Static Program Analysis Lecture 6: Dataflow Analysis V (MOP vs. Fixpoint Solution)

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- 2 Recap: Constant Propagation
- 3 Example of Constant Propagation Analysis
- 4 MOP vs. Fixpoint Solution



# The MOP Solution I

- Other solution method for dataflow systems
- MOP = Meet Over all Paths
- Analysis information for block B<sup>1</sup>
  - = least upper bound over all paths leading to /
  - = most precise information for *I* ("reference solution")

### Definition (Paths)

Let  $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$  be a dataflow system. For every  $l \in Lab$ , the set of paths up to l is given by

$$\begin{aligned} \mathsf{Path}(I) &:= \{ [l_1, \dots, l_{k-1}] \mid k \geq 1, l_1 \in E, \\ (l_i, l_{i+1}) \in F \text{ for every } 1 \leq i < k, l_k = l \}. \end{aligned}$$

For a path  $\pi = [I_1, \ldots, I_{k-1}] \in Path(I)$ , we define the transfer function  $\varphi_{\pi} : D \to D$  by

$$\varphi_{\pi} := \varphi_{I_{k-1}} \circ \ldots \circ \varphi_{I_1} \circ \mathsf{id}_D$$

(so that  $\varphi_{[]} = \mathrm{id}_D$ ).

### Definition (MOP solution)

Let  $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$  be a dataflow system where  $Lab = \{l_1, \ldots, l_n\}$ . The MOP solution for S is determined by  $mop(S) := (mop(l_1), \ldots, mop(l_n)) \in D^n$ where, for every  $l \in Lab$ ,  $mop(l) := | \{\varphi_{\pi}(\iota) \mid \pi \in Path(l)\}.$ 

#### Remark:

- *Path(1)* is generally infinite
- $\implies$  not clear how to compute mop(l)
  - In fact: MOP solution generally undecidable (later)

# 2 Recap: Constant Propagation

3 Example of Constant Propagation Analysis

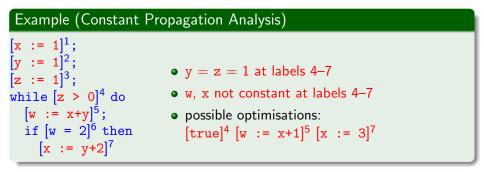
### 4 MOP vs. Fixpoint Solution



#### Constant Propagation Analysis

The goal of Constant Propagation Analysis is to determine, for each program point, whether a variable has a constant value whenever execution reaches that point.

Used for Constant Folding: replace reference to variable by constant value and evaluate constant expressions



# Formalising Constant Propagation Analysis I

The dataflow system  $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$  is given by

- set of labels  $Lab := Lab_c$ ,
- extremal labels E := {init(c)} (forward problem),
- flow relation F := flow(c) (forward problem),
- complete lattice  $(D, \sqsubseteq)$  where

• 
$$D := \{ \delta \mid \delta : Var_c \to \mathbb{Z} \cup \{ \bot, \top \} \}$$

- $\delta(x) = z \in \mathbb{Z}$ : x has constant value z
- $\delta(x) = \bot$ : x undefined
- $\delta(x) = \top$ : x overdefined (i.e., several possible values)
- $\sqsubseteq \subseteq D \times D$  defined by pointwise extension of  $\bot \sqsubseteq z \sqsubseteq \top$ (for every  $z \in \mathbb{Z}$ )

#### Example

$$Var_{c} = \{w, x, y, z\},\$$
  

$$\delta_{1} = (\underbrace{\bot}_{w}, \underbrace{1}_{x}, \underbrace{2}_{y}, \underbrace{\top}_{z}), \delta_{2} = (\underbrace{3}_{w}, \underbrace{1}_{x}, \underbrace{4}_{y}, \underbrace{\top}_{z})$$
  

$$\implies \delta_{1} \sqcup \delta_{2} = (\underbrace{3}_{w}, \underbrace{1}_{x}, \underbrace{\top}_{y}, \underbrace{\top}_{z})$$

Dataflow system  $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$  (continued):

- extremal value  $\iota := \delta_{\top} \in D$  where  $\delta_{\top}(x) := \top$  for every  $x \in Var_c$  (i.e., every x has (unknown) default value)
- transfer functions  $\{\varphi_I \mid I \in Lab\}$  defined by

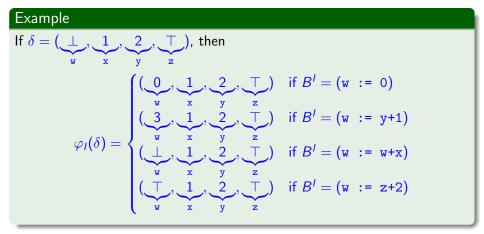
$$\varphi_{I}(\delta) := \begin{cases} \delta & \text{if } B^{I} = \text{skip or } B^{I} \in BExp\\ \delta[x \mapsto val_{\delta}(a)] & \text{if } B^{I} = (x := a) \end{cases}$$

where

$$\begin{array}{ll} \mathsf{val}_{\delta}(x) := \delta(x) \\ \mathsf{val}_{\delta}(z) := z \end{array} \quad \mathsf{val}_{\delta}(a_1 \ op \ a_2) := \begin{cases} z_1 \ op \ z_2 & \text{if } z_1, z_2 \in \mathbb{Z} \\ \bot & \text{if } z_1 = \bot \ or \ z_2 = \bot \\ \top & \text{otherwise} \end{cases}$$
  
For  $z_1 := \mathsf{val}_{\delta}(a_1) \text{ and } z_2 := \mathsf{val}_{\delta}(a_2)$ 



# Formalising Constant Propagation Analysis III





2 Recap: Constant Propagation

## 3 Example of Constant Propagation Analysis

## 4 MOP vs. Fixpoint Solution



### Example 6.1

Constant Propagation Analysis for

$$\begin{split} c &:= [x := 1]^{1}; \\ [y := 1]^{2}; \\ [z := 1]^{3}; \\ \text{while} [z > 0]^{4} \text{ do} \\ [w := x+y]^{5}; \\ \text{if} [w = 2]^{6} \text{ then} \\ [x := y+2]^{7} \end{split}$$

$$\begin{split} \varphi_1(a, b, c, d) &= (a, 1, c, d) \\ \varphi_2(a, b, c, d) &= (a, b, 1, d) \\ \varphi_3(a, b, c, d) &= (a, b, c, 1) \\ \varphi_4(a, b, c, d) &= (a, b, c, d) \\ \varphi_5(a, b, c, d) &= (b + c, b, c, d) \\ \varphi_6(a, b, c, d) &= (a, b, c, d) \\ \varphi_7(a, b, c, d) &= (a, c + 2, c, d) \end{split}$$

(for  $\delta = (\delta(\mathtt{w}), \delta(\mathtt{x}), \delta(\mathtt{y}), \delta(\mathtt{z})) \in D$ )

- Fixpoint solution (on the board)
- MOP solution (on the board)

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## Example 6.2 (Constant Propagation)

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\begin{split} c &:= \text{if} [z > 0]^1 \text{ then} \\ & [x := 2;]^2 \\ & [y := 3;]^3 \\ & \text{else} \\ & [x := 3;]^4 \\ & [y := 2;]^5 \\ & [z := x+y;]^6 \\ & [\cdots]^7 \end{split}
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Transfer functions (for  $\delta = (\delta(\mathbf{x}), \delta(\mathbf{y}), \delta(\mathbf{z})) \in D$ ):  $\varphi_1(a, b, c) = (a, b, c)$   $\varphi_2(a, b, c) = (2, b, c)$   $\varphi_3(a, b, c) = (a, 3, c)$   $\varphi_4(a, b, c) = (3, b, c)$   $\varphi_5(a, b, c) = (a, 2, c)$  $\varphi_6(a, b, c) = (a, b, a + b)$  • Fixpoint solution:  $\begin{array}{l} \mathsf{CP}_1 = \iota & = (\mathsf{T},\mathsf{T},\mathsf{T}) \\ \mathsf{CP}_2 = \varphi_1(\mathsf{CP}_1) & = (\mathsf{T},\mathsf{T},\mathsf{T}) \\ \mathsf{CP}_3 = \varphi_2(\mathsf{CP}_2) & = (2,\mathsf{T},\mathsf{T}) \\ \mathsf{CP}_4 = \varphi_1(\mathsf{CP}_1) & = (\mathsf{T},\mathsf{T},\mathsf{T}) \\ \mathsf{CP}_5 = \varphi_4(\mathsf{CP}_4) & = (3,\mathsf{T},\mathsf{T}) \\ \mathsf{CP}_6 = \varphi_3(\mathsf{CP}_3) \sqcup \varphi_5(\mathsf{CP}_5) \\ & = (2,3,\mathsf{T}) \sqcup (3,2,\mathsf{T}) = (\mathsf{T},\mathsf{T},\mathsf{T}) \\ \mathsf{CP}_7 = \varphi_6(\mathsf{CP}_6) & = (\mathsf{T},\mathsf{T},\mathsf{T}) \end{array}$ 

# MOP vs. Fixpoint Solution II

### Theorem 6.3 (MOP vs. Fixpoint Solution)

Let  $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$  be a dataflow system. Then  $mop(S) \sqsubseteq fix(\Phi_S)$ 

Reminder: by Definition 4.9,

$$\Phi_{S}: D^{n} \to D^{n}: (d_{1}, \dots, d_{n}) \mapsto (d'_{1}, \dots, d'_{n})$$
  
where  $Lab = \{1, \dots, n\}$  and, for each  $l \in Lab$ ,  
$$d'_{l} := \begin{cases} \iota & \text{if } l \in E \\ \bigsqcup \{\varphi_{l'}(d_{l'}) \mid (l', l) \in F\} & \text{otherwise} \end{cases}$$

Proof.

on the board

**Remark:** as Example 6.2 shows,  $mop(S) \neq fix(\Phi_S)$  is possible

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