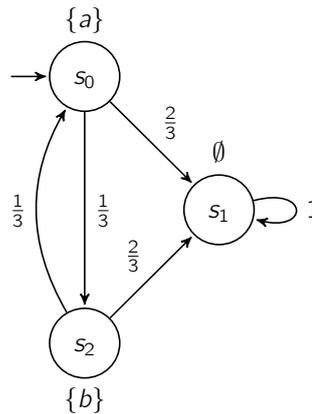


**Exercise 1 ( $\omega$ -regular Properties):**

**(4 points)**

Non-deterministic Büchi automata are strictly more expressive than deterministic ones. Recall the “powerset construction”, which is used to compute a deterministic finite automaton from a non-deterministic one.

- a) Construct a non-deterministic Büchi automaton for the language  $(a+b)^* a^\omega$ , apply the powerset construction to determinize this automaton, and compare the languages of the resulting automaton and the original one by either showing their equivalence or giving a counterexample separating the languages.
- b) Consider the following DTMC  $D$ :



Give a formal definition for the cross-product between a *non-deterministic* Büchi-Automaton and a DTMC. Apply this definition to the DTMC  $D$  and the NBA from a). What problems arise?

**Exercise 2 (Probabilities vs. Qualitative Properties):**

**(3 points)**

Let  $D = (S, \mathbf{P}, s_{init}, AP, L)$  be a finite DTMC,  $s \in S$ ,  $a, b \in AP$ . For each of the following statements, explain informally whether it is correct or not. If it is not correct, give a counterexample and indicate which of the implications (if any) hold.

- a)  $Pr(s \models \Box a) = 1$  if and only if  $s \models \forall \Box a$
- b)  $Pr(s \models \Diamond a) < 1$  if and only if  $s \not\models \forall \Diamond a$
- c)  $Pr(s \models \Box a) > 0$  if and only if  $s \models \exists \Box a$

**Exercise 3 (Algorithms for Qualitative Model Checking):**

**(3 points)**

Let  $D = (S, \mathbf{P}, s_{init}, AP, L)$  be a finite DTMC and  $a, b \in AP$ . Give algorithms (in pseudocode) to compute the following sets and briefly explain the complexity of your algorithm.

- a)  $S_{=0} = \{s \in S \mid Pr(s \models aUb) = 0\}$
- b)  $S_{=1} = \{s \in S \mid Pr(s \models aUb) = 1\}$