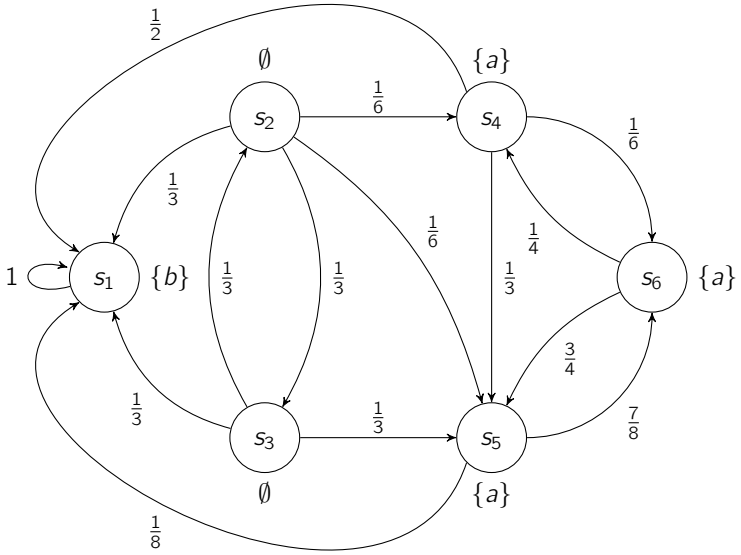
Exercise 1 (Bisimulation Minimization): (7 points)

a) Develop an algorithm in pseudocode that computes the coarsest bisimulation for a given finite DTMC $D = (S, \mathbf{P}, s_{init}, AP, L)$ in polynomial time (in the number of states and transitions). Briefly explain the ideas behind your algorithm and why it guarantees the complexity bound.

b) Apply your algorithm from **a)** to the following DTMC.



c) Suppose you are given another DTMC $D' = (S, \mathbf{P}, s_{init}, \{a, b\}, L)$ and a property $\varphi = a \mathsf{U} b$ ($a, b \in AP$) and are only interested in the probabilities $Pr(s \models \varphi)$ for all $s \in S$. Therefore, you want a quotient model that certainly preserves the probabilities for φ but not necessarily all PCTL path formulae. Indicate how your algorithm from **a)** can be improved to potentially (i.e. for some models) obtain a smaller quotient model while preserving the satisfaction probabilities for φ.

d) Suppose you are interested in preserving only properties in $\mathrm{PCTL}_{\setminus\{\bigcirc,\, \mathsf{U}^{\leq k}\}}$, i.e. all PCTL properties that do not involve the $\bigcirc$ and $\mathsf{U}^{\leq k}$ operators. Modify the definition of bisimulation from the lecture in a way that preserves this fragment of PCTL but potentially (i.e. for some models) allows for coarser quotients.

Hint: Try to find a notion of bisimulation that in particular treats the states $\{s_4, s_5, s_6\}$ of D as bisimilar.

Exercise 2 (Markov Decision Processes): (3 points)

Reconsider the Truel between Mr. Black, Mr. Gray and Mr. White.

- The truel proceeds in rounds, Mr. Black shoots first, then Mr. Gray, then Mr. White.
- If it's Mr. Black's turn, he can decide to shoot in the air or at one of the enemies that are still alive.

- If aiming at an enemy, Mr. Black will hit with a probability of $\frac{1}{3}$.
- Mr. Gray will aim at Mr. White if he's still alive and otherwise and Mr. Black.
- Mr. White will aim at Mr. Gray if he's still alive and otherwise and Mr. Black.
- Mr. Gray has a probability of $\frac{2}{3}$ of hitting an enemy and Mr. White will kill the enemy with probability 1.

a) Model the Truel as an MDP in the PRISM language.

b) Use PRISM to determine the minimal and maximal probability for the event "Mr. Black wins".

c) Use PRISM to obtain the schedulers (also called adversaries) that lead to the "worst-case" and "best-case" probability, respectively. Does your result match the discussion in the last tutorial?

Hint: Look at the command-line options of PRISM. In particular, options to generate adversaries and options to export the model.

d) Why is it not a smart idea to also model the (nondeterministic) choices of Mr. Gray and Mr. White (in the sense that they themselves can pick the enemy they are aiming at or to shoot into the air) when considering extremal (min, max) probabilities?