Theoretical Foundations of the UML

Lecture 8: Communicating Finite-State Machines

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Outline

1. Introduction
2. Communicating Finite-State Machines
3. Semantics of Communicating Finite-State Machines
4. Emptiness Problem for CFMs

Overview

1. Introduction
2. Communicating Finite-State Machines
3. Semantics of Communicating Finite-State Machines
4. Emptiness Problem for CFMs

Specification to implementation

- Consider an MSGs as complete system specifications
  - they describe a full set of possible system scenarios
- Can we obtain "realisations" that exhibit precisely these scenarios?
- Map MSGs, i.e., scenarios onto an executable model
  - model each process by a finite-state automaton
  - that communicate via unbounded directed FIFO channels

$\Rightarrow$ This yields Communicating Finite-state Machines

Intuition

The need for synchronisation messages

Overview

Preliminaries

Definition

Let

- $\mathcal{P}$ be a finite set of at least two (sequential) processes
- $\mathcal{C}$ be a finite set of message contents

Definition (communication actions, channels)

- $Act_p^! := \{!(p,q,a) \mid q \in \mathcal{P} \setminus \{p\},\ a \in \mathcal{C}\}$
  the set of send actions by process p
- $Act_p^? := \{?(p,q,a) \mid q \in \mathcal{P} \setminus \{p\},\ a \in \mathcal{C}\}$
  the set of receive actions by process p
- $Act_p := Act_p^! \cup Act_p^?$
- $Act := \bigcup_{p \in \mathcal{P}} Act_p$
- $Ch := \{(p,q) \mid p,q \in \mathcal{P},\ p \neq q\}$ “channels“

Communicating finite-state machines

Definition

A communicating finite-state machine (CFM) over $\mathcal{P}$ and $\mathcal{C}$ is a structure

$$\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$$

where

- $\mathbb{D}$ is a nonempty finite set of synchronization messages (or data)
- for each $p \in \mathcal{P}$:
  - S_p is a non-empty finite set of local states (the S_p are disjoint)
  - $\Delta_p \subseteq S_p \times Act_p \times \mathbb{D} \times S_p$ is a set of local transitions
- $s_{init} \in S_{\mathcal{A}}$ is the global initial state
  - where $S_{\mathcal{A}} := \prod_{p \in \mathcal{P}} S_p$ is the set of global states of $\mathcal{A}$
- $F \subseteq S_{\mathcal{A}}$ is the set of global final states

We often write $s \xrightarrow{\sigma, m}_p s'$ instead of $(s, \sigma, m, s') \in \Delta_p$

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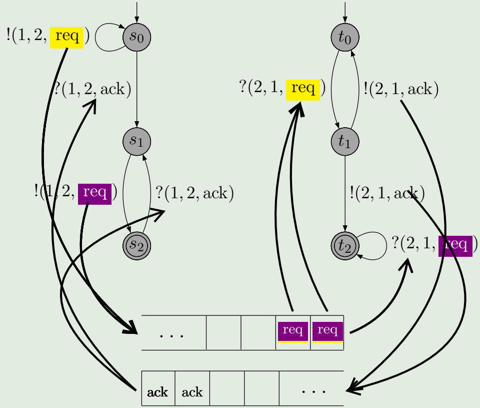
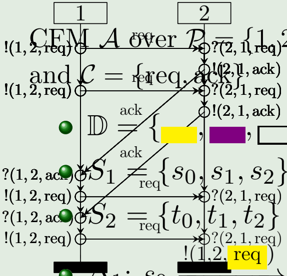
Example



CFM $\mathcal{A}$ over $\mathcal{P} = \{1, 2\}$ and $\mathcal{C} = \{\text{req}, \text{ack}\}$



- $\mathbb{D} = \{\ ,\ ,\ \}$
- $S_1 = \{s_0, s_1, s_2\}$
- $S_2 = \{t_0, t_1, t_2\}$
- Δ_1: $s_0 \xrightarrow{!(1,2,\text{req})}_1 s_0 \ \ldots$

  Δ_2: $t_0 \xrightarrow{?(2,1,\text{req})}_2 t_1 \ \ldots$
- $s_{init} = (s_0, t_0)$
- $F = \{(s_2, t_2)\}$

$!(1,2,\text{req})\ !(1,2,\text{req})\ ?(2,1,\text{req})\ !(2,1,\text{ack})\ ?(2,1,\text{req})\ !(2,1,\text{ack})\ ?(1,2,\text{ack})\ !(1,2,\text{req})\ ?(1,2,\text{ack})\ !(1,2,\text{req})\ ?(2$

Overview

Formal semantics of CFMs

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Definition (configurations)

Configurations of $\mathcal{A}$: $\mathit{Conf}_{\mathcal{A}} := S_{\mathcal{A}} \times \{\eta \mid \eta : \mathit{Ch} \to (\mathcal{C} \times \mathbb{D})^*\}$

Definition (global step)

$\Longrightarrow_{\mathcal{A}} \subseteq \mathit{Conf}_{\mathcal{A}} \times \mathit{Act} \times \mathbb{D} \times \mathit{Conf}_{\mathcal{A}}$ is defined as follows:

- sending a message: $((\overline{s}, \eta), !(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], !(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta' = \eta[(p, q) := (a, m) \cdot \eta((p, q))]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$
- receipt of a message: $((\overline{s}, \eta), ?(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], ?(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta((q, p)) = w \cdot (a, m) \neq \epsilon$ and $\eta' = \eta[(q, p) := w]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$

Example

Linearizations of a CFM

Let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

Definition (accepting runs)

A run ρ of CFM $\mathcal{A}$ on word $w = \sigma_1 \ldots \sigma_n \in Act^*$ is an alternating sequence $\rho = \gamma_0 \, m_1 \, \gamma_1 \ldots \gamma_{n-1} \, m_n \, \gamma_n$ such that

1. $\gamma_0 = (s_{init}, \eta_\varepsilon)$ with η_ε mapping any channel to ε
2. $\gamma_{i-1} \xRightarrow{\sigma_i, m_i}_{\mathcal{A}} \gamma_i$ for any $i \in \{1, \ldots, n\}$

The run ρ is accepting if $\gamma_n \in F \times \{\eta_\varepsilon\}$.

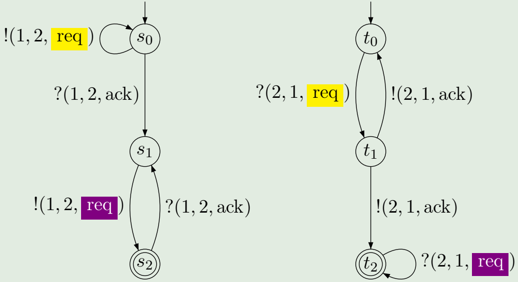
Definition (linearization of a CFM)

The (word) language of CFM $\mathcal{A}$ is defined by:

$Lin(\mathcal{A}) := \{ w \in Act^* \mid \text{there is an accepting run of } \mathcal{A} \text{ on } w \}$

Linearizations of an example CFM

Example



CFM $\mathcal{A}$ over $\{1,2\}$ and $\{\text{req}, \text{ack}\}$

$$Lin(\mathcal{A}) = \Big\{ w \in Act^* \mid \text{there is } n \geqslant 1 \text{ such that:}$$

$$w \upharpoonright 1 = !(1,2,\text{req}))^n \ (?(1,2,\text{ack}) \ !(1,2,\text{req}))^n$$

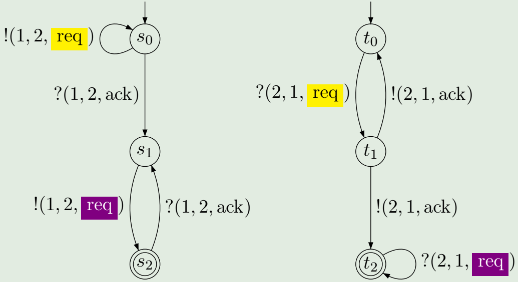
$$w \upharpoonright 2 = (?(2,1,\text{req}) \ !(2,1,\text{ack}))^n \ (?(2,1,\text{req}))^n$$

for any $u \in Pref(w)$ and $(p,q) \in Ch$:

$$\sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \geqslant 0 \Big\}$$

Linearizations and MSCs of an example CFM

Example



CFM $\mathcal{A}$ over $\{1,2\}$ and $\{\text{req}, \text{ack}\}$

$$Lin(\mathcal{A}) = \Big\{ w \in Act^* \mid \text{there is } n \geqslant 1 \text{ such that:}$$

$$w \restriction 1 = (!(1,2,\text{req}))^n \; (?(1,2,\text{ack}) \; !(1,2,\text{req}))^n$$

$$w \restriction 2 = (?(2,1,\text{req}) \; !(2,1,\text{ack}))^n \; (?(2,1,\text{req}))^n$$

for any $u \in Pref(w)$ and $(p,q) \in Ch$:

$$\sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \geqslant 0 \Big\}$$

Overview

Elementary questions are undecidable for CFMs

Emptiness of CFMs is undecidable [Brand & Zafiropulo 1983]

The following problem is undecidable (even if $\mathcal{C}$ is a singleton):

INPUT: CFM $\mathcal{A}$ over processes $\mathcal{P}$ and message contents $\mathcal{C}$
QUESTION: Is $\mathcal{L}(\mathcal{A})$ empty?

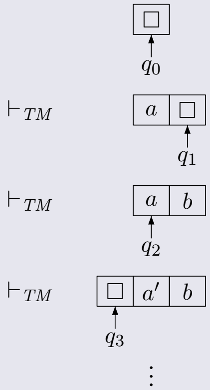
Proof (sketch)

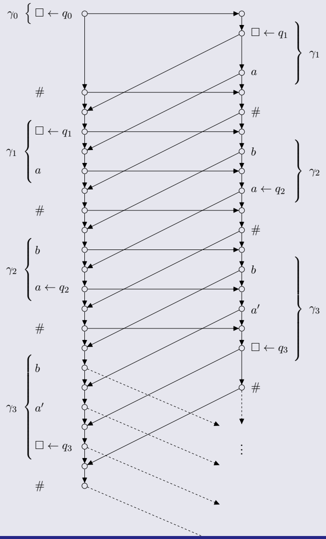
Reduction from the halting problem for Turing machine $TM = (Q, \Sigma, \Delta, \Box, q_0, q_f)$ to emptiness for a CFM with two processes. Build CFM $\mathcal{A} = ((\mathcal{A}_1, \mathcal{A}_2), \mathbb{D}, s_{init}, F)$ over $\{1, 2\}$ and some singleton set $\mathcal{C}$ such that $\mathcal{L}(\mathcal{A}) \neq \varnothing$ iff TM can reach q_f, i.e., TM accepts.

- Process 1 sends current configurations to process 2
- Process 2 chooses successor configurations and sends them to 1
- $\mathbb{D} = \Big((\Sigma \cup \{\Box\}) \times (Q \cup \{\_\}) \Big) \cup \{\#\}$

A CFM simulating a Turing machine

Proof (contd.)





A CFM simulating a Turing machine

Proof (contd.)

- Left or standstill transition: Process 2 may just wait for a symbol containing a state of TM and to alter it correspondingly. In the example, the left-moving transition (q_2, a, a', L, q_3) is applied so that process 2
  - sends b unchanged back to process 1
  - detects (receives) $a \leftarrow q_2$
  - sends a' to process 1 entering a state indicating that the symbol to be sent next has to be equipped with q_3
  - receives $\#$ so that the symbol $\Box \leftarrow q_3$ has to be inserted before returning $\#$
- Right transition: Process 2 has to guess what the position right before the head is. For example, provided process 2 decided in favor of (q_2, a, a', R, q_3) while reading b, it would have to
  - send $b \leftarrow q_3$ instead of just b, entering some state $t(a \leftarrow q_2)$
  - receive $a \leftarrow q_2$ (no other symbol can be received in state $t(a \leftarrow q_2)$)
  - send a' back to process 1

A CFM simulating a Turing machine

Proof (contd.)

- Introduce local final states s_f and t_f, one for process 1 and one for process 2, respectively (i.e., $F = \{(s_f, t_f)\}$ and $\mathcal{A}$ is locally accepting).
- At any time, process 1 may switch into s_f, in which arbitrary and arbitrarily many messages can be received to empty channel $(2, 1)$.
- Process 2 is allowed to move into t_f and to empty the channel $(1, 2)$ as soon as it receives a letter $c \leftarrow q_f$ for some c.
- As process 2 modifies a configuration of TM locally, finitely many states are sufficient in $\mathcal{A}$. $\square$