Concurrency Theory

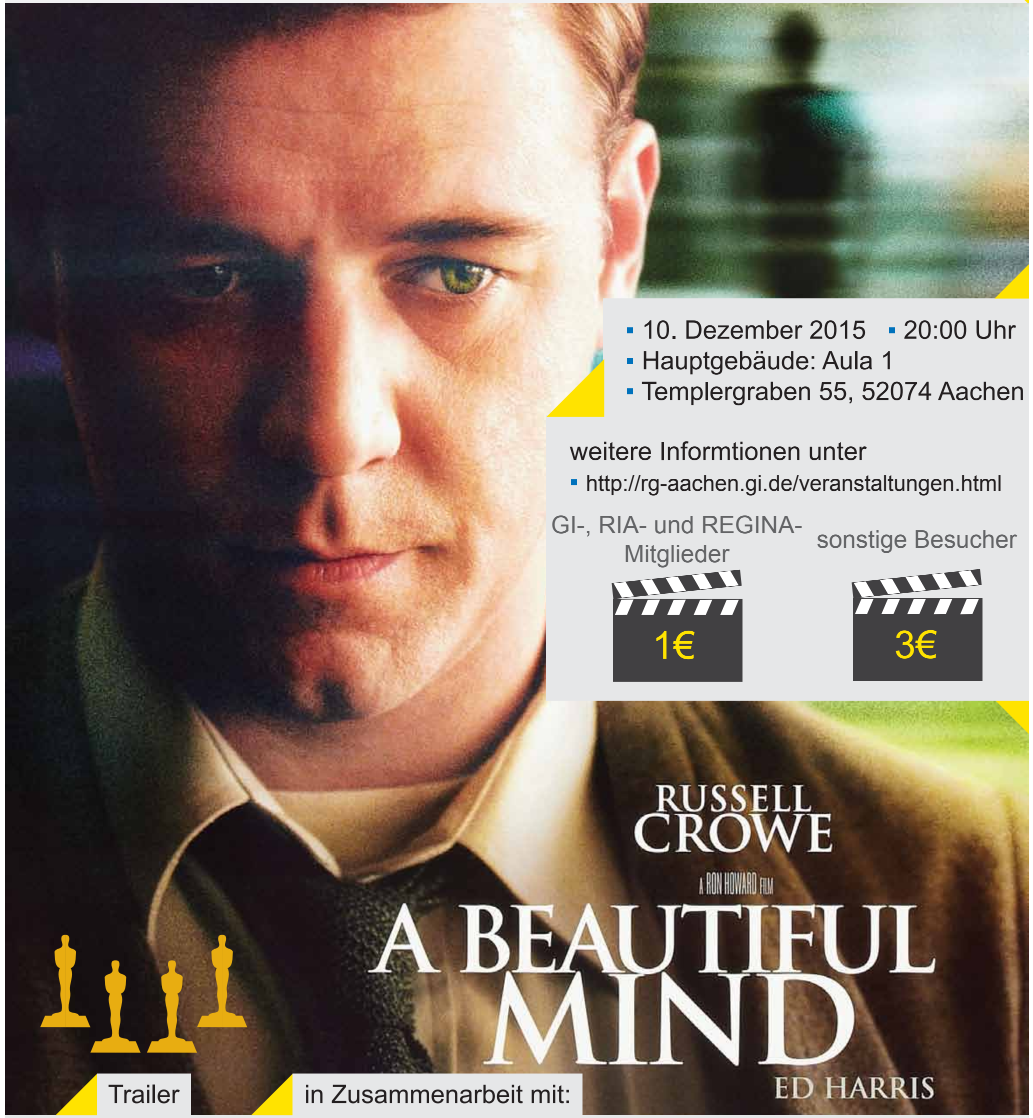
Winter Semester 2015/16

Lecture 11: Trace Equivalence

Joost-Pieter Katoen and Thomas Noll
Software Modeling and Verification Group
RWTH Aachen University

http://moves.rwth-aachen.de/teaching/ws-1516/ct/

GI - Filmaufführungen



- 10. Dezember 2015
- 20:00 Uhr
- Hauptgebäude: Aula 1
- Templergraben 55, 52074 Aachen

weitere Informtionen unter

- http://rg-aachen.gi.de/veranstaltungen.html

GI-, RIA- und REGINA-Mitglieder	sonstige Besucher
1€	3€

Trailer

in Zusammenarbeit mit:

INFORMATIK RWTH AACHEN

Filmstudio an der RWTH Aachen e.V.

Fachschaft Mathematik - Physik - Informatik · RWTH Aachen

RIA GI

Regionalgruppe Informatik Aachen der Gesellschaft für Informatik (GI)

„John Nash ist ein genialer Mathematiker mit einer großen Breite (Nash-Gleichgewicht in der Spieltheorie, reelle algebraische Mannigfaltigkeiten, Differentialgeometrie, partielle Differentialgleichungen), ausgebildet und tätig an den Elite-Universitäten im Osten der USA. Er ist aber auch etwas seltsam: Kommunikationsarm, hochnäsig und mit wenig Empathie. Nach seinem steilen Aufstieg zu Ruhm beginnt eine absonderliche Filmgeschichte, die man auf den ersten Blick dem üblichen Hollywood-Klamauk zuordnet...“

Introduction

- When using process algebras like CCS, an important approach is to model both the specification and implementation as CCS processes, say *Spec* and *Impl*.
- This gives rise to the natural question: when are two CCS processes behaving the same?
- As there are many different interpretations of “behaving the same”, different behavioural equivalences have emerged.

Behavioural Equivalence

Implementation

$$\begin{aligned} CM &= coin.\overline{coffee}.CM \\ CS &= \overline{pub}.\overline{coin}.coffee.CS \\ Uni &= (CM \parallel CS) \setminus \{coin, coffee\} \end{aligned}$$

Specification

$$Spec = \overline{pub}.Spec$$

Question

Are the specification *Spec* and implementation *Uni* behaviourally equivalent:

$$Spec \stackrel{?}{\equiv} Uni$$

Equivalence Relations

Some reasonable required properties

- Reflexivity: $P \equiv P$ for every process P
- Symmetry: $P \equiv Q$ if and only if $Q \equiv P$
- Transitivity: $\mathit{Spec}_0 \equiv \ldots \equiv \mathit{Spec}_n \equiv \mathit{Impl}$ implies that $\mathit{Spec}_0 \equiv \mathit{Impl}$

Definition 11.1 (Equivalence)

A binary relation $\equiv \subseteq S \times S$ over a set S is an equivalence if

- it is reflexive: $s \equiv s$ for every $s \in S$,
- it is symmetric: $s \equiv t$ implies $t \equiv s$ for every $s, t \in S$,
- it is transitive: $s \equiv t$ and $t \equiv u$ implies $s \equiv u$ for every $s, t, u \in S$.

Isomorphism: An Example Behavioural Equivalence

Isomorphism

Two LTSs $T_1 = (S_1, Act_1, \longrightarrow_1)$ and $T_2 = (S_2, Act_2, \longrightarrow_2)$ are isomorphic, denoted $T_1 \equiv_{iso} T_2$, if there exists a bijection $f : S_1 \to S_2$ such that

$$s \xrightarrow{\alpha}_1 t \quad \text{if and only if} \quad f(s) \xrightarrow{\alpha}_2 f(t).$$

It follows immediately that $\equiv_{iso}$ is an equivalence. (Why?)

Example 11.2 (Abelian monoid laws for $+$ and $\|$)

1. $P + Q \equiv_{iso} Q + P$, $P \parallel Q \equiv_{iso} Q \parallel P$
2. $(P + Q) + R \equiv_{iso} P + (Q + R)$, $(P \parallel Q) \parallel R \equiv_{iso} P \parallel (Q \parallel R)$
3. $P + \mathrm{nil} \equiv_{iso} P \parallel \mathrm{nil} \equiv_{iso} P$

Isomorphism II

Assumption

From now on, we will consider processes modulo isomorphism, i.e., we do not distinguish isomorphic CCS processes.

Caveat

But: isomorphism is very distinctive. For instance,

$$X = a.X \quad \text{and} \quad Y = a.a.Y$$

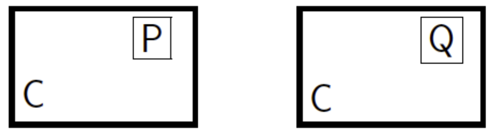
are distinguished although both can (only) execute infinitely many a-actions and should thus be considered equivalent.

The Wish List for Behavioural Equivalences

1. Less distinctive than isomorphism: an equivalence should distinguish less processes than isomorphism does, i.e., $\equiv$ should be coarser than isomorphism:
$$P \equiv_{iso} Q \implies P \equiv Q.$$
2. More distinctive than trace equivalence: an equivalence should distinguish more processes than trace equivalence does, i.e., $\equiv$ should be finer than trace equivalence:
$$P \equiv Q \implies Tr(P) = Tr(Q).$$
3. Congruence property: the equivalence must be substitutive with respect to all CCS operators (see next slide).
4. Deadlock preservation: equivalent processes should have the same deadlock behaviour, i.e., equivalent process can either both deadlock, or both cannot.[1]
5. Optional: the coarsest possible equivalence: there should be no less discriminating equivalence satisfying all these requirements.

[1] Later, we will enlarge this to a set of properties that can be expressed in a logic.

What is a Congruence?



$C(P)$ $C(Q)$

CCS contexts informally

A CCS context is a CCS process fragment with a “hole” in it (examples on the board).

CCS congruences informally

Relation $\equiv$ is a CCS congruence whenever $P \equiv Q$ implies $C(P) \equiv C(Q)$ for every CCS context C.

The Importance of Congruences

CCS congruences informally

Relation $\equiv$ is a congruence whenever $P \equiv Q$ implies $C(P) \equiv C(Q)$ for every CCS context C.

Example 11.3 (Congruence)

Let $a \equiv b$ for $a, b \in \mathbb{Z}$ whenever $a \textbf{ mod } k = b \textbf{ mod } k$, for some $k \in \mathbb{N}_+$.
Equivalence relation $\equiv$ is a congruence for addition and multiplication.

Important motivations of requiring $\equiv$ to be a congruence on processes:

1. Model-based development through refinement: replacing an abstract model *Spec* by a more detailed model *Impl*
2. Abstraction/optimisation: replacing a large (concrete) model *Impl* by a smaller (more abstract) model *Spec*.

Remark: congruences induce quotient structures with equivalence classes as elements

CCS Congruences Formally

Definition 11.4 (CCS congruence)

An equivalence relation $\equiv \subseteq Prc \times Prc$ is a CCS congruence if it is preserved by all CCS constructs, i.e., if $P, Q \in Prc$ with $P \equiv Q$ then:

$$
\begin{aligned}
\alpha.P &\equiv \alpha.Q && \text{for every } \alpha \in Act \\
P + R &\equiv Q + R && \text{for every } R \in Prc \\
P \parallel R &\equiv Q \parallel R && \text{for every } R \in Prc \\
P \setminus L &\equiv Q \setminus L && \text{for every } L \subseteq A \\
P[f] &\equiv Q[f] && \text{for every } f : A \to A
\end{aligned}
$$

Thus, a CCS congruence is substitutive for all possible CCS contexts.

Requirements on Behavioural Equivalences

Deadlocks

Definition 11.5 (Deadlock)

Let $P, Q \in Prc$ and $w \in Act^*$ such that $P \xrightarrow{w} Q$ and $Q \not\longrightarrow$. Then Q is called a w-deadlock of P.

Example 11.6

$P = a.b.\mathrm{nil} + a.\mathrm{nil}$ has an a-deadlock, whereas $Q = a.b.\mathrm{nil}$ has not.

Such properties are important as it can be crucial that a certain action is eventually possible.

Definition 11.7 (Deadlock sensitivity)

Relation $\equiv \subseteq Prc \times Prc$ is deadlock sensitive whenever:

$$P \equiv Q \text{ implies } \left(\forall w \in Act^*. \, P \text{ has a } w\text{-deadlock iff } Q \text{ has a } w\text{-deadlock}\right).$$

Trace Equivalence

Trace language (Definition 3.2)

The trace language of $P \in Prc$ is defined by:

$$Tr(P) := \{w \in Act^* \mid \exists P' \in Prc.\, P \xrightarrow{w} P'\}.$$

Trace equivalence (Definition 3.2)

$P, Q \in Prc$ are called trace equivalent iff $Tr(P) = Tr(Q)$.

Trace equivalence is evidently an equivalence relation and is less discriminative than isomorphism.

Trace Equivalence is a Congruence

Theorem 11.8

Trace equivalence is a CCS congruence.

Proof.

By structural induction over the syntax of CCS processes. For $+$ this proceeds as follows:

- Let $P, Q \in Prc$ with $Tr(P) = Tr(Q)$.
- Then for $R \in Prc$ it holds:

$$Tr(P + R) = Tr(P) \cup Tr(R) = Tr(Q) \cup Tr(R) = Tr(Q + R).$$

- Thus, $P + R$ and $Q + R$ are trace equivalent.

For the other CCS constructs, the proof goes along similar lines. Exercise: do the proof for $\|$. $\square$

Two coffee machines

Example 11.9

Consider the coffee/tea machines *CTM* and its variant *CTM'*:

$$CTM = coin.\left(\overline{coffee}.CTM + \overline{tea}.CTM\right)$$

$$CTM' = coin.\overline{coffee}.CTM' + coin.\overline{tea}.CTM'.$$

Note the difference between the two processes. Nevertheless:

$$Tr(CTM) = Tr(CTM').$$

Are we satisfied? No, as *CTM* and *CTM'* differ in the context:

$$C(\cdot) = (\underbrace{\ \cdot\ }_{\text{hole}} \parallel CA) \setminus \{coin, coffee, tea\} \text{ with } CA = \overline{coin}.coffee.CA.$$

Why? $C(CTM')$ may yield a deadlock, but $C(CTM)$ does not.

Checking Trace Equivalence

Traces by automata

For finite-state P, the trace language $Tr(P)$ of process P is accepted by the (non-deterministic) finite automaton obtained from the LTS of P with initial state P and making all states accepting (final).

Theorem 11.10

Checking trace equivalence of two finite processes is PSPACE-complete.

Proof.

Checking whether $Tr(P) = Tr(Q)$, for finite-state P and Q, boils down to deciding whether their non-deterministic automata accept the same language. As this problem in automata theory is PSPACE-complete, it follows that checking $Tr(P) = Tr(Q)$ is PSPACE-complete. $\square$

Traces and Deadlocks

Example 11.11 (Traces and deadlocks)

Traces and deadlocks are independent in the following sense:

same traces
different deadlocks

different traces
same deadlocks

But: processes with finite trace sets and identical deadlocks are trace equivalent (since every trace is a prefix of some deadlock).

Summary: Trace Equivalence

1. Trace equivalence equates processes that have the same traces, i.e., action sequences
2. Isomorphism implies trace equivalence
3. Trace equivalence trivially implies trace equivalence
4. Trace equivalence is a CCS congruence
5. Trace equivalence is not deadlock sensitive.
6. Checking trace equivalence is PSPACE-complete

Completed Trace Equivalence

Definition 11.12 (Completed traces)

A completed trace of $P \in Prc$ is a sequence $w \in Act^*$ such that:

$$P \xrightarrow{w} Q \quad \text{and} \quad Q \not\longrightarrow$$

for some $Q \in Prc$.

The completed traces of process P may be seen as capturing its deadlock behaviour, as they are precisely the action sequences that could lead to a process from which no transition is possible (i.e., is a deadlock).

Exercise

Check whether completed trace equivalence is a congruence for restriction.

Further Variations of Trace Equivalence

Definition 11.13 (Ready trace equivalence) (Baeten et al.)

A sequence $A_0\alpha_0A_1\alpha_1\ldots\alpha_nA_n$ with $A_i \subseteq Act$ and $\alpha_i \in Act$ ($i \in \mathbb{N}$) is a ready trace of process P if $P = P_0 \xrightarrow{\alpha_0} P_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} P_n$ such that $A_i = \{\alpha \in Act \mid P_i \xrightarrow{\alpha}\}$. Processes P and Q are ready-trace equivalent if they have exactly the same set of ready traces.

Definition 11.14 (Failure trace equivalence) (Reed and Roscoe)

A sequence $A_0\alpha_0A_1\alpha_1\ldots\alpha_nA_n$ with $A_i \subseteq Act$ and $\alpha_i \in Act$ ($i \in \mathbb{N}$) is a failure trace of process P if $P = P_0 \xrightarrow{\alpha_0} P_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} P_n$ such that $A_i \cap \{\alpha \in Act \mid P_i \xrightarrow{\alpha}\} = \emptyset$. Processes P and Q are failure-trace equivalent if they have exactly the same set of failure traces.

Example 11.15

$\alpha.P + \alpha.Q$ and $\alpha.P + \alpha.Q + \alpha.(P + Q)$ are failure-trace equivalent for every $P, Q \in Prc$ and $\alpha \in Act$, but not ready-trace equivalent

Summary

1. Behavioural equivalences should be
   i. less distinctive than isomorphism
   ii. more distinctive than trace equivalence
   iii. a CCS congruence
   iv. deadlock sensitive
2. Trace equivalence
   i. equates processes that have the same traces, i.e., action sequences
   ii. is a CCS congruence
   iii. is not deadlock sensitive
   iv. checking trace equivalence is PSPACE-complete
3. Variations: completed, ready, and failure traces