Theoretical Foundations of the UML

Lecture 6: Compositional Message Sequence Graphs

Joost-Pieter Katoen

Lehrstuhl für Informatik 2
Software Modeling and Verification Group

http://moves.rwth-aachen.de/teaching/ws-1415/uml/

11. November 2014

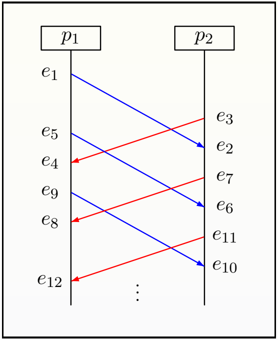
Outline

1. A non-decomposable MSC
2. Compositional Message Sequence Charts
3. Compositional Message Sequence Graphs
4. Safe Compositional Message Sequence Graphs
5. Existence of Safe Paths
6. Universality of Safe Paths

Overview

1. A non-decomposable MSC
2. Compositional Message Sequence Charts
3. Compositional Message Sequence Graphs
4. Safe Compositional Message Sequence Graphs
5. Existence of Safe Paths
6. Universality of Safe Paths

An MSC that cannot be decomposed [Yannakakis 1999]



This MSC cannot be decomposed as

$$M_1 \bullet M_2 \bullet \ldots \bullet M_n \quad \text{for } n > 1$$

This can be seen as follows:

- e_1 and $e_2 = m(e_1)$ must both belong to M_1
- $e_3 \preceq e_2$ and $e_1 \preceq e_4$ thus
  $e_3, e_4 \notin M_j$, for $j < 1$ and $j > 1$
  $\implies e_3, e_4$ must belong to M_1
- by similar reasoning: $e_5, e_6 \in M_1$ etc.

Problem:

Compulsory matching between send and receive events in the same MSG vertex (i.e., send e and receive $m(e)$ must belong to the same MSC).

Overview

1. A non-decomposable MSC
2. Compositional Message Sequence Charts
3. Compositional Message Sequence Graphs
4. Safe Compositional Message Sequence Graphs
5. Existence of Safe Paths
6. Universality of Safe Paths

Compositional MSCs [Gunter, Muscholl, Peled 2001]

Solution: drop restriction that e and $m(e)$ belong to the same MSC (= allow for incomplete message transfer)

Definition (Compositional MSC)

$M = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$ is a compositional MSC (CMSC, for short) where $\mathcal{P}, E, \mathcal{C}$ and l are defined as before, and

- $m : E_! \to E_?$ is a partial, injective function such that (as before):

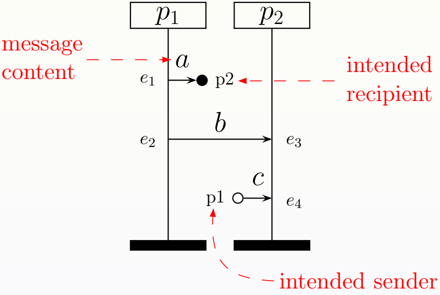
$$m(e) = e' \wedge l(e) = !(p, q, a) \quad \text{implies} \quad l(e') = ?(q, p, a)$$

- $$\preceq \;=\; \Big(\bigcup_{p \in \mathcal{P}} <_p \quad \cup \quad \underbrace{\{ (e, m(e)) \mid e \in \underbrace{dom(m)}_{\text{domain of } m} \}}_{\text{“}m(e)\text{ is defined”}} \Big)^*$$

Note:

An MSC is a CMSC where m is total and bijective.

CMSC example



$m(e_2) = e_3$
$e_1 \notin dom(m)$
$e_4 \notin rng(m)$

Concatenation of CMSCs (1)

Let $M_i = (\mathcal{P}_i, E_i, \mathcal{C}_i, l_i, m_i, \preceq_i) \in \mathbb{CM} \quad i \in \{1, 2\}$
be CMSCs with $E_1 \cap E_2 = \varnothing$

The concatenation of CMSCs M_1 and M_2 is the CMSC
$M_1 \bullet M_2 = (\mathcal{P}_1 \cup \mathcal{P}_2,\ E,\ \mathcal{C}_1 \cup \mathcal{C}_2, l, m, \preceq)$ with:

- $E = E_1 \cup E_2$
- $l(e) = l_1(e)$ if $e \in E_1$, $l_2(e)$ otherwise
- $m(e) = E_! \rightarrow E_?$ satisfies:
  1. m extends m_1 and m_2, i.e., $e \in dom(m_i)$ implies $m(e) = m_i(e)$
  2. m matches unmatched send events in M_1 with unmatched receive events in M_2 according to order on process (matching from top to bottom)
     the k-th unmatched send in M_1 is matched with
     the k-th unmatched receive in M_2 (of the same "type")
  3. $M_1 \bullet M_2$ is FIFO (when restricted to matched events)

Concatenation of CMSCs (2)

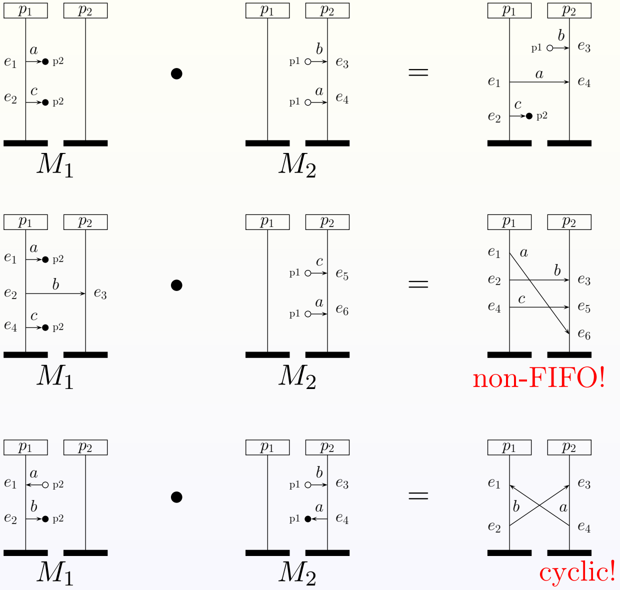
Let $M_i = (\mathcal{P}_i, E_i, \mathcal{C}_i, l_i, m_i, \preceq_i) \in \mathbb{CM} \qquad i \in \{1, 2\}$
be CMSCs with $E_1 \cap E_2 = \varnothing$

The concatenation of CMSCs M_1 and M_2 is the CMSC
$M_1 \bullet M_2 = (\mathcal{P}_1 \cup \mathcal{P}_2, E_1 \cup E_2, \mathcal{C}_1 \cup \mathcal{C}_2, l, m, \preceq)$ with:

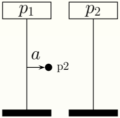
- l and m are defined as on the previous slide
- $\preceq$ is the reflexive and transitive closure of:

$$\begin{array}{lll} \left(\bigcup_{p \in \mathcal{P}} <_{p,1} \cup <_{p,2}\right) & \cup & \{(e, e') \mid e \in E_1 \cap E_p\,,\ e' \in E_2 \cap E_p\} \\ & \cup & \{(e, m(e) \mid e \in dom(m)\} \end{array}$$

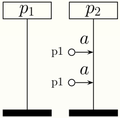
Examples



Associativity

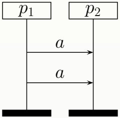


M

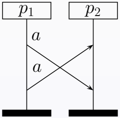


M'

$(M \bullet M) \bullet M'$:



$M \bullet (M \bullet M')$:



$\Longrightarrow$ this is non-FIFO (and thus undefined)

Note:

Concatenation of CMSCs is <u>not</u> associative.

Overview

1. A non-decomposable MSC
2. Compositional Message Sequence Charts
3. Compositional Message Sequence Graphs
4. Safe Compositional Message Sequence Graphs
5. Existence of Safe Paths
6. Universality of Safe Paths

Compositional MSG

Let $\mathbb{CM}$ be the set of all CMSCs.

Definition (Compositional MSG)

A compositional MSG (CMSG) $G = (V, \rightarrow, v_0, F, \lambda)$ with $\lambda : V \rightarrow \mathbb{CM}$, where $V, \rightarrow, v_0,$ and F as for MSGs.

The difference with an MSG is that the vertices in a CMSG are labeled with compositional MSCs (rather than "real" MSCs).

Paths

Let $G = (V, \to, v_0, F, \lambda)$ be a CMSG.

Definition (Path in a CMSG)

A path π of G is a finite sequence

$$\pi = u_0\ u_1\ \ldots\ u_n \text{ with } u_i \in V\ \ (0 \leq i \leq n) \text{ and } u_i \to u_{i+1}\ \ (0 \leq i < n)$$

Definition (Accepting path of a CMSG)

Path $\pi = u_0\ \ldots\ u_n$ is accepting if: $u_0 = v_0$ and $u_n \in F$.

Definition (CMSC of a path)

The CMSC of a path $\pi = u_0\ \ldots\ u_n$ is:

$$M(\pi) \;=\; (\ldots(\lambda(u_0) \bullet \lambda(u_1)) \bullet \lambda(u_2) \ldots) \bullet \lambda(u_n)$$

where CMSC concatenation is left associative.

The MSC language of a CMSG

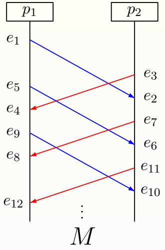
Definition (Language of a CMSG)

The (MSC) language of CMSG G is defined by:

$$L(G) = \{ \underbrace{M(\pi) \in \mathbb{M}}_{\text{only "real" MSCs}} \mid \pi \text{ is an accepting path of } G\}.$$

Accepting paths that give rise to an CMSC (which is not an MSC) are not taken into account in $L(G)$.

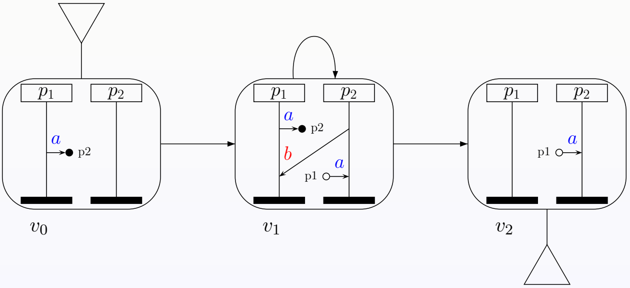
Yannakakis' example as compositional MSG



This MSC cannot be modeled for $n > 1$ by:

$$M = M_1 \bullet M_2 \bullet \ldots \bullet M_n \quad \text{with} \quad M_i \in \mathbb{M}$$

But it can be modeled as the compositional MSG:



Overview

1. A non-decomposable MSC
2. Compositional Message Sequence Charts
3. Compositional Message Sequence Graphs
4. Safe Compositional Message Sequence Graphs
5. Existence of Safe Paths
6. Universality of Safe Paths

Safe paths and CMSGs

Definition (Safe path)

Path π of CMSG G is safe whenever $M(\pi) \in \mathbb{M}$.

Definition (Safe CMSG)

CMSG G is safe if for every accepting path π of G, $M(\pi)$ is an MSC.

So:

CMSG G is safe if on any of its accepting paths there are no unmatched sends and receipts, i.e., if any of its accepting paths is indeed an MSC.

Overview

1 A non-decomposable MSC

2 Compositional Message Sequence Charts

3 Compositional Message Sequence Graphs

4 Safe Compositional Message Sequence Graphs

5 Existence of Safe Paths

6 Universality of Safe Paths

Existence of a safe accepting path

Theorem: undecidability of existence of a safe path

The decision problem "does CMSG G have at least one safe, accepting path?" is undecidable.

Proof.

By a reduction from Post's Correspondence Problem (PCP).

... black board ... □

The complement decision problem "does CMSG G have no safe, accepting path?" is undecidable too.

Overview

1. A non-decomposable MSC
2. Compositional Message Sequence Charts
3. Compositional Message Sequence Graphs
4. Safe Compositional Message Sequence Graphs
5. Existence of Safe Paths
6. Universality of Safe Paths

Universality of safe accepting paths

Theorem: undecidability of existence of a safe path

The decision problem "does CMSG G have at least one safe, accepting path?" is undecidable.

Theorem: decidability of universality of safe paths

The decision problem "are all accepting paths of CMSG G safe?" is decidable in PTIME.

Proof.

Polynomial reduction to reachability problem in (non-deterministic) pushdown automata.

... see details on the next slides ... □

Pushdown automata

Definition (Pushdown automaton)

A pushdown automaton (PDA, for short) $K = (Q, q_0, \Gamma, \Sigma, \Delta)$ with

- Q, a finite set of control states
- $q_0 \in Q$, the initial state
- Γ, a finite stack alphabet
- Σ, a finite input alphabet
- $\Delta \subseteq Q \times \Sigma \times \Gamma \times Q \times \Gamma^*$, the transition relation.

Transition relation

$(q, a, \gamma, q', \text{pop}) \in \Delta$ means: in state q, on reading input symbol a and top of stack is symbol γ, change to q' and pop γ from the stack.

Reachability in pushdown automata

Definition

A configuration c is a triple (state q, stack content Z, rest input w).

Definition

Given a transition in Δ, a (direct) successor configuration c' of c is obtained: $c \vdash c'$.

Reachability problem

For configuration c, and initial configuration c_0: $c_0 \vdash^* c$?

Theorem: [Esparza et al. 2000]

The reachability problem for PDA is decidable in PTIME.

Checking whether a CMSG is safe is decidable

- Consider any ordered pair (p_i, p_j) of processes in CMSG G
- Proof idea: construct a PDA $K_{i,j} = (Q, q_0, \Gamma, \Sigma, \Delta)$ such that

  CMSG G is not safe wrt. (p_i, p_j) iff PDA $K_{i,j}$ accepts

- For accepting path $u_0 \ldots u_k$ in G, feed $K_{i,j}$ with the word

$$\rho_0 \ldots \rho_k \;\; \text{where} \;\; \rho_i \in Lin(\lambda(u_i))$$

  such that unmatched sends (of some type) precede all unmatched receipts (of the same type)
- Possible violations that $K_{i,j}$ may encounter:
  1. nr. of unmatched $!(p_i, p_j, \cdot) \; > \;$ nr. of unmatched $?(p_j, p_i, \cdot)$
  2. type of k-th unmatched send $\neq$ type of k-th unmatched receive
  3. non-FIFO communication

The nondeterministic PDA $K_{i,j}$

Let $\{a_1, \ldots, a_k\}$ be the message contents in CMSG G for (p_i, p_j).

Nondeterministic PDA $K_{i,j} = (Q, q_0, \Gamma, \Sigma, \Delta)$ where:

- Control states $Q = \{q_0, q_{a_1}, \ldots, q_{a_k}, q_{err}, q_F\}$
- Stack alphabet $\Gamma = \{1, \#\}$
  1 counts nr. of unmatched $!(p_i, p_j, a_m)$, and # is bottom of stack
- Input alphabet $\Sigma = \begin{cases} \text{unmatched action } !(p_i, p_j, a_m) \\ \text{unmatched action } ?(p_j, p_i, a_m) \\ \text{matched actions } !?(p_i, p_j, a_m), ?!(p_j, p_i, a_m) \end{cases}$
- Transition function Δ is described on next slide

Safeness of CMSGs (2)

- Initial configuration is $(q_0, \#, w)$
  - w is linearization of actions at p_i and p_j on an accepting path of G
- On reading $!(p_i, p_j, a_m)$ in q_0, push 1 on stack
  - nondeterministically move to state q_{a_m} or stay in q_0
- On reading $?(p_j, p_i, a_m)$ in q_0, proceed as follows:
  - if 1 is on stack, pop it
  - otherwise, i.e., if stack is empty, accept (i.e., move to q_F)
- On reading matched send $!?(p_i, p_j, a_k)$ in q_0
  - stack empty (i.e., equal to $\#$)? ignore input; otherwise, accept
- Ignore the following inputs in state q_0:
  - matched send events $!?(p_j, p_i, a_k)$, and
  - unmatched sends or receipts not related to p_i and p_j
- Remaining input w empty? Accept, if stack non-empty; else reject

Safeness of CMSGs (3)

The behaviour in state q_{a_m} for $0 < m \leqslant k$:

- Ignore all actions except $?(p_j, p_i, a_\ell)$ for all $0 < \ell \leqslant k$
- On reading $?(p_j, p_i, a_\ell)$ (for some $0 < \ell \leqslant k$) in state q_{a_m} do:
  - if 1 is on top of stack, pop it
- If stack is empty:
  - if last receive differs from a_m, accept
  - otherwise reject, while ignoring the rest (if any) of the input

Safeness of CMSGs (4)

It follows: PDA $K_{i,j}$ accepts iff CMSG G is not safe wrt. (p_i, p_j)

$\Longrightarrow$ CMSG G is not safe wrt. (p_i, p_j) iff configuration $(q_F, \cdot, \cdot)$ is reachable.

$\Longrightarrow$ reachability of a configuration in a PDA is in PTIME, hence checking safeness wrt. (p_i, p_j) is in PTIME.

Time complexity

The worst-case time complexity of checking whether CMSG G is safe is in $\mathcal{O}(k^2 \cdot N^2 \cdot L \cdot |E|^2)$ where $k = |\mathcal{P}|$, $N = |V|$, and $L = |\mathcal{C}|$.

Proof.

Checking reachability in PDA $K_{i,j}$ is in $\mathcal{O}(L \cdot |E|^2)$. The number of PDAs is k^2, as we consider ordered pairs in $\mathcal{P}$. The number of paths in the CMSG G for each pair that need to be checked is in $\mathcal{O}(N^2)$, as a single traversal for each loop in G suffices. $\square$