Concurrency Theory

Winter Semester 2015/16

Lecture 12: Strong Bisimulation

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Outline of Lecture 12

Recap: Trace Equivalence

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- Our (serious) attempt today: Milner's strong bisimulation.

Robin Milner (1934–2010)

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Recap: Trace Equivalence

Bisimulation

Bisimulation and Trace Equivalence

Congruence and Deadlock Sensitivity

Buffers Revisited

Epilogue

Rationale

Observation

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Three versions will be considered in this course:

1. Strong bisimulation: ignore the special function of τ-actions
2. Weak bisimulation: treat τ-actions as invisible
3. Simulation relations: unidirectional versions of bisimulation

Strong Bisimulation I

Definition 12.1 (Strong bisimulation) (Park 1981, Milner 1989)

A binary relation $\rho \subseteq \mathit{Prc} \times \mathit{Prc}$ is a strong bisimulation whenever for every $(P, Q) \in \rho$ and $\alpha \in \mathit{Act}$:

1. if $P \xrightarrow{\alpha} P'$, then there exists $Q' \in \mathit{Prc}$ such that $Q \xrightarrow{\alpha} Q'$ and $(P', Q') \in \rho$, and
2. if $Q \xrightarrow{\alpha} Q'$, then there exists $P' \in \mathit{Prc}$ such that $P \xrightarrow{\alpha} P'$ and $(P', Q') \in \rho$.

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Relation $\sim$ is called strong bisimilarity.

Strong Bisimulation II

$$\begin{array}{ccc} P & \xrightarrow{\alpha} & P' \\ \rho & & \\ Q & & \end{array} \quad \text{can be completed to} \quad \begin{array}{ccc} P & \xrightarrow{\alpha} & P' \\ \rho & & \rho \\ Q & \xrightarrow{\alpha} & Q' \end{array}$$

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Examples

Example 12.3 (A first example)

Claim: $P \sim Q$ where

$$\begin{aligned} P &= a.P_1 + a.P_2 & \qquad Q &= a.Q_1 \\ P_1 &= b.P_2 & Q_1 &= b.Q_1 \\ P_2 &= b.P_2 & & \end{aligned}$$

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Example 12.5 (Counterexample; cf. Example 11.9)

Show on board that $CTM \not\sim CTM'$ where
$$\begin{aligned} CTM &= coin.\left(\overline{coffee}.CTM + \overline{tea}.CTM\right) \\ CTM' &= coin.\overline{coffee}.CTM' + coin.\overline{tea}.CTM'. \end{aligned}$$

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Properties of Strong Bisimilarity

Lemma 12.6 (Properties of $\sim$)

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Proof.

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Bisimulation on Paths

Lemma 12.7 (Bisimulation on paths)

Whenever we have:

$$P_0 \xrightarrow{\alpha_1} P_1 \xrightarrow{\alpha_2} P_2 \xrightarrow{\alpha_3} P_3 \xrightarrow{\alpha_4} P_4 \ldots\ldots$$

$$\rho$$

$$Q_0$$

Bisimulation on Paths

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Proof.

by induction on the length of the path □

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Take $P = a.P_1$ with $P_1 = b.\mathrm{nil} + c.\mathrm{nil}$ and $Q = a.b.nil + a.c.\mathrm{nil}$.

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Take $P = a.P_1$ with $P_1 = b.\mathrm{nil} + c.\mathrm{nil}$ and $Q = a.b.nil + a.c.\mathrm{nil}$.
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Thus, P and Q are trace equivalent.
But: $P \not\sim Q$, as there is no state in the LTS of Q that is bisimilar to P_1.
Why? No state in Q can perform both b and c. $\square$

Deterministic Transition Systems

Definition 12.9 (Determinism)

$P \in Prc$ is deterministic whenever for every of its states s it holds:

$$\left(s \xrightarrow{\alpha} t \text{ and } s \xrightarrow{\alpha} u\right) \quad \text{implies} \quad t = u.$$

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Proof.

Left as an exercise. In fact, for deterministic processes, trace equivalence, complete trace, failure trace, and ready trace equivalence all coincide. $\square$

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Bisimulation

Bisimulation and Trace Equivalence

Congruence and Deadlock Sensitivity

Buffers Revisited

Epilogue

Congruence

Theorem 12.11 (CCS congruence property of $\sim$)

Strong bisimilarity $\sim$ is a CCS congruence, that is, whenever $P, Q \in Prc$ such that $P \sim Q$,

$$\begin{array}{rcll} \alpha.P & \sim & \alpha.Q & \text{for every action } \alpha \\ P + R & \sim & Q + R & \text{for every process } R \\ P \parallel R & \sim & Q \parallel R & \text{for every process } R \\ P \setminus L & \sim & Q \setminus L & \text{for every set } L \subseteq A \\ P[f] & \sim & Q[f] & \text{for every relabelling } f : A \to A \end{array}$$

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Proof.

- for $\parallel$: on the board
- for other CCS operators: left as an exercise □

Deadlock Sensitivity of $\sim$

Definition (Deadlock; cf. Definition 11.5)

Let $P, Q \in Prc$ and $w \in Act^*$ such that $P \xrightarrow{w} Q$ and $Q \not\longrightarrow$. Then Q is called a w-deadlock of P.

Congruence and Deadlock Sensitivity

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$\sim$ *is deadlock sensitive.*

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on the board $\square$

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Two Buffers

Example 12.13 (One-place buffer)

$$\begin{aligned} B_0^1 &= in.B_1^1 \\ B_1^1 &= \overline{out}.B_0^1. \end{aligned}$$

Buffers Revisited

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Example 12.14 (Two-place buffer)

$$\begin{aligned} B_0^2 &= in.B_1^2 \\ B_1^2 &= in.B_2^2 + \overline{out}.B_0^2 \\ B_2^2 &= \overline{out}.B_1^2. \end{aligned}$$

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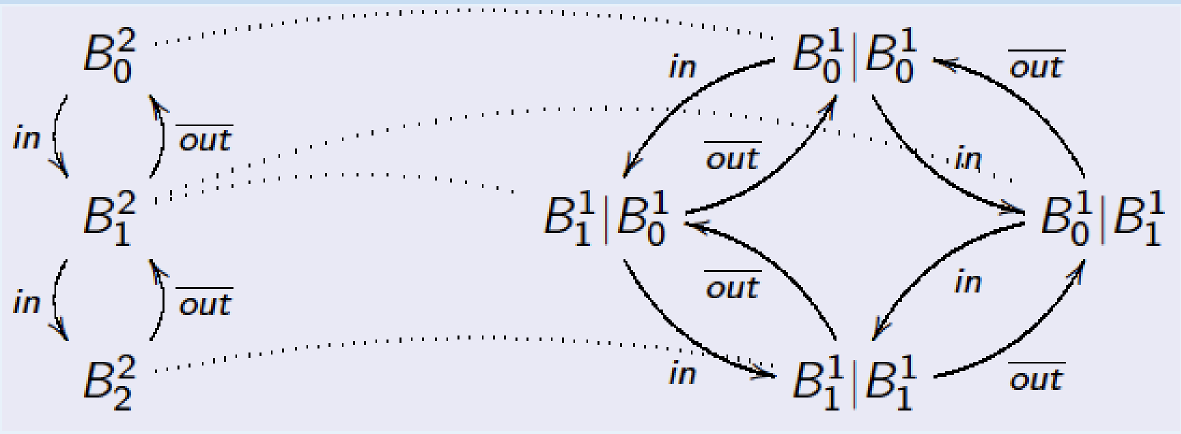
$B^2_0 \sim B^1_0 \parallel B^1_0$



Semaphores: A Generalisation

Example 12.15 (An n-ary semaphore)

Let S_i^n stand for a semaphore for n resources i of which are taken:

$$\begin{aligned} S_0^n &= get.S_1^n \\ S_i^n &= get.S_{i+1}^n + put.S_{i-1}^n \quad \text{for } 0 < i < n \\ S_n^n &= put.S_{n-1}^n \end{aligned}$$

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This process is strongly bisimilar to n parallel binary semaphores:

Lemma 12.16

For every $n \in \mathbb{N}_+$, we have: $S_0^n \sim \underbrace{S_0^1 \parallel \cdots \parallel S_0^1}_{n \text{ times}}$.

Semaphores II

Lemma

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Proof.

Consider the following binary relation where $i_1, i_2, \ldots, i_n \in \{0, 1\}$:

$$\rho = \left\{ \left(S_i^n, S_{i_1}^1 \parallel \cdots \parallel S_{i_n}^1\right) \;\middle|\; \sum_{j=1}^{n} i_j = i \right\}$$

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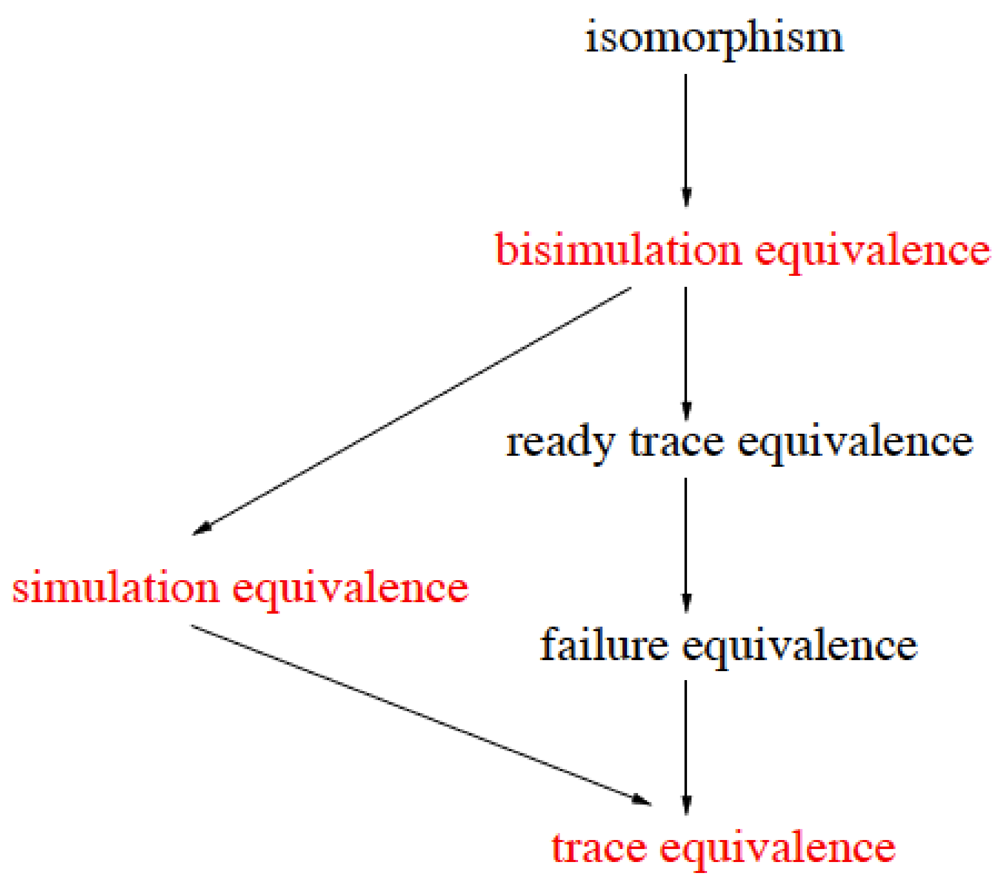
Then: ρ is a strong bisimulation and $(S_0^n, \underbrace{S_0^1 \parallel \cdots \parallel S_0^1}_{n \text{ times}}) \in \rho$. $\square$

Outline of Lecture 12

Overview of Some Behavioural Equivalences



Epilogue

Summary

- Strong bisimulation of processes is based on mutually mimicking each other

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- Strong bisimulation of processes is based on mutually mimicking each other
- Strong bisimilarity $\sim$:
  1. is the largest strong bisimulation
  2. is an equivalence
  3. is a CCS congruence
  4. is strictly finer than trace equivalence
  5. is deadlock sensitive