Prof. Dr. Ir. Joost-Pieter Katoen

Christian Dehnert, Sebastian Junges

Exercise 1 (Sigma Algebras): (3 points)

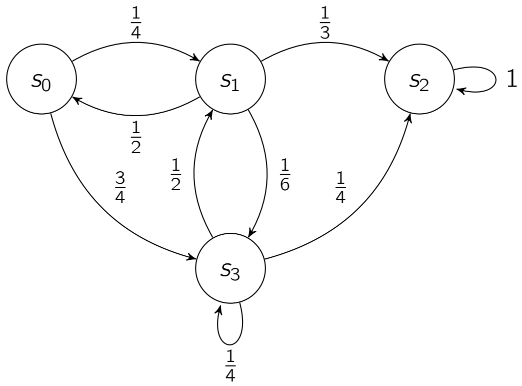
Consider a finite DTMC $D = (S, \mathbf{P}, s_{init}, AP, L)$ and subsets of states $A, B \subseteq S$. Show that the following two sets of paths are measurable, i.e. contained in the σ-algebra of D:

a) the set of paths starting in state s_{init} and remaining forever in states from A,

b) the set of paths starting in state s_{init}, remaining forever in states from A and passing through a state in B after exactly 5 time-steps.

Exercise 2 (Probabilities in DTMCs): (4 points)

Consider the following DTMC.



a) Compute the probability of going from s_0 to s_3 in *exactly* 3 steps.

b) Compute the probability of being in state s_3 after exactly 3 steps assuming a uniform initial distribution (over all states).

c) Compute the limiting probability of being in state s_3.

d) Compute the probability of going from s_0 to s_3 in *at most* 3 steps.

e) Compute the probability of reaching (without a bound on the number of steps) s_3 when starting in s_0.

Exercise 3 (Duelling Cowboys): (3 points)

We consider the following scenario.
Three Cowboys: "The Good" (G), "The Bad" (B), and "The Ugly" (U) meet each other in the desert for a famous duel.

- The three may shoot as long as anyone else is still alive. Due to differences in (re)loading times, we assume that they shoot in turns. That is, The Good shoots first, then The Bad and finally The Ugly.
- The Good has a chance of a half of hitting anyone. If he hits, he does so uniformly over the living contestants.
- The Bad has a chance of 0.9 of hitting anyone. If The Ugly is alive, then he aims for him with probability p. If The Ugly already died, then he surely aims at The Good.

- The Ugly hits The Good with a chance of q. If he does not hit The Good or The Good already died, he hits The Bad.

a) For certain values of p and q, this scenario is a stochastic process. For which values?

b) Depict a DTMC for this process. Please indicate for each state (i) who is alive and (ii) whose turn it is.