Overview

Theoretical Foundations of the UML

Lecture 3: Races

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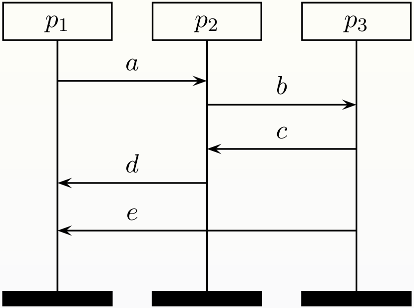
Summary of Lecture #2

1. A Message Sequence Chart is a partial order
   - between send and receive events
   - totally ordered per process — vertical ordering
   - receive events happen after their send events — horizontal ordering
   - respecting the first-in first out (FIFO) property

2. Linearizations are totally ordered extensions of partial orders
   - all linearizations of an MSC are well-formed
     1. every receive is preceded by a corresponding send
     2. respects the FIFO ordering
     3. no send events without corresponding receive

3. Every well-formed word can be transformed into an MSC
   - two linearizations of the same MSC yield isomorphic MSCs

4. Thus: there is a 1-to-1 relation between MSCs and its linearizations



These pictures are formalized using partial orders.

Message Sequence Chart (MSC) (1)

Definition

An MSC $M = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$ with:

- $\mathcal{P}$, a finite set of processes $\{p_1, p_2, \ldots, p_n\}$ with $n > 1$
- E, a finite set of events

$$E = \biguplus_{p \in \mathcal{P}} E_p = E_? \mathbin{\dot\cup} E_!$$

- $\mathcal{C}$, a finite set of message contents
- $l : E \to Act$, a labelling function defined by:

$$l(e) = \begin{cases} !(p,q,a) & \text{if} \quad e \in E_p \cap E_! \\ ?(p,q,a) & \text{if} \quad e \in E_p \cap E_? \end{cases} \text{, for } p \neq q \in \mathcal{P},\ a \in \mathcal{C}$$

Message Sequence Chart (MSC) (2)

Definition

- $m : E_! \to E_?$ a bijection ("matching function"), satisfying:

$$m(e) = e' \wedge l(e) = !(p, q, a) \text{ implies } l(e') = ?(q, p, a) \;\; (p \neq q,\; a \in \mathcal{C})$$

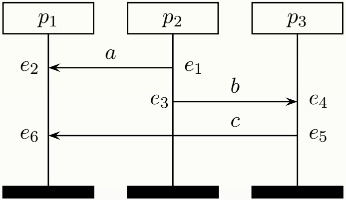
- $\preceq \subseteq E \times E$ is a partial order ("visual order") defined by:

$$\preccurlyeq \; = \Big(\underbrace{\bigcup_{p \in \mathcal{P}} <_p}_{<_p \text{ is a total order = "top-to-bottom" order on process } p} \;\cup\; \underbrace{\{(e, m(e)) \mid e \in E_!\}}_{\text{communication order } <_c} \Big)^*$$

where for relation R, R^* denotes its reflexive and transitive closure.

Example

Visual order can be misleading



can e_6 occur before e_2?

If message b takes much shorter than message a, then c might arrive at p_1 before a.

In practice, e_6 might occur before e_2, but $e_2 <_{p_1} e_6$ and thus $e_2 \preceq e_6$.

This is misleading and called a race.

What is a race?

A race condition asserts a particular order of events will occur because of the visual ordering (i.e., the partial order $\preceq$) when, in practice, this order cannot be guaranteed to hold.

Q: When are race conditions possible and how to detect them?

Causal order

Main principles:

- Send events should happen before their matching receive events
- The ordering of events wrt. sends on same process is unaffected
- Receive events on a process sent from the same process are ordered as their sends

Definition

For MSC $M = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$, relation $\ll \subseteq E \times E$ is defined by:

$$
\begin{array}{lll}
e \ll e' & \text{iff} & e' = m(e) \\
 & \text{or} & e <_p e' \text{ and } E_! \cap \{e, e'\} \neq \varnothing \\
 & \text{or} & e, e' \in E_p \cap E_? \text{ and } m^{-1}(e) <_q m^{-1}(e')
\end{array}
$$

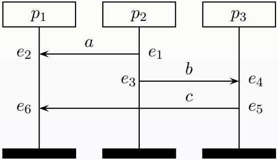
$\ll^*$ is a partial order (referred to as causal order) in which events at the same process are not necessarily ordered.

Causal order: example

Definition

For MSC $M = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$, relation $\ll \subseteq E \times E$ is defined by:

$$
\begin{array}{lll}
e \ll e' & \text{iff} & e' = m(e) \\
 & \text{or} & e <_p e' \text{ and } E_! \cap \{e, e'\} \neq \varnothing \\
 & \text{or} & e, e' \in E_p \cap E_? \text{ and } m^{-1}(e) <_q m^{-1}(e')
\end{array}
$$



Example

$e_1 \ll e_2$, $e_3 \ll e_4$, $e_5 \ll e_6$, $e_1 \ll e_3$, $e_4 \ll e_5$, not $(e_2 \ll e_6)$

Races

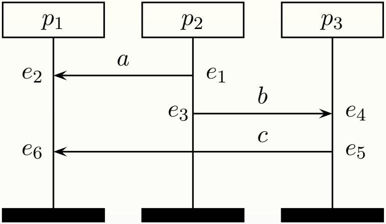
Definition

MSC M contains a race if for some $e, e' \in E_?$ and process p:

$$e <_p e' \text{ but not } (e \ll^* e')$$

where $\ll^* \subseteq E \times E$ is the reflexive and transitive closure of $\ll$.

Race: example



Visual order versus causal order

1. $e_1 \preceq e_2,\ \ e_3 \preceq e_4,\ \ e_5 \preceq e_6,\ \ e_1 \preceq e_3,\ \ e_4 \preceq e_5,\ \ e_2 \preceq e_6$
2. $e_1 \ll e_2,\ e_3 \ll e_4,\ e_5 \ll e_6,\ e_1 \ll e_3,\ e_4 \ll e_5,$ not $(e_2 \ll e_6)$

As $\ll^* \not\subseteq \preceq$, this MSC contains a race.

Other examples

Why are races problematic?

Recall: MSC M has a race if $\preceq \not\subseteq \ll^*$ or equivalently:

$$\exists e, e' \in E_? \ . \ (e <_p e' \text{ and } e \not\ll^* e')$$

Whenever $\preceq \not\subseteq \ll^*$, implementations based on $<_p$ may cause problems:

1. unspecified message reception
   - a process receives a message which by the MSC is not possible
2. deadlocks
   - a process blocking on receipt of an unexpected message may block others too
3. message loss
   - unexpectedly received messages may be ignored
4. exploiting wrong message content

Checking whether an MSC has a race

- MSC M has a race if $\preceq \not\subseteq \ll^*$

- How to check whether MSC M has a race?

  compute $\ll^*$ *and check whether* $\preceq \subseteq \ll^*$

- transitive closure $\ll^*$ is computed using Floyd-Warshall's algorithm

  - algorithm for finding shortest paths in a weighted digraph with positive or negative edge weights[1]
  - easily adapted for computing the transitive closure of digraphs
  - worst-case time complexity $\mathcal{O}(|E|^3)$
  - by using some specifics of MSC, this is reduced to $\mathcal{O}(|E|^2)$

- So: race checking can be done quadratically in the number of events

[1] for digraphs without negative cycles.

Computing $\ll^*$: Warshall's algorithm

Algorithm

$\underbrace{\text{compute } \ll^*}_{\text{Warshall's Algorithm}}$ and compare with $\preceq$

Warshall's Algorithm: <u>input:</u> $R \subseteq X \times X$ where X is a set
<u>output:</u> R^*

Idea:

Consider R and R^* as directed graphs

There is an edge $x \Rightarrow y$ in R^* iff there is a (possibly empty) sequence

$$x = x_0 \to x_1 \to x_2 \to \ldots \to x_n = y \text{ in } R$$

(our setting: $X = E, R = \ll, \ R^* = \ll^*$)

Warshall's algorithm: preliminaries

- assume: graph vertices are numbered $\{1, 2, \ldots, n\}$ where $n = |E|$
- for $j \in \{1, \ldots, n{+}1\}$ define relation $\overset{j}{\Longrightarrow}$ as follows:
  $x \overset{j}{\Longrightarrow} y$ iff $\exists$ path in R from x to y such that all vertices on the path ($\neq x, y$) have a smaller number than j
- Then:

$$\begin{array}{llll} (1) & x \Longrightarrow y & \text{iff} & x \overset{n+1}{\Longrightarrow} y \\ (2) & x \overset{1}{\Longrightarrow} y & \text{iff} & x = y \text{ or } x \ll y \\ (3) & x \overset{y+1}{\Longrightarrow} z & \text{iff} & x \overset{y}{\Longrightarrow} z \text{ or } x \overset{y}{\Longrightarrow} y \overset{y}{\Longrightarrow} z \end{array}$$

- Algorithm: determine the relations $\overset{1}{\Longrightarrow}, \ldots, \overset{n}{\Longrightarrow}, \overset{n+1}{\Longrightarrow}$ iteratively using properties (2) + (3); Result is then given by (1).
- Store $\overset{i}{\Longrightarrow}$ in a boolean matrix C
- Postcondition: $C[x, y] = \texttt{true}$ iff $(x, y) \in R^*$
- Precondition: $\forall x, y \in X \; . \; C[x, y] = \texttt{false}$

Warshall's algorithm

```
/* first compute x =1=> y                                          */
for x := 1 to n do
    for y := 1 to n do
        C[x, y] := (x = y or (x, y) ∈ R)
/* loop invariant: after the j-th iteration of                     */
/* outermost loop it holds: C[x, y] = true iff x =j+1=> y            */
for y := 1 to n do
    for x := 1 to n do
        if C[x, y] then
            for z := 1 to n do
                if C[y, z] then
                    C[x, z] := true
```

$$/^* \text{ first compute } x \xRightarrow{1} y \quad {}^*/$$

$$C[x,y] := (x = y \text{ or } \underbrace{(x,y) \in R}_{x \ll y})$$

$$/^* \text{ outermost loop it holds: } C[x,y] = \texttt{true} \text{ iff } x \xRightarrow{j+1} y \quad {}^*/$$

Correctness and complexity

Lemma: correctness

After j iterations: $x \stackrel{j+1}{\Longrightarrow} y$ iff $C[x, y] = \texttt{true}$.

Proof:

if: trivial; *only if*: by induction on j.

Complexity

Worst-case time complexity of Warshall's algorithm : $\mathcal{O}(|n|^3)$ with $n = |X|$

Proof:

follows from the facts that there is a triple-nested loop of which each loop has at most n iterations.

Efficiency improvement [Alur et al. '96]

Warshall's algorithm computes R^* for every binary relation $R \subseteq X \times X$.

Recall: our interest is in determining R^* for $R = \ll$

Using some properties of $\ll$ the complexity can be improved.

Exploit that for $\ll$:

1. $\ll$ is acyclic (as it is a partial order)
2. number of immediate predecessors of $e \in E$ under $\ll$ is at most two (why?)

   Recall that e is an immediate predecessor of e' (under $\ll$) if:

$$e \ll e' \text{ and } \neg(\exists e'' \notin \{e, e'\}.\ e \ll e'' \wedge e'' \ll e')$$

Efficiency improvement [Alur et al. '96]

The main loop of Warshall's algorithm:

for $e := 1$ to n do
    for $e' := 1$ to n do
        if $C[e', e]$ then
            for $e'' := 1$ to n do
                if $C[e, e'']$ then
                    $C[e', e''] := \texttt{true}$

The main loop of Alur *et. al.*'s algorithm for detecting races in MSCs:

for $e := 1$ to n do
    for $e' := e - 1$ downto 1 do
        if (not $C[e', e]$ and $e' \ll e$) then
            $C[e', e] := \texttt{true}$
            for $e'' := 1$ to $e' - 1$ do
                if $C[e'', e']$ then
                    $C[e'', e] := \texttt{true}$

Detecting races in MSCs

Theorem

Let M be an MSC with set E of events and $n = |E|$. Checking whether M has a race can be done in $\mathcal{O}(n^2)$.

Proof:

Since $\ll$ is acyclic, we number the events such that the numbering defines a total order that is consistent with visual order $\preceq$. This can be done in $\mathcal{O}(n)$ using a standard topological sort. Then observe that the innermost loop:

$$\begin{array}{l} \text{for } e'' := 1 \text{ to } e' - 1 \text{ do} \\ \qquad \text{if } C[e'', e'] \text{ then } C[e'', e] := \texttt{true} \end{array}$$

of the triple-nested main loop is executed for (e, e') only if e' is an immediate predecessor of e under $\ll$. As for MSCs, an event can have at most two immediate predecessors, the innermost loop is executed at most $2 \cdot n$ times. This yields a total worst-case time complexity of $n^2 + 2 \cdot n$.