Static Program Analysis Lecture 6: Dataflow Analysis V (MOP vs. Fixpoint Solution)

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1 Recap: The MOP Solution

- 2 Recap: Constant Propagation
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- 4 MOP vs. Fixpoint Solution



The MOP Solution I

- Other solution method for dataflow systems
- MOP = Meet Over all Paths
- Analysis information for block B¹
 - = least upper bound over all paths leading to /
 - = most precise information for *I* ("reference solution")

Definition (Paths)

Let $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ be a dataflow system. For every $l \in Lab$, the set of paths up to l is given by

$$\begin{aligned} \mathsf{Path}(I) &:= \{ [l_1, \dots, l_{k-1}] \mid k \geq 1, l_1 \in E, \\ (l_i, l_{i+1}) \in F \text{ for every } 1 \leq i < k, l_k = l \}. \end{aligned}$$

For a path $\pi = [I_1, \ldots, I_{k-1}] \in Path(I)$, we define the transfer function $\varphi_{\pi} : D \to D$ by

$$\varphi_{\pi} := \varphi_{I_{k-1}} \circ \ldots \circ \varphi_{I_1} \circ \mathsf{id}_D$$

(so that $\varphi_{[]} = \mathrm{id}_D$).

Definition (MOP solution)

Let $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ be a dataflow system where $Lab = \{l_1, \ldots, l_n\}$. The MOP solution for S is determined by $mop(S) := (mop(l_1), \ldots, mop(l_n)) \in D^n$ where, for every $l \in Lab$, $mop(l) := | \{\varphi_{\pi}(\iota) \mid \pi \in Path(l)\}.$

Remark:

- *Path(1)* is generally infinite
- \implies not clear how to compute mop(l)
 - In fact: MOP solution generally undecidable (later)

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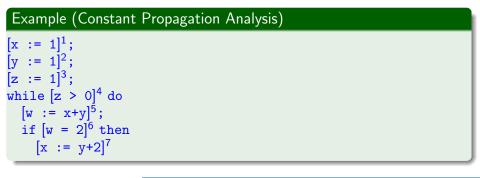
The goal of Constant Propagation Analysis is to determine, for each program point, whether a variable has a constant value whenever execution reaches that point.



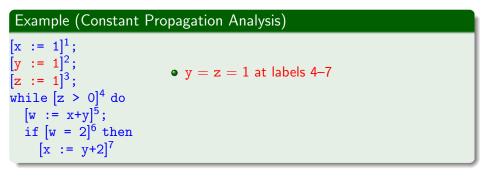
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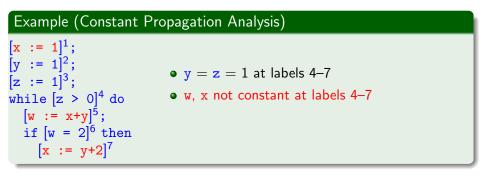
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Used for Constant Folding: replace reference to variable by constant value and evaluate constant expressions

Example (Constant Propagation Analysis) $\begin{bmatrix} x & := 1 \end{bmatrix}^{1};$ $\begin{bmatrix} y & := 1 \end{bmatrix}^{2};$ $\begin{bmatrix} z & := 1 \end{bmatrix}^{3};$ while $\begin{bmatrix} z > 0 \end{bmatrix}^{4}$ do $\begin{bmatrix} w & := x+y \end{bmatrix}^{5};$ if $\begin{bmatrix} w & = 2 \end{bmatrix}^{6}$ then $\begin{bmatrix} true \end{bmatrix}^{4} \begin{bmatrix} w & := x+1 \end{bmatrix}^{5} \begin{bmatrix} x & := 3 \end{bmatrix}^{7}$ $\begin{bmatrix} x & := y+2 \end{bmatrix}^{7}$

Formalising Constant Propagation Analysis I

The dataflow system $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ is given by

- set of labels $Lab := Lab_c$,
- extremal labels E := {init(c)} (forward problem),
- flow relation F := flow(c) (forward problem),
- complete lattice (D, \sqsubseteq) where

•
$$D := \{ \delta \mid \delta : Var_c \to \mathbb{Z} \cup \{ \bot, \top \} \}$$

- $\delta(x) = z \in \mathbb{Z}$: x has constant value z
- $\delta(x) = \bot$: x undefined
- $\delta(x) = \top$: x overdefined (i.e., several possible values)
- $\sqsubseteq \subseteq D \times D$ defined by pointwise extension of $\bot \sqsubseteq z \sqsubseteq \top$ (for every $z \in \mathbb{Z}$)

Example

$$Var_{c} = \{w, x, y, z\},\$$

$$\delta_{1} = (\underbrace{\bot}_{w}, \underbrace{1}_{x}, \underbrace{2}_{y}, \underbrace{\top}_{z}), \delta_{2} = (\underbrace{3}_{w}, \underbrace{1}_{x}, \underbrace{4}_{y}, \underbrace{\top}_{z})$$

$$\implies \delta_{1} \sqcup \delta_{2} = (\underbrace{3}_{w}, \underbrace{1}_{x}, \underbrace{\top}_{y}, \underbrace{\top}_{z})$$

Dataflow system $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ (continued):

- extremal value $\iota := \delta_{\top} \in D$ where $\delta_{\top}(x) := \top$ for every $x \in Var_c$ (i.e., every x has (unknown) default value)
- transfer functions $\{\varphi_I \mid I \in Lab\}$ defined by

$$\varphi_{I}(\delta) := \begin{cases} \delta & \text{if } B^{I} = \text{skip or } B^{I} \in BExp\\ \delta[x \mapsto val_{\delta}(a)] & \text{if } B^{I} = (x := a) \end{cases}$$

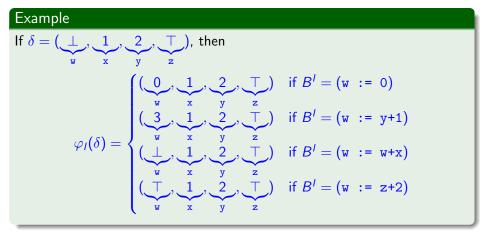
where

$$\begin{array}{ll} \mathsf{val}_{\delta}(x) := \delta(x) \\ \mathsf{val}_{\delta}(z) := z \end{array} \quad \mathsf{val}_{\delta}(a_1 \ op \ a_2) := \begin{cases} z_1 \ op \ z_2 & \text{if } z_1, z_2 \in \mathbb{Z} \\ \bot & \text{if } z_1 = \bot \text{ or } z_2 = \bot \\ \top & \text{otherwise} \end{cases}$$

For $z_1 := \mathsf{val}_{\delta}(a_1) \text{ and } z_2 := \mathsf{val}_{\delta}(a_2)$



Formalising Constant Propagation Analysis III





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Example 6.1

Constant Propagation Analysis for

$$\begin{split} c &:= [x := 1]^{1}; \\ [y := 1]^{2}; \\ [z := 1]^{3}; \\ \text{while} [z > 0]^{4} \text{ do} \\ [w := x+y]^{5}; \\ \text{if} [w = 2]^{6} \text{ then} \\ [x := y+2]^{7} \end{split}$$

$$\begin{split} \varphi_1(a, b, c, d) &= (a, 1, c, d) \\ \varphi_2(a, b, c, d) &= (a, b, 1, d) \\ \varphi_3(a, b, c, d) &= (a, b, c, 1) \\ \varphi_4(a, b, c, d) &= (a, b, c, d) \\ \varphi_5(a, b, c, d) &= (b + c, b, c, d) \\ \varphi_6(a, b, c, d) &= (a, b, c, d) \\ \varphi_7(a, b, c, d) &= (a, c + 2, c, d) \end{split}$$

(for $\delta = (\delta(\mathtt{w}), \delta(\mathtt{x}), \delta(\mathtt{y}), \delta(\mathtt{z})) \in D$)

- Fixpoint solution (on the board)
- MOP solution (on the board)

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MOP vs. Fixpoint Solution I

Example 6.2 (Constant Propagation)

```
c := if [z > 0]^{1} then 
 [x := 2;]^{2} 
 [y := 3;]^{3} 
 else 
 [x := 3;]^{4} 
 [y := 2;]^{5} 
 [z := x+y;]^{6} 
 [...]^{7}
```



MOP vs. Fixpoint Solution I

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Transfer functions (for $\delta = (\delta(\mathbf{x}), \delta(\mathbf{y}), \delta(\mathbf{z})) \in D$): $\varphi_1(a, b, c) = (a, b, c)$ $\varphi_2(a, b, c) = (2, b, c)$ $\varphi_3(a, b, c) = (a, 3, c)$ $\varphi_4(a, b, c) = (3, b, c)$ $\varphi_5(a, b, c) = (a, 2, c)$ $\varphi_6(a, b, c) = (a, b, a + b)$

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\begin{split} c &:= \text{if} [z > 0]^1 \text{ then} \\ & [x := 2;]^2 \\ & [y := 3;]^3 \\ & \text{else} \\ & [x := 3;]^4 \\ & [y := 2;]^5 \\ & [z := x+y;]^6 \\ & [\cdots]^7 \end{split}
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$$\begin{array}{l} \textbf{G} \quad \text{Fixpoint solution:} \\ CP_1 = \iota & = (T, T, T) \\ CP_2 = \varphi_1(CP_1) & = (T, T, T) \\ CP_3 = \varphi_2(CP_2) & = (2, T, T) \\ CP_4 = \varphi_1(CP_1) & = (T, T, T) \\ CP_5 = \varphi_4(CP_4) & = (3, T, T) \\ CP_6 = \varphi_3(CP_3) \sqcup \varphi_5(CP_5) \\ & = (2, 3, T) \sqcup (3, 2, T) = (T, T, T) \\ CP_7 = \varphi_6(CP_6) & = (T, T, T) \end{array}$$



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\begin{split} c &:= \text{if} [z > 0]^1 \text{ then} \\ & [x := 2;]^2 \\ & [y := 3;]^3 \\ & \text{else} \\ & [x := 3;]^4 \\ & [y := 2;]^5 \\ & [z := x+y;]^6 \\ & [\cdots]^7 \end{split}
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MOP vs. Fixpoint Solution II

Theorem 6.3 (MOP vs. Fixpoint Solution)

Let $S = (Lab, E, F, (D, \sqsubseteq), \iota, \varphi)$ be a dataflow system. Then $mop(S) \sqsubseteq fix(\Phi_S)$

Reminder: by Definition 4.9,

 $\Phi_{S}: D^{n} \to D^{n}: (d_{1}, \dots, d_{n}) \mapsto (d'_{1}, \dots, d'_{n})$ where $Lab = \{1, \dots, n\}$ and, for each $l \in Lab$, $d'_{l} := \begin{cases} \iota & \text{if } l \in E \\ \bigsqcup \{\varphi_{l'}(d_{l'}) \mid (l', l) \in F\} & \text{otherwise} \end{cases}$



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$$d'_{l} := \begin{cases} \iota & \text{if } l \in E \\ \bigsqcup \{\varphi_{l'}(d_{l'}) \mid (l', l) \in F\} & \text{otherwise} \end{cases}$$

Proof.

on the board

Remark: as Example 6.2 shows, $mop(S) \neq fix(\Phi_S)$ is possible

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