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Exercise 1 (Positional Policies): (2 points)

Let $\mathcal{M}$ be a finite MDP with state space S and initial state s_0.

a) Prove or disprove: For $T \subseteq S$ and for any $k \in \mathbb{N}$ there exists a positional policy such that

$$\Pr_{\sigma}^{\mathcal{M}}(s_0 \models \Diamond^{\leq k} T) = \Pr_{\min}^{\mathcal{M}}(s_0 \models \Diamond^{\leq k} T)$$

Exercise 2 (Randomized Policies): (5 points)

Let $\mathcal{M}$ be a finite MDP with state space S and initial state s_0. A positional randomized policy is given by a function $\sigma\colon S \to \mathrm{Dist}(Act)$.

a) Define the induced DTMC of a positional randomized policy on an MDP.

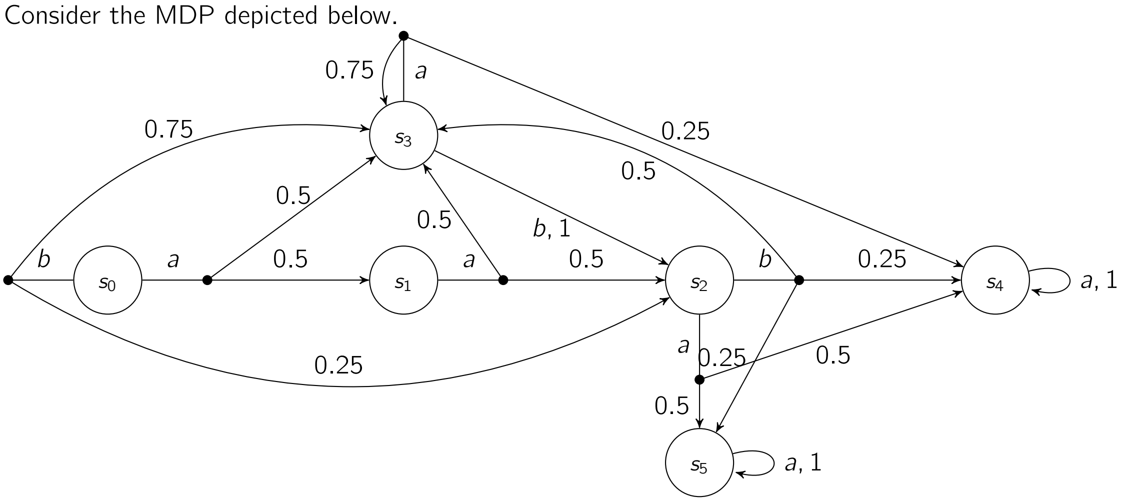
b) Prove or disprove: Let P be an ω-regular property s.t. there exists no deterministic positional policy σ with $\Pr_{\sigma}^{\mathcal{M}}(s_0 \models P) > 0$. Then there exists no randomized positional policy σ' such that $\Pr_{\sigma'}^{\mathcal{M}}(s_0 \models P) > 0$

c) Prove or disprove: For $T \subseteq S$ and for every randomized positional policy σ exists a deterministic positional policy σ' such that

$$\Pr_{\sigma}^{\mathcal{M}}(s_0 \models \Diamond T) \leq \Pr_{\sigma'}^{\mathcal{M}}(s_0 \models \Diamond T)$$

Exercise 3 (Value Iteration): (3 points)

Consider the MDP depicted below.



Use value iteration to compute the probabilities $\Pr_{max}^{\mathcal{M}}(s \models \Diamond s_4)$ for all $s \in S$. You may abort the process after 6 iteration. Provide the final policy as described in the lecture.