

Static Program Analysis

Lecture 21: Shape Analysis & Final Remarks

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)



noll@cs.rwth-aachen.de

<http://moves.rwth-aachen.de/teaching/ws-1415/spa/>

Winter Semester 2014/15

- 1 Recap: Pointer Analysis
- 2 Shape Analysis
- 3 Further Topic in Program Analysis
- 4 Final Remarks

The Shape Analysis Approach

- **Goal:** determine the **possible shapes of a dynamically allocated data structure** at given program point
- **Interesting information:**
 - **data types** (to avoid type errors, such as dereferencing `nil`)
 - **aliasing** (different pointer variables having same value)
 - **sharing** (different heap pointers referencing same location)
 - **reachability** of nodes (garbage collection)
 - **disjointness** of heap regions (parallelizability)
 - **shapes** (lists, trees, absence of cycles, ...)
- **Concrete questions:**
 - Does `x.next` point to a shared element?
 - Does a variable `p` point to an allocated element every time `p` is dereferenced?
 - Does a variable point to an acyclic list?
 - Does a variable point to a doubly-linked list?
 - Can a loop or procedure cause a memory leak?
- **Here:** basic outline; details in [Nielson/Nielson/Hankin 2005, Sct. 2.6]

Syntactic categories:

Category	Domain	Meta variable
Arithmetic expressions	$AExp$	a
Boolean expressions	$BExp$	b
Selector names	Sel	sel
Pointer expressions	$PExp$	p
Commands (statements)	Cmd	c

Context-free grammar:

$a ::= z \mid x \mid a_1 + a_2 \mid \dots \mid p \mid \text{nil} \in AExp$

$b ::= t \mid a_1 = a_2 \mid b_1 \wedge b_2 \mid \dots \mid \text{is-nil}(p) \in BExp$

$p ::= x \mid x.sel$

$c ::= [\text{skip}]' \mid [p := a]' \mid c_1 ; c_2 \mid \text{if } [b]' \text{ then } c_1 \text{ else } c_2 \mid$
 $\text{while } [b]' \text{ do } c \mid [\text{malloc } p]' \in Cmd$

Approach: representation of (infinitely many) concrete heap states by (finitely many) abstract **shape graphs**

- **abstract nodes** X = sets of variables
- interpretation: $x \in X$ iff x points to concrete node represented by X
- \emptyset represents all concrete nodes that are **not directly addressed** by pointer variables
- $x, y \in X$ (with $x \neq y$) indicate **aliasing** (as x and y point to the same concrete node)
- if $x.sel$ and y refer to the same heap address and if X, Y are abstract nodes with $x \in X$ and $y \in Y$, this yields **abstract edge** $X \xrightarrow{sel} Y$
- **transfer functions** transform (sets of) shape graphs

Definition (Shape graph)

A **shape graph** $G = (S, H)$ consists of

- a set $S \subseteq 2^{\text{Var}}$ of **abstract locations** and
- an **abstract heap** $H \subseteq S \times \text{Sel} \times S$
 - notation: $X \xrightarrow{\text{sel}} Y$ for $(X, \text{sel}, Y) \in H$

with the following properties:

Disjointness: $X, Y \in S \implies X = Y$ or $X \cap Y = \emptyset$

(a variable can refer to at most one heap location)

Determinacy: $X \neq \emptyset$ and $X \xrightarrow{\text{sel}} Y$ and $X \xrightarrow{\text{sel}} Z \implies Y = Z$

(target location is unique if source node is unique)

SG denotes the set of all shape graphs.

Remark: the following example shows that determinacy requires $X \neq \emptyset$:

Concrete: $y \longrightarrow \bullet \xleftarrow{\text{sel}} \bullet$ Abstract: $Y = \{y\} \xleftarrow{\text{sel}} X = \emptyset \xrightarrow{\text{sel}} Z = \{z\}$
 $z \longrightarrow \bullet \xleftarrow{\text{sel}} \bullet$

Example

Let $G = (S, H)$ be a shape graph. Then the following concrete heap properties can be expressed as conditions on G :

- $x \neq \text{nil}$
 $\iff \exists X \in S : x \in X$
- $x = y \neq \text{nil}$ (aliasing)
 $\iff \exists Z \in S : x, y \in Z$
- $x.\text{sel1} = y.\text{sel2} \neq \text{nil}$ (sharing)
 $\implies \exists X, Y, Z \in S : x \in X, y \in Y, X \xrightarrow{\text{sel1}} Z \xleftarrow{\text{sel2}} Y$
(“ $\xleftarrow{\quad}$ ” only valid if $Z \neq \emptyset$)

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Shape Analysis

The goal of **Shape Analysis** is to determine, for each program point, a set of **shape graphs** that represent **all heap structures** which can occur during program execution at that point.

- **Forward** analysis
- **Domain:** $(D, \sqsubseteq) := (2^{SG}, \subseteq)$
 - Var, Sel finite $\implies SG$ finite $\implies 2^{SG}$ finite $\implies ACC$
- **Extremal value:** $\iota := \{\text{shape graphs for possible initial values of } Var\}$

Example 21.1 (List reversal; cf. Example 20.4)

- Variables: $Var = \{x, y, z\}$
- Assumption: x points to any (finite, non-cyclic) list, $y = z = \text{nil}$

$$\implies \iota = \left\{ \underbrace{(\emptyset, \emptyset)}_{\text{empty}}, \underbrace{\boxed{\{x\}}}_{1 \text{ elem.}}, \underbrace{\boxed{\{x\}} \xrightarrow{\text{next}} \boxed{\emptyset}}_{2 \text{ elem.}}, \underbrace{\boxed{\{x\}} \xrightarrow{\text{next}} \boxed{\emptyset} \xrightarrow{\text{next}} \boxed{\emptyset}}_{\geq 3 \text{ elem.}} \right\}$$

The Transfer Functions

Transfer functions: $\varphi_I : 2^{SG} \rightarrow 2^{SG}$ (monotonic)

- Transform each single shape graph into a set of shape graphs:

$$\varphi_I(\{G_1, \dots, G_n\}) = \bigcup_{i=1}^n \varphi_I(G_i)$$

- $\varphi_I(G)$ determined by B^I (where $G = (S, H)$):

- $[\text{skip}]^I$: $\varphi_I(G) := \{G\}$

- $[b]^I$: $\varphi_I(G) := \{G\}$

- $[p := a]^I$: case-by-case analysis w.r.t. p and a

- [Nielsen/Nielsen/Hankin 2005, Sct. 2.6.3]: 12 cases

- may involve (high degree of) non-determinism

- see example on following slide

- $[\text{malloc } x]^I$: $\varphi_I(G) := \{(S' \cup \{\{x\}\}, H')\}$ where

- $S' := \{X \setminus \{x\} \mid X \in S\}$

- $H' := H \cap S' \times \text{Sel} \times S'$

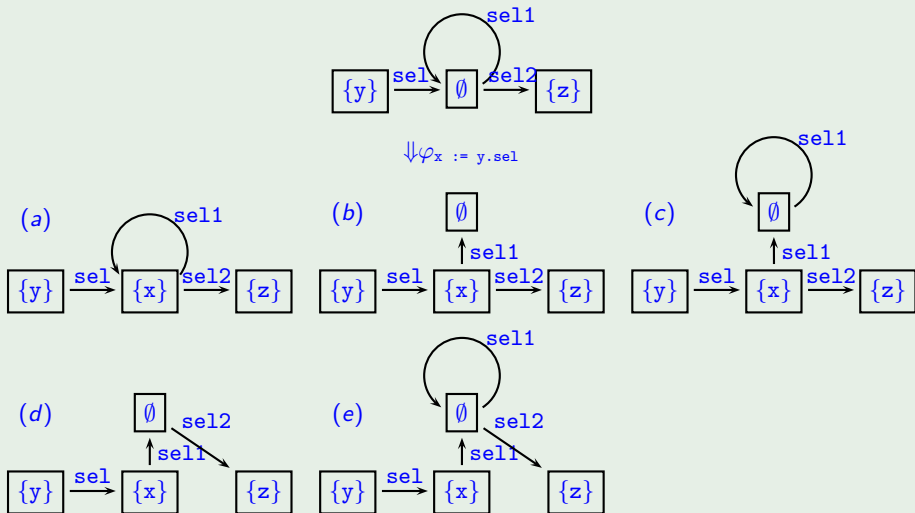
- $[\text{malloc } x.\text{sel}]^I$: equivalent to $[\text{malloc } t]^{I_1}; [x.\text{sel} := t]^{I_2}; [t := \text{nil}]^{I_3}$;
(with fresh $t \in \text{Var}$ and $I_1, I_2, I_3 \in \text{Lab}$)

- Crucial for **soundness**: **safety of approximation**

If shape graph G approximates heap h and $h \xrightarrow{B^I} h'$,

then there exists $G' \in \varphi_I(G)$ such that G' approximates h'

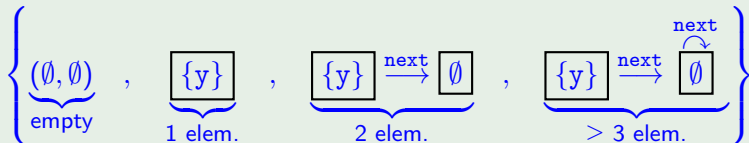
Example 21.2



Application to List Reversal

Example 21.3 (List reversal; cf. Example 20.4)

Shape analysis of list reversal program yields **final result**



Interpretation:

- + Result again a **finite list**
- but potentially **cyclic** (may be a “lasso”, but not a ring)
- also “**reversal**” **property** not guaranteed

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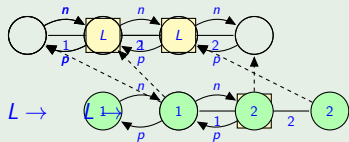
Dedicated Algorithms for Pointer Analysis

- **nil Pointer Analysis:** checks whether dereferencing operations possibly involve **nil** pointers
 - with shape analysis: possible for $x \in Var$ if there exists (reachable) $G = (S, H)$ such that $x \notin \bigcup_{x \in S} X$
- **Points-To Analysis:** yields function pt that for each $x \in Var$ returns set $pt(x)$ of possible pointer targets
 - x and y may be aliases if $pt(x) \cap pt(y) \neq \emptyset$
 - with shape analysis: there exists (reachable) $G = (S, H)$ and $Z \in S$ such that $x, y \in Z$
- Usually **faster** and sometimes **more precise** than shape analysis, but **less general** (only “shallow” properties)
- Fastest algorithms are **flow-insensitive** (points-to edges only added but never removed)

Graph Grammar Approaches to Pointer Analysis

- e.g., J. Heinen, C. Jansen, J.-P. Katoen, T. Noll: *Verifying Pointer Programs using Graph Grammars*. Science of Computer Programming 97, 157–162, 2015
- idea: specify data structures by **graph production rules**
- **concretization** by forward application
- **abstraction** by backward application
- all pointer operations remain **concrete**
⇒ avoids complicated definition of transfer functions

Example 21.4 (Doubly-linked lists)



- **So far:** **semantics** and **dataflow analysis** of programs considered independently (formal soundness proofs only for abstract interpretation; cf. Lecture 13)
- Of course both are (and should be) related!
- To this aim: compare results of **concrete semantics** (Definition 11.9) with **outcome of analysis**
- **Example:** **correctness of Constant Propagation**

Let $c \in \text{Cmd}$ with $l_0 = \text{init}(c)$, and let $l \in \text{Lab}_c$, $x \in \text{Var}$, and $z \in \mathbb{Z}$ such that $\text{CP}_l(x) = z$. Then for all $\sigma_0, \sigma \in \Sigma$ such that $\langle l_0, \sigma_0 \rangle \rightarrow^* \langle l, \sigma \rangle$, $\sigma(x) = z$.

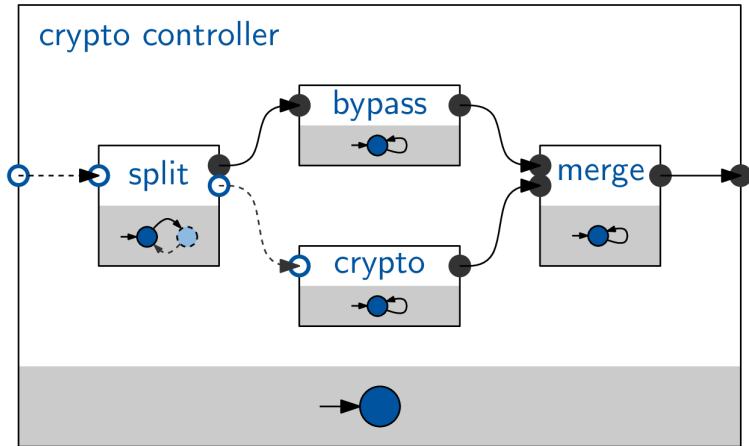
- see [Nielson/Nielson/Hankin 2005, Sct. 2.2]

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- **Schedule** online
 - 12 + 24 March, 8 April
 - see <http://moves.rwth-aachen.de/teaching/ws-1415/spa/>
- **Q&A session** on Tuesday, 24 February, 14:00–15:30, AH 6
 - please submit questions beforehand to dehnert@cs.rwth-aachen.de or benjamin.kaminski@cs.rwth-aachen.de
 - contact me in case of unresolved/later questions

- **Computer security**: system architectures that disallow sensitive information to be “leaked” to unauthorised entities
- Critical: **covert channels** that expose information
- Requires analysis of **information flows** within and between architectural components
- Standard approaches (non-interference, slicing) ignore **encryption**
- Goal: **analysis of cryptographically-masked information flows using slicing techniques**

Crypto controller



Introduction to Model Checking [Katoen; V3 Ü2]

- 1 Labelled transition systems
- 2 Classification of properties: safety, liveness, fairness
- 3 Temporal logics LTL and CTL
- 4 Model checking algorithms
- 5 Abstraction using (bi-)simulation

Semantics and Verification of Software [Noll; V3 Ü2]

- 1 The imperative model language WHILE
- 2 Operational, denotational and axiomatic semantics of WHILE
- 3 Equivalence of the semantics
- 4 Applications: compiler correctness, ...
- 5 Extensions: procedures, non-determinism, concurrency