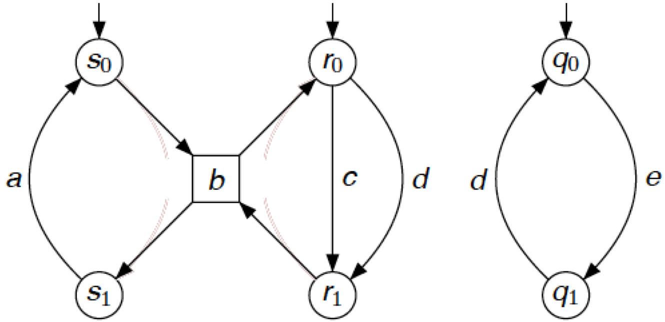

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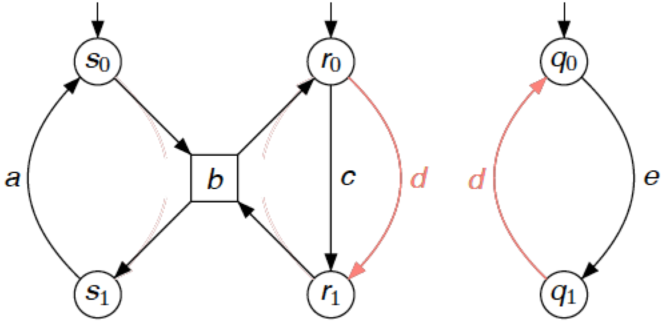

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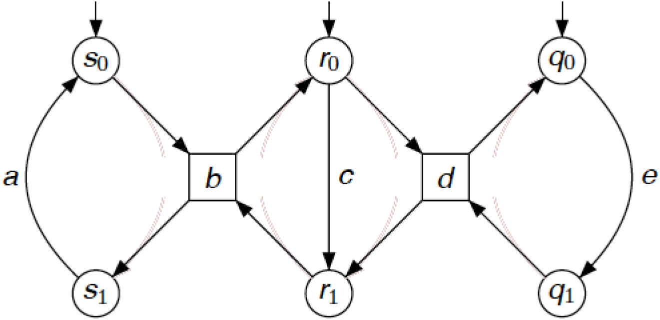

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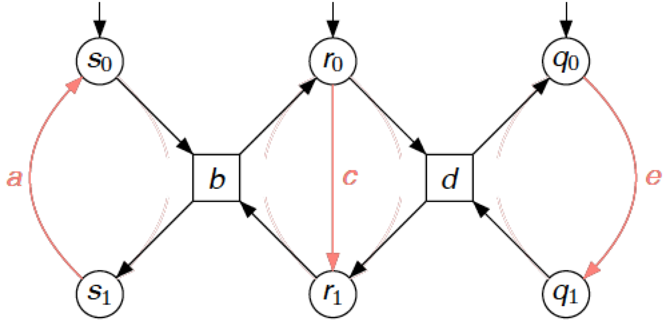

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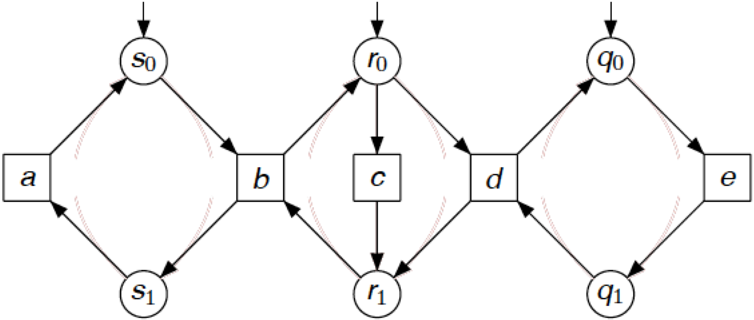

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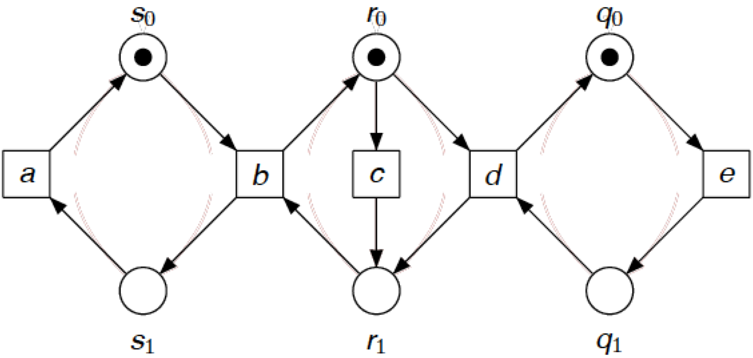

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A Petri net is a structure with two kinds of elements: places and transitions. They are connected by arcs.

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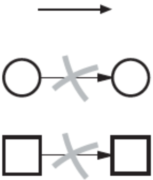

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Nets

[2] F is also called the flow relation.

Nets

Net

A Petri net N is a triple (P, T, F) where:

- P is the finite set of places
- T is the finite set of transitions with $P \cap T = \varnothing$
- $F \subseteq (P \times T) \cup (T \times P)$ are the arcs[2]

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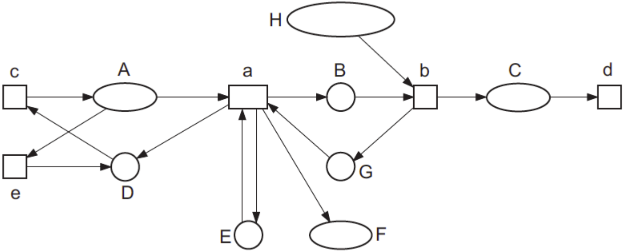

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The pre- and post-sets

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Let node $x \in P \cup T$.
The pre-set of x is defined by: ${}^\bullet x = \{ y \mid (y, x) \in F \}$.
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Two nodes $x, y \in N$ form a loop if $x \in {}^\bullet y$ and $y \in {}^\bullet x$.

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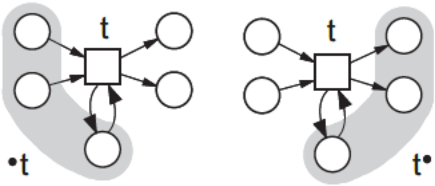

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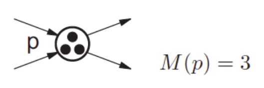

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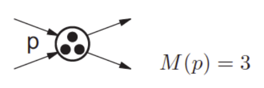

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Note: a marking is a multiset.

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$M(p) = 3$

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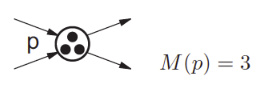

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Note: a marking is a multiset. It defines a distribution of tokens across places. Tokens are depicted as black dots.

Transition firing

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Enabling and occurrence of a transition

Let (P, T, F, M) be an elementary system net. Marking M enables a transition t if $M(p) \geqslant 1$ for each place $p \in {}^\bullet t$.

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Transition t can occur in marking M if t is enabled at M. Its occurrence leads to marking M', denoted $M \xrightarrow{t} M'$, defined for place $p \in P$ by:

$$M'(p) \ = \ M(p) - F(p, t) + F(t, p).$$

where we represent F by its characteristic function.

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Transition t is enabled whenever every $p \in^\bullet t$ holds at least one token. On t's occurrence, one token is removed from each place in $^\bullet t$, and one token is put in each place in $t^\bullet$:

$$M'(p) \;=\; \begin{cases} M(p) - 1 & \text{if } p \in {}^\bullet t \text{ and } p \notin t^\bullet \\ M(p) + 1 & \text{if } p \in t^\bullet \text{ and } p \notin^\bullet t \\ M(p) & \text{otherwise} \end{cases}$$

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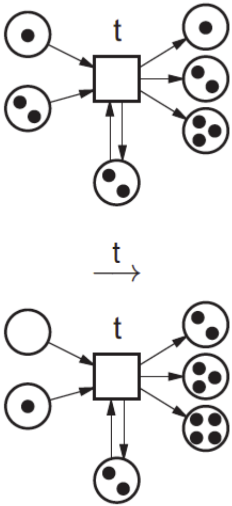

Overview

The interleaving semantics of Petri nets

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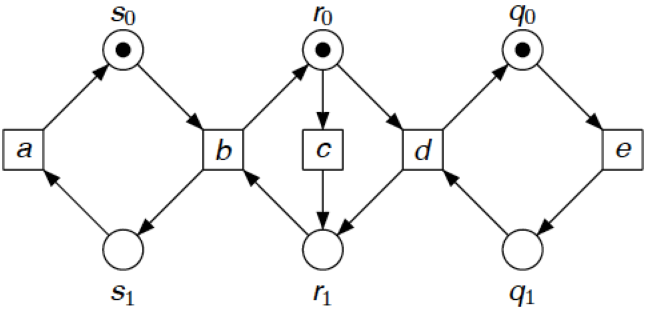
An execution semantics

State: marking (distribution of tokens over the net)

Transitions: $M \xrightarrow{t} M'$

Sequential runs: $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} \ldots$

The interleaving semantics of Petri nets



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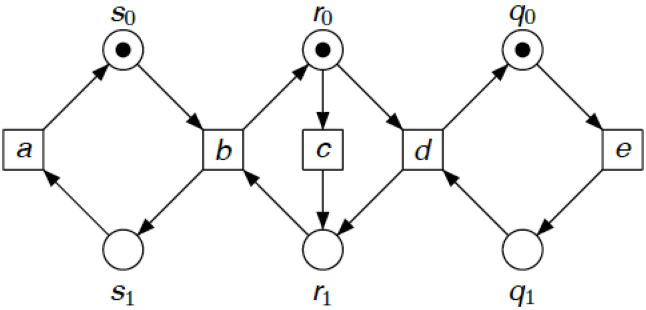

$$\begin{array}{c} s_1 \\ r_1 \\ q_1 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

As the marking for s_0 is the complement of s_1, the marking for s_0 is omitted. The same applies to the places r_0 and q_0.

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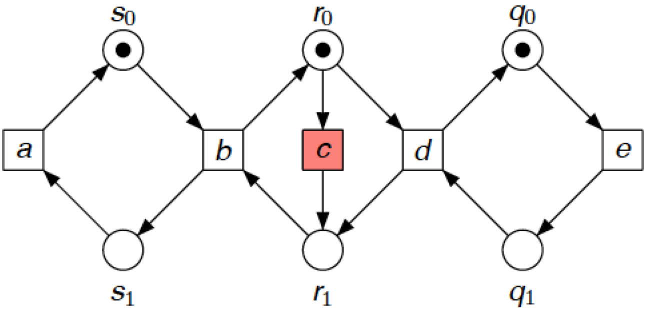

$$\begin{array}{c} s_1 \\ r_1 \\ q_1 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c}$$

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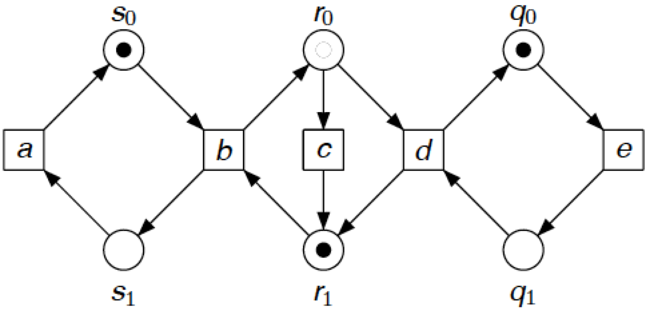

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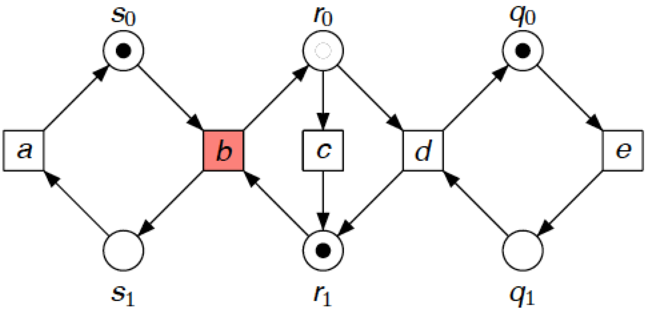

$$\begin{array}{c} s_1 \\ r_1 \\ q_1 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b}$$

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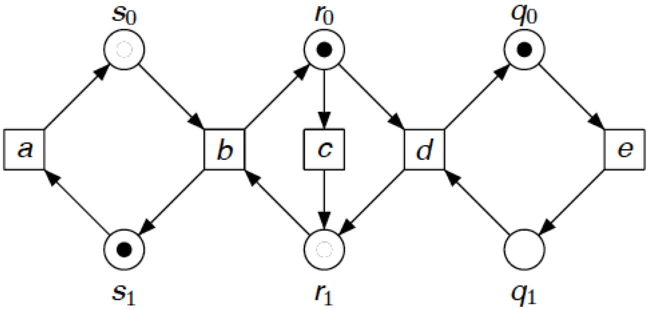

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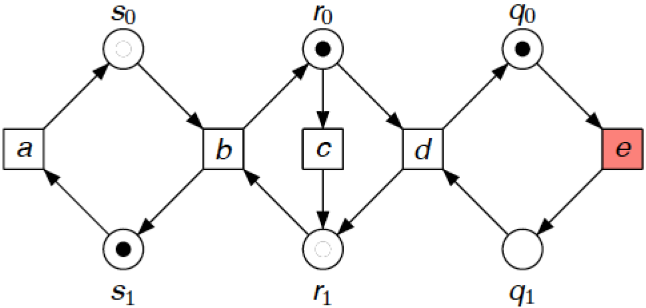

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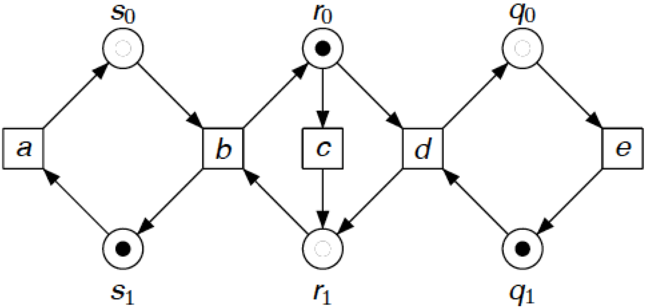

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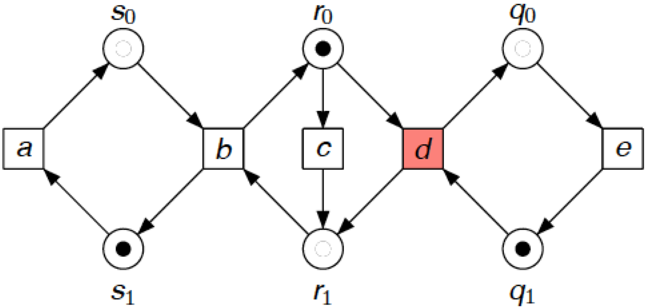

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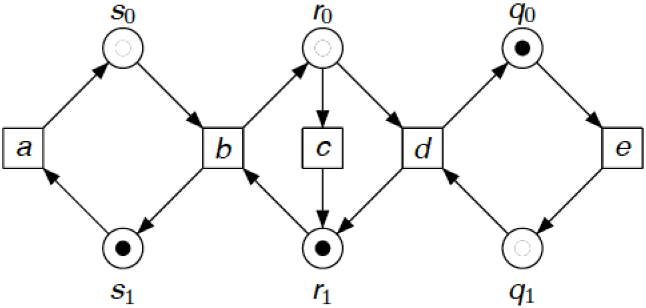

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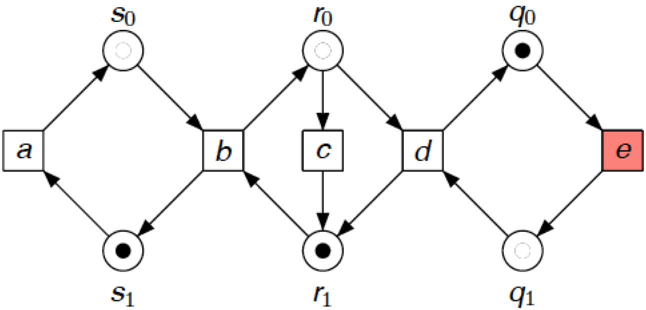

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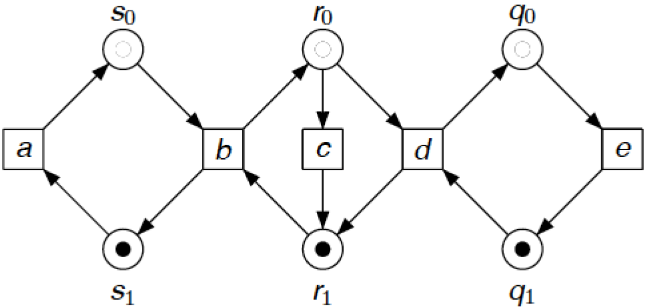

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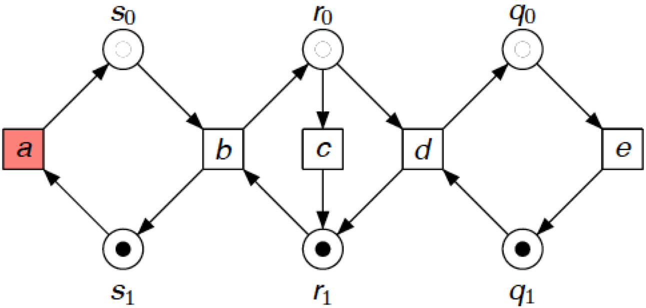

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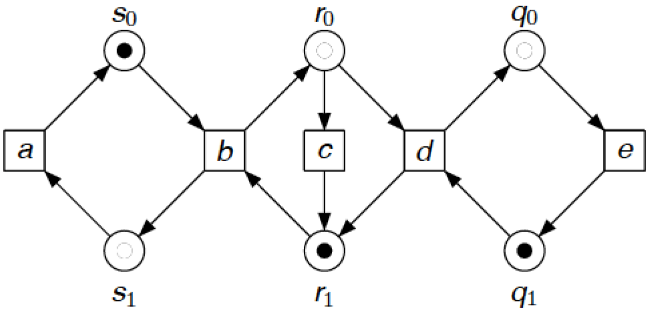

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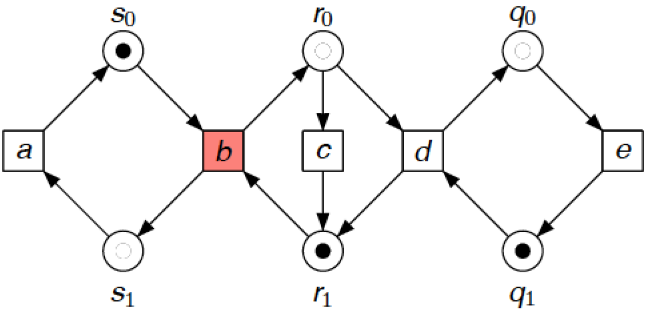
The interleaving semantics of Petri nets



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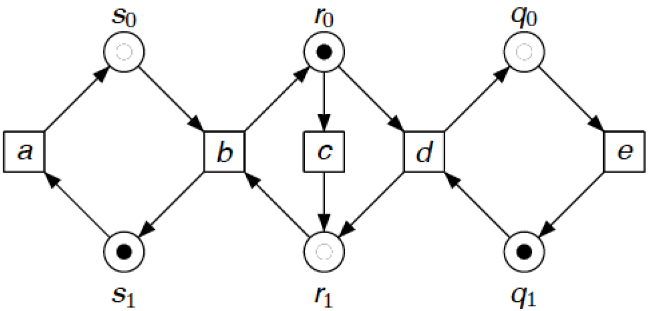

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Reachable markings

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Step sequence

A sequence of transitions $\sigma = t_1 \, t_2 \ldots t_n$ is an step sequence if there exist markings M_1 through M_n such that:

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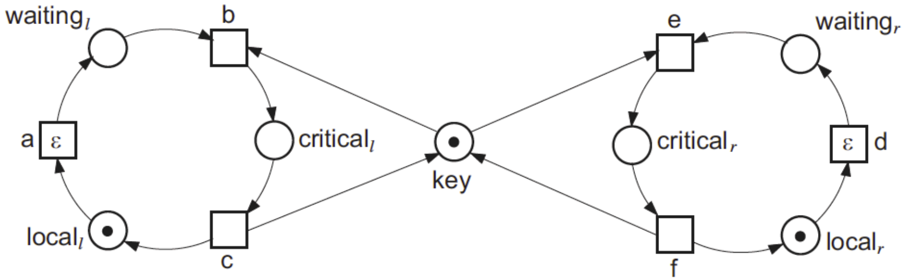
M is a reachable marking if there exists a step sequence σ with $M_0 \xrightarrow{\sigma} M$.

Mutual exclusion

Two processes cycling through the states local, waiting and critical.



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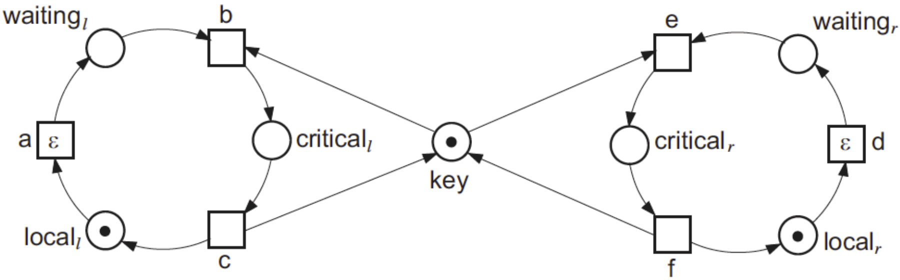
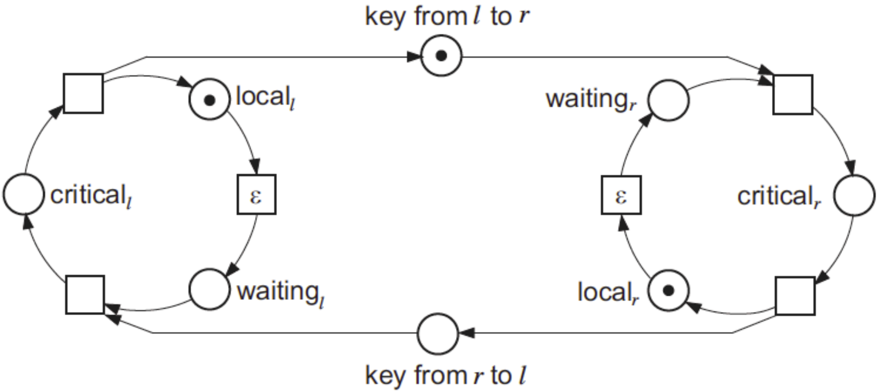


Between transitions b and e a conflict can arise infinitely often. No strategy has been modeled to solve this conflict.

Mutual exclusion

A strategy where processes are acquired access in an alternating fashion:



One-bounded elementary system nets

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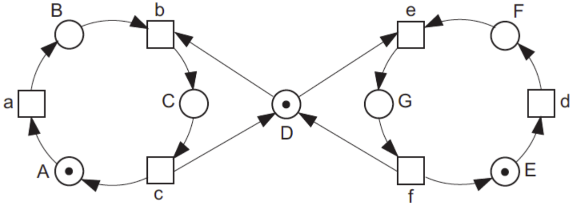
Markings of 1-bounded elementary net systems can be described as a string of marked places, e.g., ADE. Two steps begin with this marking: $ADE \xrightarrow{a} BDE$ and $ADE \xrightarrow{d} ADF$.



Overview

Sequential runs

Sequential run

Let N be an elementary net system. A sequential run of N is a sequence

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \cdots$$

of steps of N starting with the initial marking M_0. A run can be finite or infinite. A finite run $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_1} \cdot \xrightarrow{t_n} M_n$ is complete if M_n does not enable any transition.

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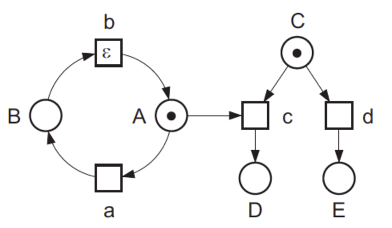

A sample complete run is:

$$AC \xrightarrow{a} BC \xrightarrow{b} AC \xrightarrow{c} D$$

A sample incomplete run is:

$$AC \xrightarrow{d} AE \xrightarrow{a} BE$$

Marking graph

The marking graph of N has as nodes the reachable markings of N and as edges the reachable steps of N.

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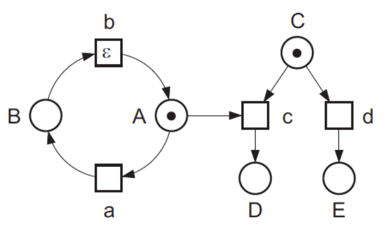

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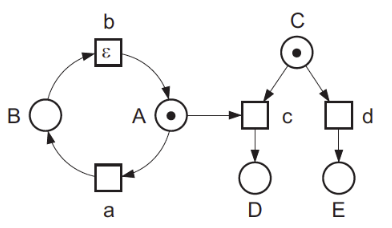

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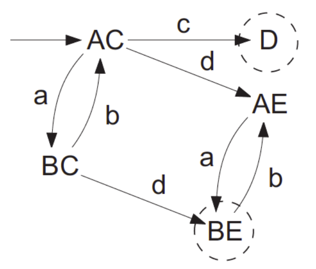

Its marking graph

Overview

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- The marking graph is the interleaving semantics of a net.