Static Program Analysis

Lecture 9: Dataflow Analysis VIII (Conditional Interval Analysis & Java Virtual Machine)

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Outline

1 Recap: Taking Conditional Branches into Account

2 Interval Analysis with Assertions

3 The Java Virtual Machine

4 The Java Bytecode Verifier

5 The Type-Level Abstract Interpreter

Taking Conditional Branches into Account II

- **Solution:** introduce transfer functions for branches
- **First approach:** attach (negated) conditions as labels to control flow edges
  - advantage: no language modification required
  - disadvantage: entails extension of DFA framework
  - will not further be considered here
- **Second approach:** encode conditions as assertions (statements)
  - advantage: DFA framework can be reused
  - disadvantage: entails extension of WHILE language
  - the way we will follow

Definition (Labelled WHILE programs with assertions)

The syntax of labelled WHILE programs with assertions is defined by the following context-free grammar:

$$
\begin{aligned}
a &::= z \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \in AExp \\
b &::= t \mid a_1 = a_2 \mid a_1 > a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in BExp \\
c &::= [\texttt{skip}]^l \mid [x \texttt{ := } a]^l \mid c_1 ; c_2 \mid \\
&\quad\ \texttt{if } [b]^l \texttt{ then } c_1 \texttt{ else } c_2 \mid \texttt{while } [b]^l \texttt{ do } c \mid [\texttt{assert } b]^l \in Cmd
\end{aligned}
$$

To be done:

- Definition of transfer functions for `assert` blocks (depending on analysis problem)
- Idea: assertions as filters that let only "valid" information pass

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Interval Analysis with Assertions I

So far:

- The domain $(Int, \subseteq)$ of intervals over $\mathbb{Z}$ is defined by
  $$Int := \{[z_1, z_2] \mid z_1 \in \mathbb{Z} \cup \{-\infty\}, z_2 \in \mathbb{Z} \cup \{+\infty\}, z_1 \leq z_2\} \cup \{\emptyset\}$$
  where
  - $-\infty \leq z$, $z \leq +\infty$, and $-\infty \leq +\infty$ (for all $z \in \mathbb{Z}$)
  - $\emptyset \subseteq J$ (for all $J \in Int$)
  - $[y_1, y_2] \subseteq [z_1, z_2]$ iff $y_1 \geq z_1$ and $y_2 \leq z_2$
- Transfer functions $\{\varphi_l \mid l \in Lab\}$ are defined by
  $$\varphi_l(\delta) := \begin{cases} \delta & \text{if } B^l = \texttt{skip} \text{ or } B^l \in BExp \\ \delta[x \mapsto val_\delta(a)] & \text{if } B^l = (x \texttt{ := } a) \end{cases}$$
  where
  $$val_\delta(x) := \delta(x) \qquad val_\delta(a_1\texttt{+}a_2) := val_\delta(a_1) \oplus val_\delta(a_2)$$
  $$val_\delta(z) := [z, z] \qquad val_\delta(a_1\texttt{-}a_2) := val_\delta(a_1) \ominus val_\delta(a_2)$$
  $$val_\delta(a_1\texttt{*}a_2) := val_\delta(a_1) \odot val_\delta(a_2)$$
  with
  $$\emptyset \oplus J := J \oplus \emptyset := \emptyset \ominus J := \ldots := \emptyset$$
  $$[y_1, y_2] \oplus [z_1, z_2] := [y_1 + z_1, y_2 + z_2]$$
  $$[y_1, y_2] \ominus [z_1, z_2] := [y_1 - z_2, y_2 - z_1]$$
  $$[y_1, y_2] \odot [z_1, z_2] := [\bigsqcap\{y_1z_1, y_1z_2, y_2z_1, y_2z_2\}, \bigsqcup\{y_1z_1, y_1z_2, y_2z_1, y_2z_2\}]$$

Interval Analysis with Assertions II

Additionally for $B^l = (\texttt{assert}\ b)$, $\delta : Var_c \to Int$ and $x \in Var_c$:

$$\varphi_l(\delta)(x) := \begin{cases} \emptyset & \text{if } Z = \emptyset \\ \left[\bigsqcap_{\mathbb{Z}\cup\{-\infty\}} Z, \bigsqcup_{\mathbb{Z}\cup\{+\infty\}} Z\right] & \text{otherwise} \end{cases}$$

where

- $Z := \{\sigma(x) \mid \sigma \in \Sigma_\delta, val_\sigma(b) = \text{true}\}$
- $\Sigma_\delta := \{\sigma : Var_c \to \mathbb{Z} \mid \forall y \in Var_c : \sigma(y) \in \delta(y)\}$
  (and thus $\Sigma_\delta = \emptyset$ iff $\delta(y) = \emptyset$ for some $y \in Var_c$)
- $val_\sigma : BExp \to \mathbb{B}$ as before

Interval Analysis with Assertions III

Example 9.1

$$Var_c = \{\mathtt{x}, \mathtt{y}\},\ \delta = (\underbrace{[-\infty, 2]}_{\mathtt{x}}, \underbrace{[0, +\infty]}_{\mathtt{y}})$$

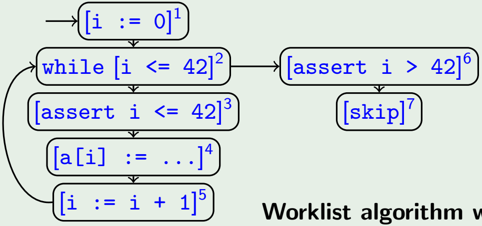
$$\Longrightarrow \begin{aligned} \varphi_{\mathtt{assert\ x>0}}(\delta) &= ([1, 2], [0, +\infty]) \\ \varphi_{\mathtt{assert\ x=y}}(\delta) &= ([0, 2], [0, 2]) \\ \varphi_{\mathtt{assert\ x>y}}(\delta) &= ([1, 2], [0, 1]) \\ \varphi_{\mathtt{assert\ x<y}}(\delta) &= ([-\infty, 2], [0, +\infty]) \end{aligned}$$

Remarks:

- Again for $B^l = (\mathtt{assert}\ b)$ and $\delta : Var_c \to Int$, $\varphi_l(\delta) \sqsubseteq \delta$ and hence $\Sigma_{\varphi_l(\delta)} \subseteq \Sigma_\delta$ ("filter")
- Again if $\mathsf{AI}_l(x) = \emptyset$ for some $l \in Lab_c$ and $x \in Var_c$, then l is unreachable (and $\mathsf{AI}_l(y) = \emptyset$ for all $y \in Var_c$)

Interval Analysis with Assertions IV

Example 9.2 (Interval analysis for array index; cf. Example 7.6)



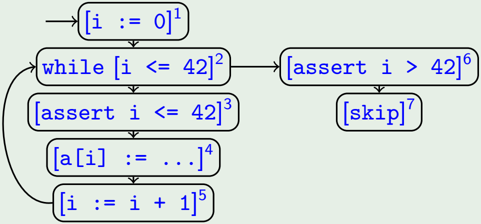
$$\begin{aligned}
\varphi_1(J) &= [0,0] \\
\varphi_2(J) &= J \\
\varphi_3(J) &= J \cap [-\infty, 42] \\
\varphi_4(J) &= J \\
\varphi_5(\emptyset) &= \emptyset \\
\varphi_5([i_1, i_2]) &= [i_1 + 1, i_2 + 1] \\
\varphi_6(J) &= J \cap [43, +\infty]
\end{aligned}$$

Worklist algorithm with widening:

W	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
12, 23, 34, 45, 52, 26, 67	$[-\infty, +\infty]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
23, 34, 45, 52, 26, 67	$[-\infty, +\infty]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
34, 45, 52, 26, 67	$[-\infty, +\infty]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
45, 52, 26, 67	$[-\infty, +\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$	$\emptyset$
52, 26, 67	$[-\infty, +\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
23, 26, 67	$[-\infty, +\infty]$	$[0, +\infty]$	$[0,0]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
34, 26, 67	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0,0]$	$[0,0]$	$\emptyset$	$\emptyset$
45, 26, 67	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0,0]$	$\emptyset$	$\emptyset$
52, 26, 67	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$\emptyset$	$\emptyset$
26, 67	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$\emptyset$	$\emptyset$
67	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$\emptyset$
ε	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[43, +\infty]$

Interval Analysis with Assertions V

Example 9.2 (Interval analysis for array index; continued)



$$
\begin{aligned}
\varphi_1(J) &= [0, 0]\\
\varphi_2(J) &= J\\
\varphi_3(J) &= J \cap [-\infty, 42]\\
\varphi_4(J) &= J\\
\varphi_5(\emptyset) &= \emptyset\\
\varphi_5([i_1, i_2]) &= [i_1 + 1, i_2 + 1]\\
\varphi_6(J) &= J \cap [43, +\infty]
\end{aligned}
$$

Narrowing:

	AI_1	AI_2	AI_3	AI_4	AI_5	AI_6	AI_7
$\mathrm{fix}^\nabla(\Phi_S)$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[43, +\infty]$
$\Phi_S(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, 42]$	$[0, +\infty]$	$[0, +\infty]$	$[43, +\infty]$
$\Phi_S^2(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty, +\infty]$	$[0, +\infty]$	$[0, +\infty]$	$[0, 42]$	$[0, 42]$	$[0, +\infty]$	$[43, +\infty]$
$\Phi_S^3(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty, +\infty]$	$[0, 43]$	$[0, +\infty]$	$[0, 42]$	$[0, 42]$	$[0, +\infty]$	$[43, +\infty]$
$\Phi_S^4(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty, +\infty]$	$[0, 43]$	$[0, 43]$	$[0, 42]$	$[0, 42]$	$[0, 43]$	$[43, +\infty]$
$\Phi_S^5(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty, +\infty]$	$[0, 43]$	$[0, 43]$	$[0, 42]$	$[0, 42]$	$[0, 43]$	$[43, 43]$
$\Phi_S^6(\mathrm{fix}^\nabla(\Phi_S))$	$[-\infty, +\infty]$	$[0, 43]$	$[0, 43]$	$[0, 42]$	$[0, 42]$	$[0, 43]$	$[43, 43]$

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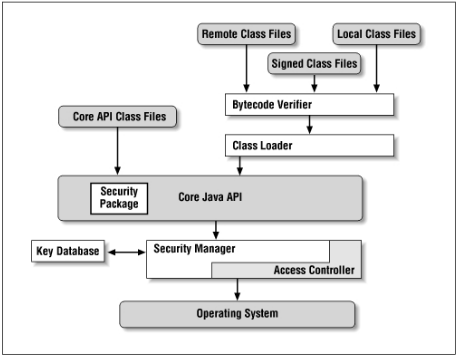
Java Bytecode

- Intermediate language between high-level language and machine code
- Execution on Java Virtual Machine (JVM)
- **Advantages:**
  - architecture independency (especially for web applications)
  - faster than pure (i.e., source code) interpretation
- **Problem:** security issues
  - destruction of data
  - modification of data
  - disclosure of personal information
  - modification of other programs

Java Security: the Sandbox

- Insulation layer providing indirect access to system resources
- Hardware access via API classes and methods
- Bytecode verification upon uploading
    - well-typedness
    - proper object referencing
    - proper control flow



The Java Virtual Machine

- Conventional stack-based abstract machine
- Supports object-oriented features: classes, methods, etc.
- Stack for intermediate results of expression evaluations
- Registers for source-level local variables and method parameters
- Both part of method activation record
  (and thus preserved across method calls)
- Method entry point specifies required number of registers (m_r) and stack slots (m_s; for memory allocation)
- (Most) instructions are typed

Example: Factorial Function

Example 9.3 (Factorial function)

Java source code:

```
static int factorial(int n)
{ int res;
  for (res = 1; n > 0; n--) res = res * n;
  return res; }
```

Corresponding JVM bytecode:

```
method static int factorial(int), 2 registers, 2 stack slots
   1: istore 0    // store n in register 0
   2: iconst_1    // push constant 1
   3: istore 1    // store res in register 1
   4: iload 0     // push n
   5: ifle 12     // if <= 0, go to end
   6: iload 1     // push res
   7: iload 0     // push n
   8: imul        // res * n on top of stack
   9: istore 1    // store in res
  10: iinc 0, -1  // decrement n
  11: goto 4      // go to loop header
  12: iload 1     // push res
  13: ireturn     // return res to caller
```

JVM Instruction Set (excerpt)

`iload` n: push integer from register n

`istore` n: pop integer into register n

`iconst_`z: push integer z

`aconst_null`: push null reference

`iadd`: add two topmost integers on stack and push sum

`getfield` C f τ: pops reference to object (of class C) and pushes value of field f (of type τ)

`putfield` C f τ: pops value v (of type τ) and reference to object o (of class C) and assigns v to field f of o

`new` C: creates new object (of class C) and pushes reference

`invoke` C M $\tau_0(\tau_1, \ldots, \tau_n)$: pops values $v_1, \ldots, v_n$ (of type $\tau_1, \ldots, \tau_n$) and reference to object (of class C), calls method M with parameters $v_1, \ldots, v_n$, and pushes return value (of type τ_0)

`if_icmpeq` l: pop two topmost integers from stack and jump to line l if equal

`ireturn`: return to caller with integer result on top of stack

($\approx$ 200 instructions in total)

Malicious Bytecode

Example 9.4 (Malicious bytecode)

```
1: iconst_5
2: iconst_1
3: putfield A f int
```

interprets second stack entry (5) as reference to object of class `A` and assigns first stack entry (1) to field `f` of this object

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Correctness of Bytecode

Conditions to ensure proper operation:

Type correctness: arguments of instructions always of expected type

No stack over-/underflow: never push to full stack or pop from empty stack

Code containment: PC must always point into the method code

Register initialization: load from non-parameter register only after store

Object initialization: constructor must be invoked before using class instance

Access control: operations must respect visibility modifiers (`private`/`protected`/`public`)

Options:

- dynamic checking at execution time ("defensive JVM approach")
  - expensive, slows down execution
- static checking at loading time (here)
  - verified code executable at full speed without extra dynamic checks

The Java Bytecode Verifier

Summary: dataflow analysis applied to type-level abstract interpretation of JVM

1. Association of type information with register and stack contents
   - set of types forms a complete lattice
2. Simulation of execution of instructions at type level
3. Use dataflow analysis to cover all concrete executions
4. Modularity: analysis proceeds method per method

(see X. Leroy: *Java Bytecode Verification: Algorithms and Formalizations*, Journal of Automated Reasoning 30(3-4), 2003, 235–269)

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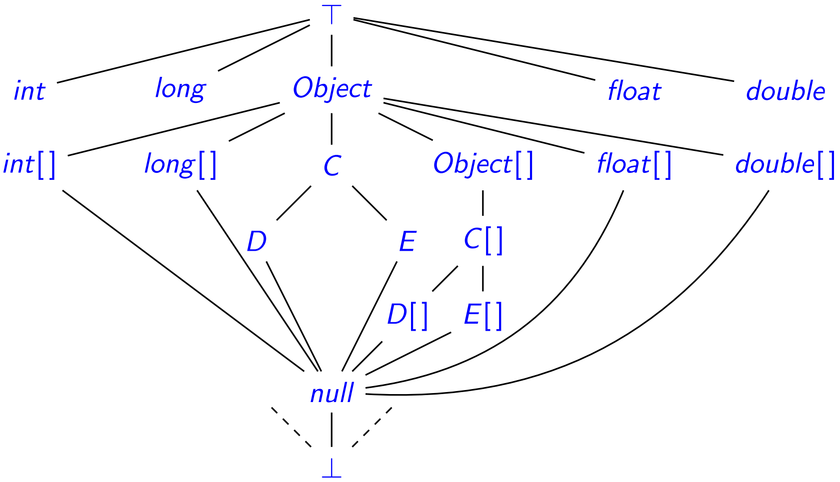
Types

The set of types, *Typ*, is composed of

- Primitive types:
  - *int* (covering *boolean*, *byte*, *char*, *short*)
  - *long*
  - *float*
  - *double*
- Object reference types: C for every class name C
- Array types: $\tau[\,]$ for every primitive or object reference type τ
- Method types: $\tau_0(\tau_1, \ldots, \tau_n)$ for $n \in \mathbb{N}$, $\tau_i \in$ *Typ*
- Special types:
  - *null* (null reference)
  - *Object* (any object)
  - $\top$ (contents of uninitialized registers, i.e., any value)
  - $\perp$ (absence of any value)

The Subtyping Relation (excerpt)

(C, D, E user-defined classes; D, E extending C)



Notation: $\tau_1 \sqsubseteq_t \tau_2$

The Type-Level Abstract Interpreter I

- **Idea:** execute JVM instructions on types (rather than concrete values)
  - stack type $S \in Typ^{\leq m_s}$ (top to the left)
  - register type $R : \{0, \ldots, m_r - 1\} \to Typ$
- Represented as transition relation

$$i : (S, R) \to (S', R')$$

  where
  - i: current instruction
  - (S, R): stack/register type before execution
  - (S', R'): stack/register type after execution
- Errors (type mismatch, stack over-/underflow, ...) denoted by absence of transition

The Type-Level Abstract Interpreter II

Some transition rules:

`iconst_`z : $(S, R) \to (int.S, R)$ if $|S| < m_s$

`aconst_null` : $(S, R) \to (null.S, R)$ if $|S| < m_s$

`iadd` : $(int.int.S, R) \to (int.S, R)$

`if_icmpeq` l : $(int.int.S, R) \to (S, R)$

`iload` n : $(S, R) \to (int.S, R)$ if $0 \leq n < m_r, R(n) = int, |S| < m_s$

`aload` n : $(S, R) \to (R(n).S, R)$ if $0 \leq n < m_r, R(n) \sqsubseteq_t Object, |S| < m_s$

`istore` n : $(int.S, R) \to (S, R[n \mapsto int])$ if $0 \leq n < m_r$

`astore` n : $(\tau.S, R) \to (S, R[n \mapsto \tau])$ if $0 \leq n < m_r, \tau \sqsubseteq_t Object$

`getfield` C f τ : $(D.S, R) \to (\tau.S, R)$ if $D \sqsubseteq_t C$

`putfield` C f τ : $(\tau'.D.S, R) \to (S, R)$ if $\tau' \sqsubseteq_t \tau, D \sqsubseteq_t C$

`invoke` C M σ : $(\tau'_n \ldots \tau'_1.\tau'.S, R) \to (\tau_0.S, R)$ if $\sigma = \tau_0(\tau_1, \ldots, \tau_n), \tau'_i \sqsubseteq_t \tau_i$ for $1 \leq i \leq n, \tau' \sqsubseteq_t C$

Some Theoretical Properties

Lemma 9.5

1. $(Typ, \sqsubseteq_t)$ *is a complete lattice satisfying ACC.*
2. *(Determinacy) The transitions of the abstract interpreter define a partial function: If* $i : (S, R) \to (S_1, R_1)$ *and* $i : (S, R) \to (S_2, R_2)$*, then* $S_1 = S_2$ *and* $R_1 = R_2$*.*
3. *(Soundness) If* $i : (S, R) \to (S', R')$*, then for all concrete states* (s, r) *matching* (S, R)*, the defensive JVM will not stop with a run-time type exception when applying* i *to* (s, r) *(but rather change to some* (s', r') *matching* (S', R')*).*

Proof.

see X. Leroy: *Java Bytecode Verification: Algorithms and Formalizations* □