Theorem 2. *L is safely realizable iff L is weakly closed under $\models$ and closed under $\models^{df}$.*

$\Longrightarrow$. Assume L is safely realizable. Then:

1. L is realizable, and by the previous theorem, L is closed under $\models$, and thus L is weakly closed under $\models$.
2. There exists a deadlock-free weak CFM A with $Lin(A) = L$. As A is weak and deadlock-free, it follows that $Lin(A) = L$ is closed under $\models^{df}$.

$\Longleftarrow$. Assume L is weakly closed under $\models$ and closed under $\models^{df}$. Let $L_p = \{w \upharpoonright p \mid w \in L\}$ for any process $p \in P$. Since L is finite, L_p is regular. Let DFA A_p (with state set Q_p, initial state s^p_{init} and set F_p of accepting states) be such that $L(A_p) = L_p$. W.l.o.g. we assume that all states in A_p are productive, i.e., for any state q in A_p it is possible to reach a state in F_p. Now consider the weak CFM: $A = ((A_p)_{p\in P}, s_{init}, F)$ with: $s_{init} = \prod_{p\in P} s^p_{init}$, thus $s_{init} = (s^{p_1}_{init}, \ldots, s^{p_n}_{init})$, $F = \prod_{p\in P} F_p$ with $F_p \subseteq Q_p$.

Claim: A is deadlock-free and $Lin(A) = L$. Obviously, then L is safely realizable. The proof of this claim goes as follows:

1. $Lin(A) = L$. This is proven by:
   - $\supseteq$. Let $w \in L$. Then, for every process p, $w \upharpoonright p \in L_p$. Thus, DFA A_p has an accepting run for $w \upharpoonright p$ and as $F = \prod_{p\in P} F_p$, CFM A has an accepting run for w. So, $w \in Lin(A)$.
   - $\subseteq$. Let $w \in Lin(A)$. As every word in $Lin(A)$ is well-formed, w is well-formed. Since $F = \prod_{p\in P} F_p$, $w \upharpoonright p \in L_p$ for each process p. Thus $L \models w$. Since L is weakly closed under $\models$, it holds $w \in L$.
2. A is deadlock-free. This is proven as follows. Assume A has successfully read the input word $w \in \mathtt{Act}^*$. The word w may be either accepted or not. If it is accepted, there is nothing to prove. Assume w is not accepted. As A has successfully read w, for every process p, $w \upharpoonright p$ is a prefix of a word in L_p. Since L is closed under $\models^{df}$, it follows that $w \in pref(L)$. Let $w.u \in L$ for $u \neq \epsilon$. As A_p is deterministic, it has a unique (local) accepting run for $w.u \upharpoonright p$. This applies to every process p. As $F = \prod_{p\in P} F_p$, it follows that CFM A has a unique accepting run for $w.u$. As this applies to every input word w, A is deadlock-free.