Concurrency Theory

Winter Semester 2015/16

Lecture 4: Hennessy-Milner Logic with Recursion

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http://moves.rwth-aachen.de/teaching/ws-1516/ct/

Written Exams in Concurrency Theory

1. Friday, 26.02.2016 11:30–14:00, AH 2
2. Tuesday, 29.03.2016 10:00–12:30, AH 1

Online registration via CampusOffice is enabled.

Software Consulting – Praxis-Workshop

Bringen Sie Informatik zur Wirkung!

Ein erheblicher Teil der Informatiker arbeitet im Beratungsumfeld. In der Beratung lösen Sie kontinuierlich neue Fragestellungen bei verschiedenen Kunden und erlangen in den Projekten breites Wissen. Erfahren Sie aus erster Hand, welche spannenden Möglichkeiten Software-Beratung bietet, und probieren Sie aus, ob dieses Berufsfeld zu Ihnen passt!

Inhalte des Workshops:

- Wir diskutieren mit Ihnen, was ein Software Consultant genau macht und warum es sich lohnt, Berater zu sein.
- Sie bearbeiten im Team eine anspruchsvolle IT-Fallaufgabe im Rahmen eines realen Software-Migrationsprojektes unter Berücksichtigung der technischen, ökonomischen und organisatorischen Rahmenbedingungen. Bei der Lösung unterstützen Sie unsere erfahrenen Kollegen.

Dienstag, **15.12.2015**, 09.00-16.00 Uhr

RWTH Aachen, Informatik E3, Ahornstraße 55, 2. OG, Raumnr. 222

Melden Sie sich unter Angabe Ihres Semesters bis zum **07.12.2015** an.
Wir freuen uns auf Sie!

Kontakt: Anne-Kristin Hauk (**hauk@itestra.de**) – www.itestra.com

Outline of Lecture 4

Syntax of HML

Definition (Syntax of HML)

The set *HMF* of Hennessy-Milner formulae over a set of actions *Act* is defined by the following syntax:

$$
\begin{array}{rcll}
F & ::= & \mathrm{tt} & \text{(true)} \\
 & \mid & \mathrm{ff} & \text{(false)} \\
 & \mid & F_1 \wedge F_2 & \text{(conjunction)} \\
 & \mid & F_1 \vee F_2 & \text{(disjunction)} \\
 & \mid & \langle \alpha \rangle F & \text{(diamond)} \\
 & \mid & [\alpha] F & \text{(box)}
\end{array}
$$

where $\alpha \in Act$.

Abbreviations for $L = \{\alpha_1, \ldots, \alpha_n\}$ ($n \in \mathbb{N}$):

- $\langle L \rangle F := \langle \alpha_1 \rangle F \vee \ldots \vee \langle \alpha_n \rangle F$
- $[L]F := [\alpha_1]F \wedge \ldots \wedge [\alpha_n]F$
- In particular, $\langle \emptyset \rangle F :=$ ff and $[\emptyset]F :=$ tt

Semantics of HML

Definition (Semantics of HML)

Let $(S, Act, \longrightarrow)$ be an LTS and $F \in HMF$. The set of processes in S that satisfy F, $[\![F]\!] \subseteq S$, is defined by:

$$[\![tt]\!] := S \qquad [\![ff]\!] := \emptyset$$
$$[\![F_1 \wedge F_2]\!] := [\![F_1]\!] \cap [\![F_2]\!] \qquad [\![F_1 \vee F_2]\!] := [\![F_1]\!] \cup [\![F_2]\!]$$
$$[\![\langle\alpha\rangle F]\!] := \langle\cdot\alpha\cdot\rangle([\![F]\!]) \qquad [\![[\alpha]F]\!] := [\cdot\alpha\cdot]([\![F]\!])$$

where $\langle\cdot\alpha\cdot\rangle, [\cdot\alpha\cdot] : 2^S \to 2^S$ are given by

$$\langle\cdot\alpha\cdot\rangle(T) := \{s \in S \mid \exists s' \in T : s \xrightarrow{\alpha} s'\}$$
$$[\cdot\alpha\cdot](T) := \{s \in S \mid \forall s' \in S : s \xrightarrow{\alpha} s' \implies s' \in T\}$$

We write $s \models F$ iff $s \in [\![F]\!]$. Two HML formulae are equivalent (written $F \equiv G$) iff they are satisfied by the same processes in every LTS.

Recap: Hennessy-Milner Logic

Process Traces

Goal: reduce processes to the action sequences they can perform

Definition (Trace language)

For every $P \in Prc$, let

$$Tr(P) := \{w \in Act^* \mid \text{ex. } P' \in Prc \text{ such that } P \xrightarrow{w} P'\}$$

be the trace language of P (where $\xrightarrow{w} := \xrightarrow{a_1} \circ \ldots \circ \xrightarrow{a_n}$ for $w = a_1 \ldots a_n$).

$P, Q \in Prc$ are called trace equivalent if $Tr(P) = Tr(Q)$.

Example (One-place buffer)

$B = in.\overline{out}.B$

$\implies Tr(B) = (in \cdot \overline{out})^* \cdot (in + \varepsilon)$

Outline of Lecture 4

Recap: Hennessy-Milner Logic

HML and Process Traces

Adding Recursion to HML

HML with One Recursive Variable

HML and Process Traces

Lemma 4.1

Let $(Prc, Act, \longrightarrow)$ *be an LTS, and let* $P, Q \in Prc$ *satisfy the same HMF (i.e.,* $\forall F \in HMF : P \models F \iff Q \models F$*). Then* $Tr(P) = Tr(Q)$*.*

HML and Process Traces

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Proof.

on the board □

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Proof.

on the board □

Remark: the converse does *not* hold.

Example 4.2

- Let $P := a.(b.\text{nil} + c.\text{nil}) \in Prc$, $Q := a.b.\text{nil} + a.c.\text{nil} \in Prc$
- Then $Tr(P) = Tr(Q) = \{\varepsilon, a, ab, ac\}$

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- Then $Tr(P) = Tr(Q) = \{\varepsilon, a, ab, ac\}$
- Let $F := [a](\langle b\rangle\text{tt} \wedge \langle c\rangle\text{tt}) \in HMF$
- Then $P \models F$ but $Q \not\models F$
- [Later: $P, Q \in Prc$ HML-equivalent iff bismilar]

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HML and Process Traces

Adding Recursion to HML

HML with One Recursive Variable

Finiteness of HML

Observation: HML formulae only describe finite part of process behaviour

- each modal operator ($[.]$, $\langle . \rangle$) talks about *one* step
- only finite nesting of operators (modal depth)

Adding Recursion to HML

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Example 4.3

- $F := (\langle a \rangle [a] \mathrm{ff}) \vee \langle b \rangle \mathrm{tt} \in HMF$ has modal depth 2
- Checking F involves analysis of all behaviours of length ≤ 2

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- Checking F involves analysis of all behaviours of length ≤ 2

But: sometimes necessary to refer to arbitrarily long computations (e.g., “no deadlock state reachable”)

- possible solution: support infinite conjunctions and disjunctions

Infinite Conjunctions

Example 4.4

- Let $C = a.C$, $D = a.D + a.\text{nil}$
- Then $C \models [a]\langle a\rangle\text{tt}$ but $D \not\models [a]\langle a\rangle\text{tt}$

Infinite Conjunctions

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- Let $C = a.C$, $D = a.D + a.\text{nil}$
- Then $C \models [a]\langle a\rangle\text{tt}$ but $D \not\models [a]\langle a\rangle\text{tt}$
- Now redefine D as $D_n = a.D_n + a.E_n$ where $n \in \mathbb{N}$, $E_k = a.E_{k-1}$ $(1 \leq k \leq n)$, $E_0 = \text{nil}$
- Then (for $[\alpha]^k F := \underbrace{[\alpha] \ldots [\alpha]}_{k \text{ times}} F$ where $F \in HMF$):
  - $C \models [a]^k\langle a\rangle\text{tt}$ for all $k \in \mathbb{N}$
  - $D_n \models [a]^k\langle a\rangle\text{tt}$ for all $0 \leq k \leq n$
  - $D_n \not\models [a]^k\langle a\rangle\text{tt}$ for all $k > n$

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  - $D_n \not\models [a]^k \langle a\rangle \mathrm{tt}$ for all $k > n$
- Conclusion: no single HML formula can distinguish C and all D_n
- Generally: invariant property "always $\langle a\rangle \mathrm{tt}$" not expressible
- Requires infinite conjunction:

$$Inv(\langle a\rangle \mathrm{tt}) = \langle a\rangle \mathrm{tt} \wedge [a]\langle a\rangle \mathrm{tt} \wedge [a][a]\langle a\rangle \mathrm{tt} \wedge \ldots = \bigwedge_{k \in \mathbb{N}} [a]^k \langle a\rangle \mathrm{tt}$$

Infinite Disjunctions

Dually: possibility properties expressible by infinite disjunctions

Example 4.5

- Let $C = a.C$, $D = a.D + a.\text{nil}$ as before
- C has no possibility to terminate
- D has the option to terminate (i.e., to eventually satisfy $[a]\text{ff}$) at any time by choosing the $a.\text{nil}$ branch

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- Representable by infinite disjunction:

$$Pos([a]\text{ff}) = [a]\text{ff} \vee \langle a\rangle[a]\text{ff} \vee \langle a\rangle\langle a\rangle[a]\text{ff} \vee \ldots = \bigvee_{k\in\mathbb{N}} \langle a\rangle^k[a]\text{ff}$$

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Problem: infinite formulae not easy to handle

Introducing Recursion

Solution: employ recursion!

- $\mathit{Inv}(\langle a\rangle \mathrm{tt}) \equiv \langle a\rangle \mathrm{tt} \wedge [a]\, \mathit{Inv}(\langle a\rangle \mathrm{tt})$
- $\mathit{Pos}([a]\mathrm{ff}) \equiv [a]\mathrm{ff} \vee \langle a\rangle\, \mathit{Pos}([a]\mathrm{ff})$

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Solution: employ recursion!

- $Inv(\langle a\rangle \mathrm{tt}) \equiv \langle a\rangle \mathrm{tt} \wedge [a]\, Inv(\langle a\rangle \mathrm{tt})$
- $Pos([a]\mathrm{ff}) \equiv [a]\mathrm{ff} \vee \langle a\rangle\, Pos([a]\mathrm{ff})$

Interpretation: the sets of states $X, Y \subseteq S$ satisfying the respective formula should solve the corresponding equation, i.e.,

- $X = \langle\cdot a\cdot\rangle(S) \cap [\cdot a\cdot](X)$
- $Y = [\cdot a\cdot](\emptyset) \cup \langle\cdot a\cdot\rangle(Y)$

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Solution: employ recursion!

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Open questions

- Do such recursive equations (always) have solutions?
- If so, are they unique?
- How can we compute whether a process satisfies a recursive formula?

Existence of Solutions

Example 4.6

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  - $X = \emptyset$ is a solution (as no process can satisfy both $\langle a\rangle\mathrm{tt}$ and $[a]\mathrm{ff}$)
  - but we expect $C \in X$ (as C can perform a invariantly)
  - in fact, $X = \{C\}$ also solves the equation (and is the greatest solution w.r.t. $\subseteq$)

$\Longrightarrow$ write $X \overset{max}{=} \langle a\rangle\mathrm{tt} \wedge [a]X$

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$\Longrightarrow$ write $X \overset{max}{=} \langle a \rangle \text{tt} \wedge [a]X$

- Possibility: $Y \equiv [a]\text{ff} \vee \langle a \rangle Y$
  - greatest solution: $Y = \{C, D, \text{nil}\}$
  - but we expect $C \notin Y$ (as C cannot terminate at all)
  - here: least solution w.r.t. $\subseteq$: $Y = \{D, \text{nil}\}$

$\Longrightarrow$ write $Y \overset{min}{=} [a]\text{ff} \vee \langle a \rangle Y$

Uniqueness of Solutions

Uniqueness of solutions

- Use greatest solutions for properties that hold unless the process has a finite computation that disproves it.
- Use least solutions for properties that hold if the process has a finite computation that proves it.

Adding Recursion to HML

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Example 4.7

Let $(S, Act, \longrightarrow)$ be an LTS, $s \in S$, and $F \in HMF$.

- Invariant: $Inv(F) \equiv X$ for $X \stackrel{max}{=} F \wedge [Act]X$
  - $s \models Inv(F)$ if all states reachable from s satisfy F

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  - $s \models Inv(F)$ if all states reachable from s satisfy F
- Possibility: $Pos(F) \equiv Y$ for $Y \stackrel{min}{=} F \vee \langle Act \rangle Y$
  - $s \models Pos(F)$ if a state satisfying F is reachable from s

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- Possibility: $Pos(F) \equiv Y$ for $Y \stackrel{min}{=} F \vee \langle Act \rangle Y$
  - $s \models Pos(F)$ if a state satisfying F is reachable from s
- Safety: $Safe(F) \equiv X$ for $X \stackrel{max}{=} F \wedge ([Act]\mathrm{ff} \vee \langle Act \rangle X)$
  - $s \models Safe(F)$ if s has a complete (i.e., infinite or terminating) transition sequence where each state satisfies F

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  - $s \models Safe(F)$ if s has a complete (i.e., infinite or terminating) transition sequence where each state satisfies F
- Eventuality: $Evt(F) \equiv Y$ for $Y \stackrel{min}{=} F \vee (\langle Act \rangle \mathrm{tt} \wedge [Act]Y)$
  - $s \models Evt(F)$ if each complete transition sequence starting in s contains a state satisfying F

Outline of Lecture 4

Syntax of HML with One Recursive Variable

Initially: only one variable

Later: mutual recursion

Syntax of HML with One Recursive Variable

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Definition 4.8 (Syntax of HML with one variable)

The set HMF_X of Hennessy-Milner formulae with one variable X over a set of actions Act is defined by the following syntax:

$$
\begin{array}{rll}
F ::= & X & \text{(variable)} \\
\mid & \mathrm{tt} & \text{(true)} \\
\mid & \mathrm{ff} & \text{(false)} \\
\mid & F_1 \land F_2 & \text{(conjunction)} \\
\mid & F_1 \lor F_2 & \text{(disjunction)} \\
\mid & \langle \alpha \rangle F & \text{(diamond)} \\
\mid & [\alpha] F & \text{(box)}
\end{array}
$$

where $\alpha \in Act$.

Semantics of HML with One Recursive Variable I

So far: $[\![F]\!] \subseteq S$ for $F \in HMF$ and LTS $(S, Act, \longrightarrow)$

Now: semantics of formula depends on states that (are assumed to) satisfy X

Semantics of HML with One Recursive Variable I

So far: $[\![F]\!] \subseteq S$ for $F \in HMF$ and LTS $(S, Act, \longrightarrow)$

Now: semantics of formula depends on states that (are assumed to) satisfy X

Definition 4.9 (Semantics of HML with one variable)

Let $(S, Act, \longrightarrow)$ be an LTS and $F \in HMF_X$. The semantics of F,

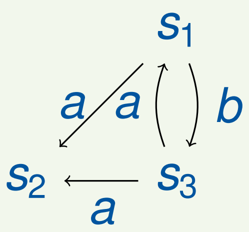
$$[\![F]\!] : 2^S \to 2^S,$$

is defined by

$$\begin{aligned}
[\![X]\!](T) &:= T \\
[\![\mathrm{tt}]\!](T) &:= S \\
[\![\mathrm{ff}]\!](T) &:= \emptyset \\
[\![F_1 \wedge F_2]\!](T) &:= [\![F_1]\!](T) \cap [\![F_2]\!](T) \\
[\![F_1 \vee F_2]\!](T) &:= [\![F_1]\!](T) \cup [\![F_2]\!](T) \\
[\![\langle\alpha\rangle F]\!](T) &:= \langle\cdot\alpha\cdot\rangle([\![F]\!](T)) \\
[\![[\alpha]F]\!](T) &:= [\cdot\alpha\cdot]([\![F]\!](T))
\end{aligned}$$

Semantics of HML with One Recursive Variable II

Example 4.10



Let $S := \{s_1, s_2, s_3\}$.

Semantics of HML with One Recursive Variable II

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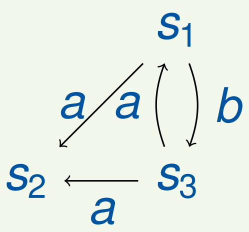

Let $S := \{s_1, s_2, s_3\}$.

- $[\![\langle a\rangle X]\!](\{s_1\}) = \{s_3\}$

Semantics of HML with One Recursive Variable II

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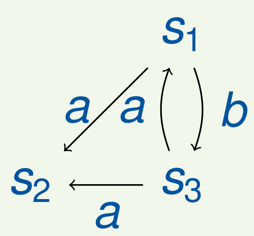

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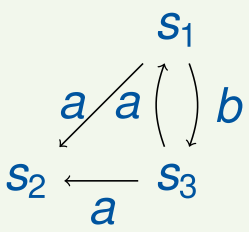

Let $S := \{s_1, s_2, s_3\}$.

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- $[\![\langle a\rangle X]\!](\{s_1, s_2\}) = \{s_1, s_3\}$
- $[\![[b]X]\!](\{s_2\}) = \{s_2, s_3\}$

Semantics of HML with One Recursive Variable III

- Idea underlying the definition of

$$[\![.]\!] : HMF_X \to (2^S \to 2^S) :$$

  if $T \subseteq S$ gives the set of states that satisfy X, then $[\![F]\!](T)$ will be the set of states that satisfy F

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- How to determine this T?
- According to previous discussion: as solution of recursive equation of the form $X = F_X$ where $F_X \in HMF_X$

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- But: solution not unique; therefore write:

$$X \overset{min}{=} F_X \qquad \text{or} \qquad X \overset{max}{=} F_X$$

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$$X \overset{min}{=} F_X \qquad \text{or} \qquad X \overset{max}{=} F_X$$
- In the following we will see:
  1. Equation $X = F_X$ always solvable
  2. Least and greatest solutions are unique and can be obtained by fixed-point iteration