Exercise 1 (Extending the Java Analysis with Array Operations): (3 Points)

In this exercise we will extend the JVM instructions given in the lecture by the following instructions.

- `aaload`, pops reference to an array a and an integer i from stack and pushes a reference to the i-th entry of a
- `aastore`, pops reference to an array a, and integer i and a reference to an object o from stack and assigns o to the i-th enty of a
- `newarray` τ, pops an integer $count$ and pushes a reference to an array of size $count$ and type τ onto the stack

a) Provide the transitions rules of the type-level abstract interpreter for the instructions given above.

b) Perform a *type correctness* analysis for the following Java bytecode.

```
iconst 10
istore 1
iload 1
newarray A
astore 2
iconst 1
istore 3
iload 1
iload 3
if_icmgeq 1
aload 2
iload 3
aload 1
aastore
iload 3
iconst 1
iadd
istore 3
```

Exercise 2 (Galois Connections): (3 Points)

Let $\Sigma = \{a, b, c\}$, Σ^* be the set of finite words over the alphabet Σ and $\preceq$ be the prefix relation on Σ^*, i.e. $w \preceq v$ if and only if w is a prefix of v. Let $v \sqsubseteq_\Sigma w \Leftrightarrow w \preceq v$.

a) Prove or disprove that $(\Sigma^*, \sqsubseteq_\Sigma)$ is a complete lattice.

b) Consider $L = (2^{\Sigma^*}, \subseteq)$ and $M = (\Sigma^* \cup \{\bot\}, \sqsubseteq_M)$ where

$$\text{(i) } v \sqsubseteq_M w \text{ if } w \preceq v \text{ and } v, w \in \Sigma^* \text{ and (ii) } \bot \sqsubseteq_M v \text{ for all } v \in M.$$

Define a Galois connection (α, γ), $\alpha\colon L \to M$, $\gamma\colon M \to L$ between the two complete lattices L and M such that the image of α is infinite and prove that (α, γ) is indeed a Galois connection.

Exercise 3 (Properties of Galois Connections): (4 Points)

Let $(L, \sqsubseteq_L)$ and $(M, \sqsubseteq_M)$ be complete lattices with $\bot_K$ be the least element of lattice K. Further let (α, γ) be a Galois connection with $\alpha : L \to M$ and $\gamma : M \to L$. Prove or disprove the following statements.

a) $\alpha(\bot_L) = \bot_M$

b) $\gamma(\bot_M) = \bot_L$

c) $\alpha \circ \gamma \circ \alpha = \alpha$

d) $\gamma \circ \alpha \circ \gamma = \gamma$