Modeling and Verification of Probabilistic Systems

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http://moves.rwth-aachen.de/teaching/ws-1516/movep15/

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Overview

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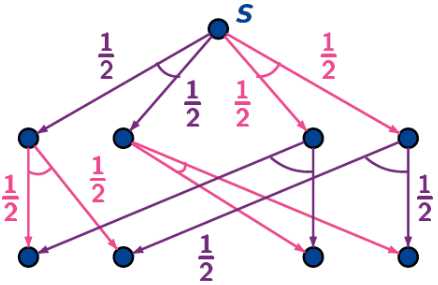

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Beware

Nondeterminism is not the same as a uniform distribution!

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Randomized distributed algorithms are typically appropriately modeled by MDPs, as probabilities affect just a small part of the algorithm and nondeterminism is used to model concurrency between processes by means of interleaving.

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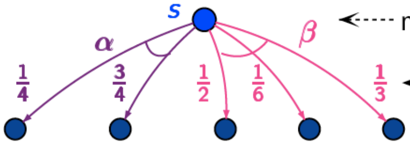

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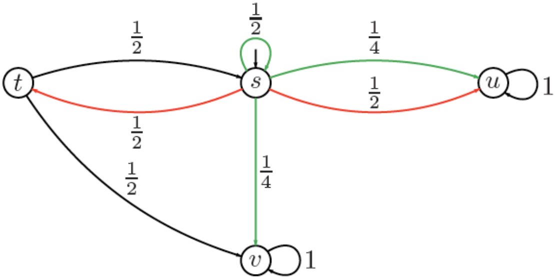

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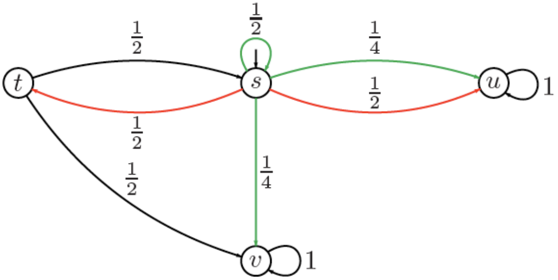

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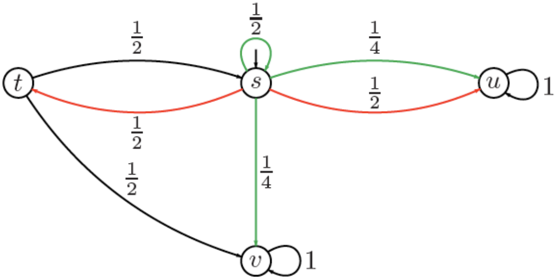

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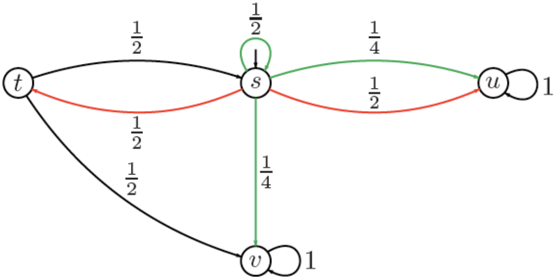

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- $Act(t) = \{ \alpha \}$ with $\mathbf{P}(t, \alpha, s) = \mathbf{P}(t, \alpha, u) = \frac{1}{2}$ and 0 otherwise

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- **2** concurrent processes $\mathcal{P}_1$, $\mathcal{P}_2$ with **3** phases:

  n_i noncritical actions of process $\mathcal{P}_i$
  w_i waiting phase of process $\mathcal{P}_i$
  c_i critical section of process $\mathcal{P}_i$

- competition of both processes are waiting
- resolved by a randomized arbiter who tosses a coin

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- competition if both processes are waiting
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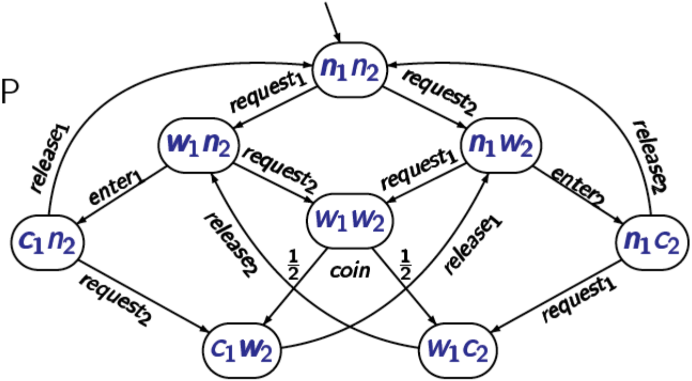

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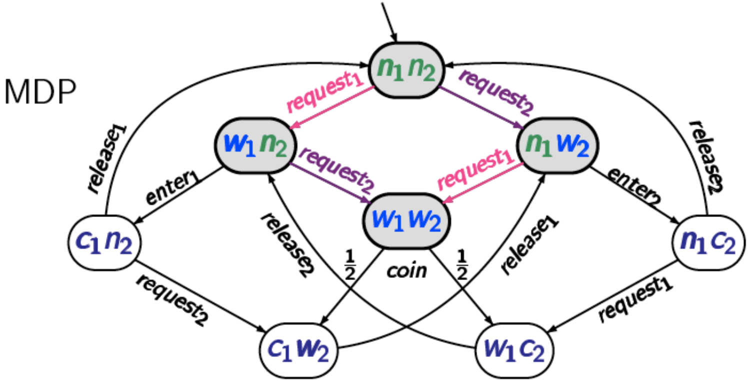

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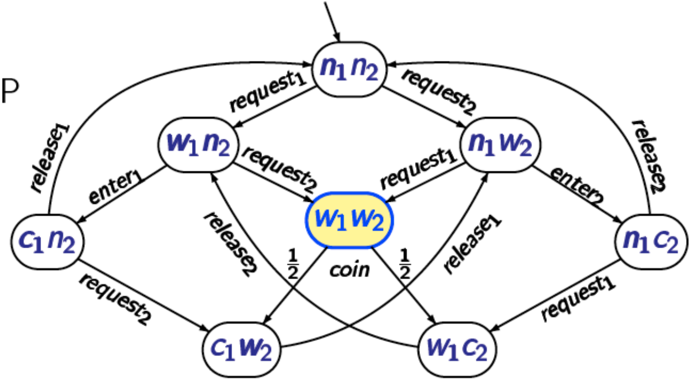

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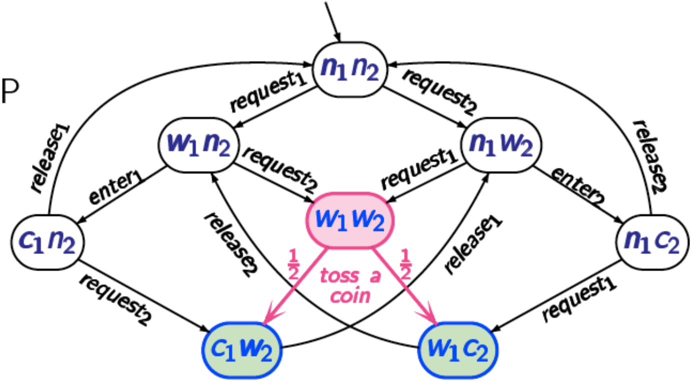

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5. Continue with step 2.

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State graph

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$$\pi \;=\; s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots,$$

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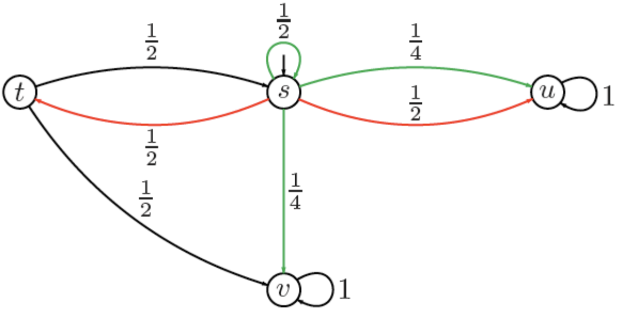

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- Reasoning about probabilities of sets of paths of an MDP relies on the resolution of nondeterminism. This resolution is performed by a policy.[1] A policy chooses in any state s one of the actions $\alpha \in \mathit{Act}(s)$.

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Overview

Policies

Policy

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Induced Markov chain

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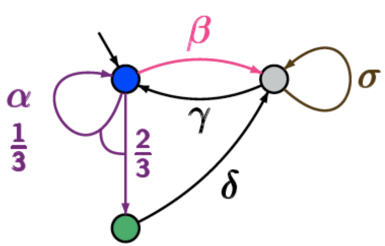
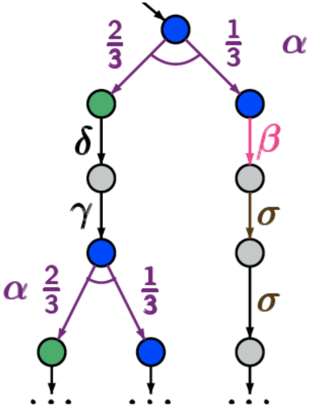


MDP



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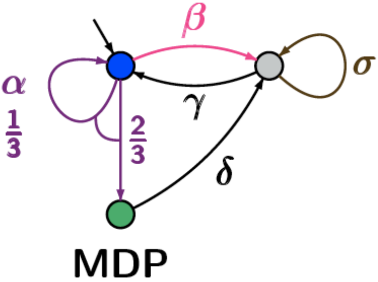

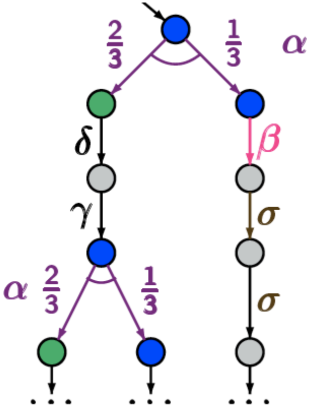

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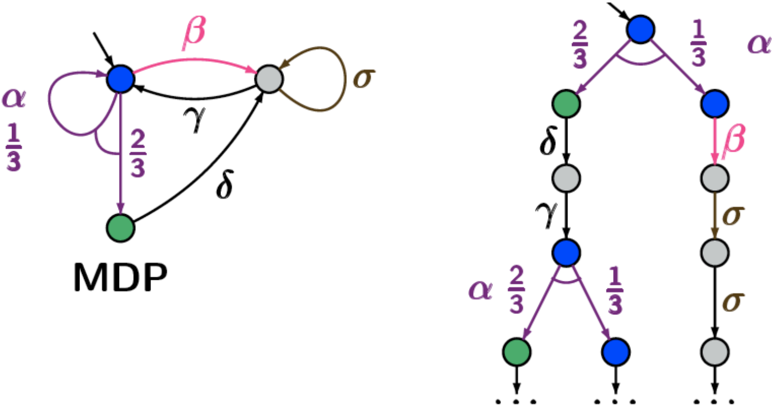

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Example MDP

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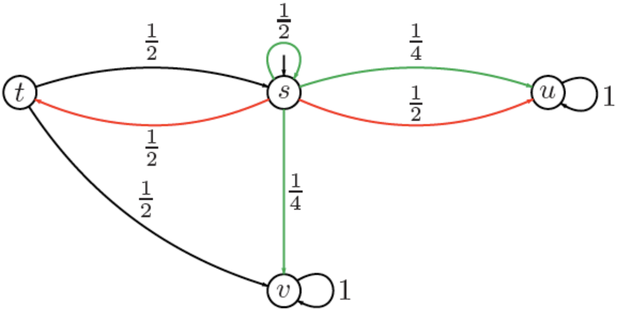

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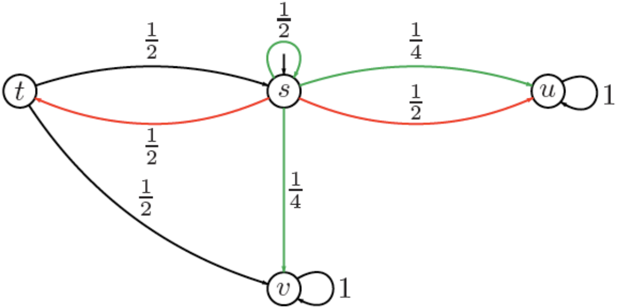

Consider a policy that alternates between selecting red and green, starting with red.

Example induced DTMC

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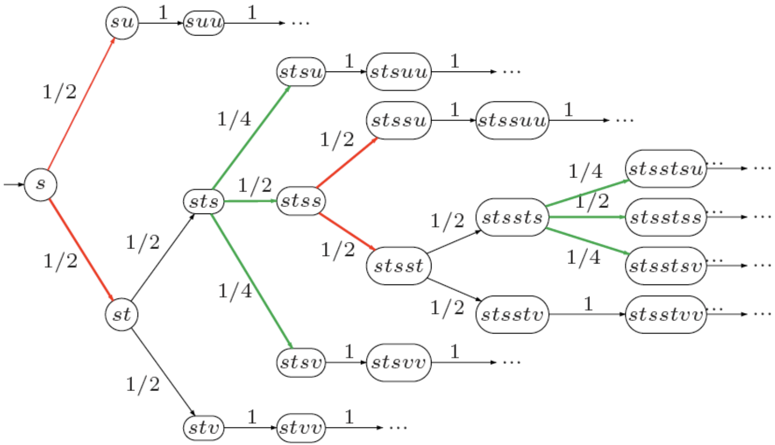

Induced DTMC for a policy that alternates between selecting red and green.

MDP paths versus paths in the induced DTMC

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$$s_0 \xrightarrow{\mathfrak{S}(\widehat{\pi}_0)} s_1 \xrightarrow{\mathfrak{S}(\widehat{\pi}_1)} s_2 \xrightarrow{\mathfrak{S}(\widehat{\pi}_2)} \ldots$$

is a $\mathfrak{S}$-path in $\mathcal{M}$.

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Similarly, for fixed state s of $\mathcal{M}$, which is considered as the unique starting state,

$$Pr^{\mathfrak{S}}(s \models P) \;=\; Pr^{\mathcal{M}_{\mathfrak{S}}}_{s}\{\, \pi \in Paths(s) \mid trace(\pi) \in P \,\}$$

where we identify the paths in $\mathcal{M}_{\mathfrak{S}}$ with the corresponding $\mathfrak{S}$-paths in $\mathcal{M}$.

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