Introduction to Model Checking 2015: Exercise 3.

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We will use the following nomenclature in the rest of the exercise. For a set A, $\mathcal{P}(A)$ is the power set of A. The universe of infinite words (or strings) is Σ^ω where Σ is the alphabet. The set of finite words is Σ^+. The i^{th} element of a word t is denoted by $t[i]$. A finite word x is a *prefix* of a word y (finite or infinite) is denoted as $x \prec y$. The linear time closure operator $\mathsf{cl} : \mathcal{P}(\Sigma^\omega) \to \mathcal{P}(\Sigma^\omega)$ is defined as:

$$\mathsf{cl}(T) = \{\ t \in \Sigma^\omega \mid \forall x \prec t\ \exists t' \in T:\ x \prec t'\}.$$

We say, $A \subseteq \Sigma^\omega$ is *safe* iff $\mathsf{cl}(A) = A$ and A is *live* iff $\mathsf{cl}(A) = \Sigma^\omega$.

Exercise 1 *(4 points)*

Prove the following simple *equivalences:*

1. $\mathsf{cl}(\emptyset) = \emptyset$.
2. $A \subseteq B$ *implies* $\mathsf{cl}(A) \subseteq \mathsf{cl}(B)$.
3. $T \subseteq \mathsf{cl}(T)$.
4. $\mathsf{cl}(T) = \mathsf{cl}(\mathsf{cl}(T))$.
5. $\mathsf{cl}(A \cup B) = \mathsf{cl}(A) \cup \mathsf{cl}(B)$.

Exercise 2 *(2 points)*

Show the following:

1. *A singleton set is safe.*
2. *Consider the set* $R_a = \{\ t \in \Sigma^\omega \mid \exists i \in \mathbb{N}:\ t[i] = a\}$, *where* $a \in \Sigma$. R_a *is not safe but it is live.*
3. $A \cup \overline{cl(A)}$ *is live for any set* A. *(where* $\overline{B}$ *is the set complement of* B.*)*

Exercise 3 *(4 points)*

Give examples of:

1. *If* A *and* B *are live then* $A \cap B$ *is not live.*
2. *A set* T *such that* $T \subsetneq \mathsf{cl}(T)$.
3. *The distributiveness of* safe *over* $\cup$ *(union) can be easily extended to finite union that is: Finite union of safe sets are safe.*
$$\mathsf{cl}(A_1 \cup \ldots \cup A_n) = A_1 \cup \ldots \cup A_n.$$
Show by an example that the statement does not hold for countable union, i.e.:
$$\mathsf{cl}\left(\bigcup_{i \in \omega} A_i\right) \quad \neq \quad \bigcup_{i \in \omega} A_i.$$
Hint: Exercise 2.2.