Concurrency Theory WS 2019/2020

— Exercise 3 —

Hand in until October 31th before the exercise class.

Exercise 1 (20 Points)

(a) Suppose it holds for some arbitrary but fixed HML formula F that $[\![F^c]\!] = S \setminus [\![F]\!]$ for every LTS $(S,\ Act,\ \rightarrow)$. Prove that $[\![\ ([\alpha]F)^c\ ]\!] = S \setminus [\![[\alpha]F]\!]$.

(b) Prove or disprove: $(F^c)^c$ and F is semantically equivalent for every HMF formula F.

(c) Prove or disprove: $(F^c)^c$ and F is syntactically equivalent for every HMF formula F.

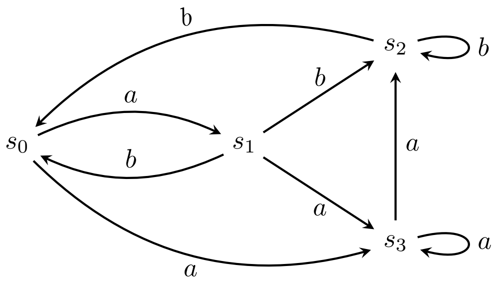
Exercise 2 (40 Points)

Semantics of HML with recursion Let $(S, Act, \rightarrow)$ be an LTS, the semantics of a HML formula $F \in HMF_X$ is defined in lecture 5 as a function

$$[\![F]\!] : 2^S \rightarrow 2^S.$$

1) Show that $[\![F]\!]$ is monotonic over the complete lattice $(2^S, \subseteq)$.

2) An LTS $(S, Act, \rightarrow)$ is given as follows:



Consider following questions:

- compute $[\![\langle b\rangle[a]\mathtt{tt} \wedge \langle b\rangle[b]X]\!](\{s_0, s_2\})$.
- compute the set of processes satisfying following property

$$X \overset{\min}{=} \langle b\rangle\langle a\rangle\mathtt{tt} \vee \langle b\rangle[b]X$$

- compute the sets of processes satisfying following equational systems

$$\begin{aligned} A &\overset{\max}{=} [a]B \\ B &\overset{\max}{=} \langle a\rangle C \wedge [b]B \\ C &\overset{\max}{=} [b]B \end{aligned}$$

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Exercise 3 (40 Points)

Complete Lattices Let $(L, \sqsubseteq)$ and $(M, \sqsubseteq)$ be complete lattices, and M finite.

1. $\gamma\colon M \to L$ is monotone,
2. $\gamma(\top) = \top$, and
3. for each $m_1, m_2 \in M$ with $m_1 \not\sqsubseteq m_2$ and $m_2 \not\sqsubseteq m_1$ it holds that $\gamma(\bigsqcap\{m_1, m_2\}) = \bigsqcap\{\gamma(m_1), \gamma(m_2)\}$

For each of the following, give an M, L and γ s.t.:

- (i) and (ii) hold, but (iii) not.
- (i) and (iii) hold, but (ii) not.
- (ii) and (iii) hold, but (i) not.

Show that (i)—(iii) are jointly equivalent to $\gamma\colon M \to L$ satisfying

$$\gamma\left(\bigsqcap Y\right) = \bigsqcap\{\gamma(l) \mid l \in Y\}$$

for each $Y \subseteq M$.