Overview

- Introduction
- **Modelling parallel systems**
  - Transition systems
  - Modeling hard- and software systems
  - Parallelism and communication $\longleftarrow$
- Linear Time Properties
- Regular Properties
- Linear Temporal Logic
- Computation-Tree Logic
- Equivalences and Abstraction

Channel systems

Channel systems

representation of data-dependent parallel systems with

- communication over shared variables
- synchronous message passing
- asynchronous message passing

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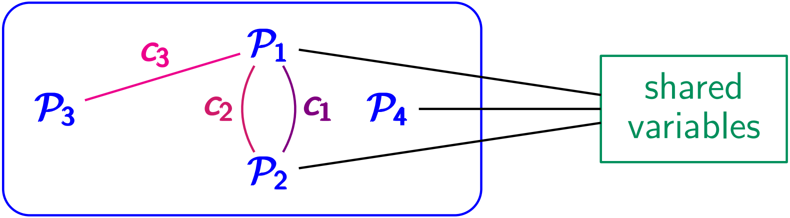

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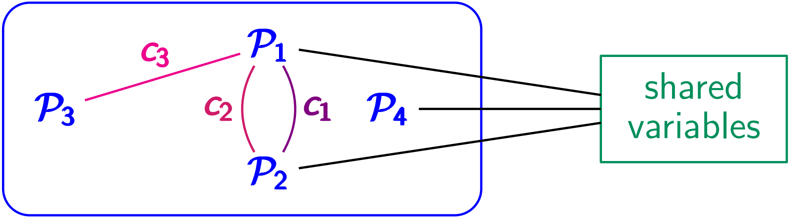

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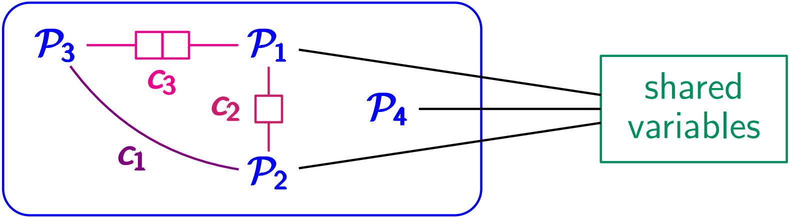


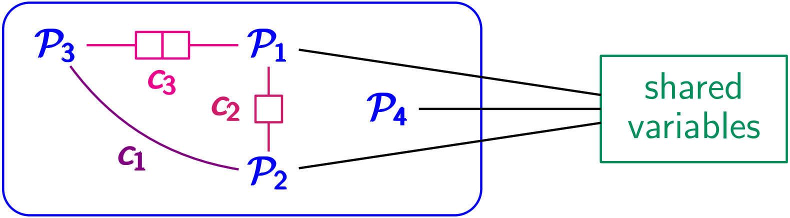
channel types: synchronous or FIFO

↑

capacity $\widehat{=}$ number of buffer cells

representation of data-dependent parallel systems with

- communication over shared variables
- synchronous message passing ⟵ capacity **0**
- asynchronous message passing ⟵ capacity $\geq$ **1**



channel types: synchronous or FIFO

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- with conditional transitions $\ell_i \xrightarrow{g:\alpha} \ell_i'$ (as before)
- and communication actions

$\ell_i \xrightarrow{c!v} \ell_i'$	sending value v via channel c
$\ell_i \xrightarrow{c?x} \ell_i'$	receiving a value for variable x via channel c

Typed variables and channels

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s.t. $\xi(c)$ is a word over $Dom(c)$ of length $\leq cap(c)$

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Effect of communication actions in CS

asynchronous message passing via channels of capacity ≥ 1

	enabled if ...	effect
sending $c!v$		
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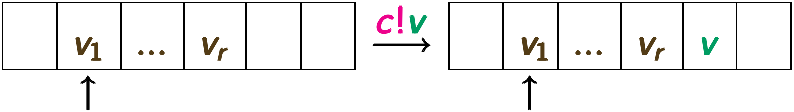

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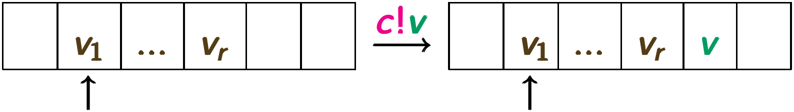

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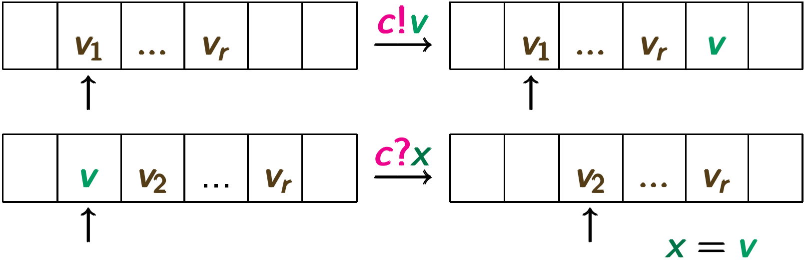

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synchronous message passing via channels of capacity $\mathbf{0}$

- $c!v$ and $c?x$ are executed at the same time
- effect $x := v$

TS-semantics of channel systems

channel system over $(Var, Chan)$
$$\mathcal{C} = [\mathcal{P}_1 | \ldots | \mathcal{P}_n]$$

$$\Downarrow$$

transition system $\mathcal{T}_{\mathcal{C}}$

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transition system $\mathcal{T}_{\mathcal{C}}$

states of $\mathcal{T}_{\mathcal{C}}$ have the form

$$\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle$$

locations of $\mathcal{P}_1, \ldots, \mathcal{P}_n$

variable valuation

channel evaluation

TS-semantics of channel systems

states $\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle$ where

- ℓ_i location of program graph $\mathcal{P}_i$,
- $\eta \in \mathit{Eval}(\mathit{Var})$ variable evaluation
- $\xi \in \mathit{Eval}(\mathit{Chan})$ channel evaluation

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variable evaluation:

$$\eta : Var \longrightarrow \bigcup_{x \in Var} Dom(x) \quad \text{with } \eta(x) \in Dom(x)$$

channel evaluation:

$$\xi : Chan \rightarrow \bigcup_{c \in Chan} Dom(c)^* \quad \text{with } \xi(c) \in Dom(c)^* \text{ and } |\xi(c)| \leq cap(c)$$

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only channels c with $\mathit{cap}(c) \geq 1$ are relevant

Transition relation of channel systems

states $\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle$ where $\ell_i \in Loc_i$, $\eta \in Eval(Var)$,

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interleaving rule for actions $\alpha \in \mathit{Act}_i$:

$$\frac{\ell_i \overset{g:\alpha}{\hookrightarrow}_i \ell_i' \;\wedge\; \eta \models g}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\alpha} \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \mathit{Effect}_i(\alpha, \eta), \xi \rangle}$$

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$\uparrow$

does not affect the channel evaluation ξ

SOS-rules for asynchronous message passing

for channel c with $cap(c) \geq 1$

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receiving a message:

$$\frac{\ell_i \xhookrightarrow{c?x}_i \ell'_i \;\wedge\; \xi(c) = v_1 v_2 \ldots v_k \;\wedge\; k \geq 1}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \xrightarrow{\tau} \langle \ell_1, \ldots, \ell'_i, \ldots, \ell_n, \eta', \xi' \rangle}$$

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where $\eta' = \eta[x{:=}v_1]$

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sending a message:

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How many states...

has a transition system for a channel system with ... **?**

- **2** processes with **2** locations each
- **2** Boolean variables
- **2** channels of capacity **10** and Boolean values

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answer:

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note: $2^{11} - 1 = 1 + 2 + 2^2 + \ldots + 2^{10}$

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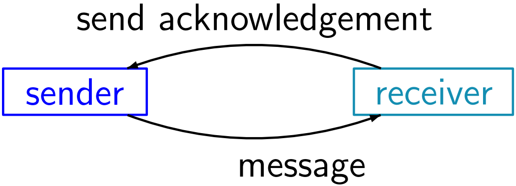
Alternating bit protocol (ABP)



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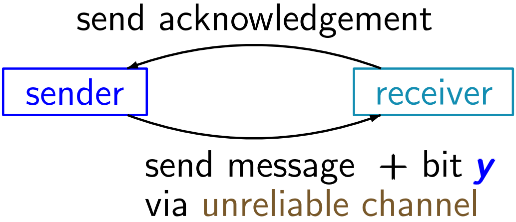

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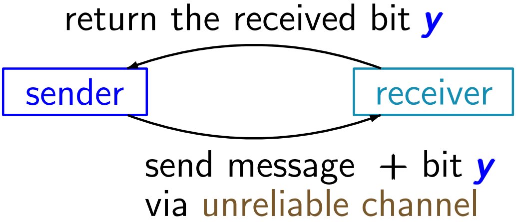

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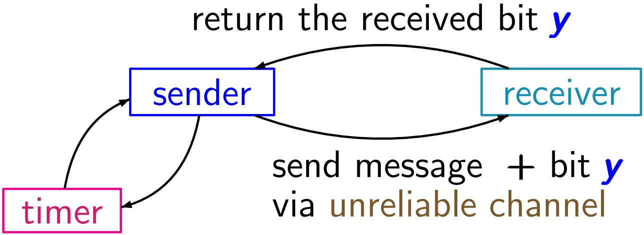
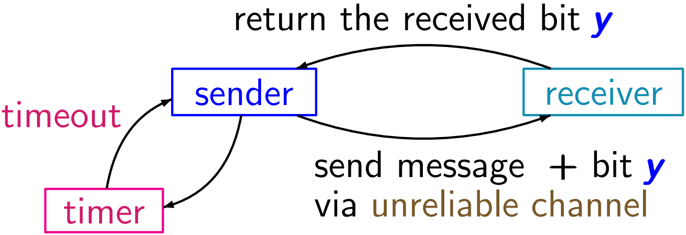


Protocol for the sender




```
LOOP FOREVER
(1)  send message + bit y and activate timer
(2)  AWAIT timeout or acknowledgement DO
          IF timeout THEN goto (1)
                     ELSE turn off timer; y:=¬y
          FI
     OD
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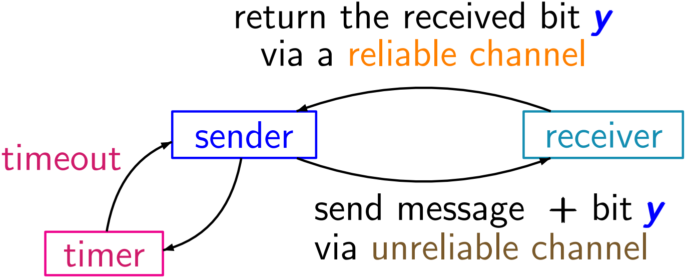


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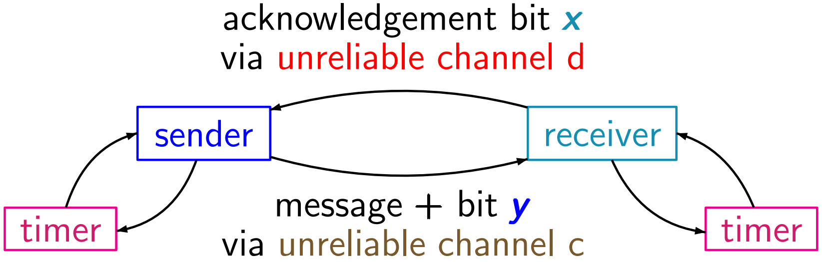

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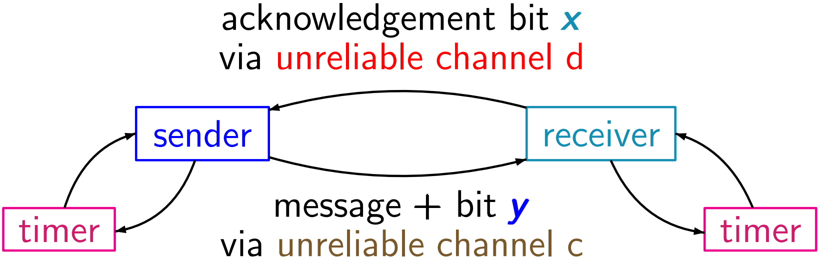


```
LOOP FOREVER
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(2)  AWAIT  timeout or acknowledgement x DO
          IF  timeout THEN  goto (1)
           ELSE  IF  x=y THEN  turn off timer; y:=¬y
                            ELSE  ignore x
                 FI
          FI
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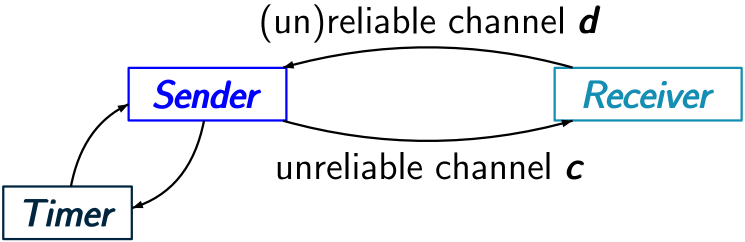
Alternating bit protocol (ABP)



channel system: [*Sender* | *Timer* | *Receiver*]

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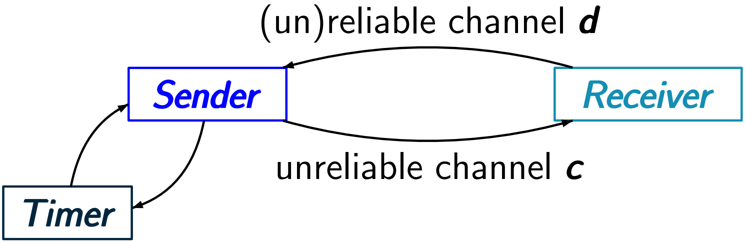
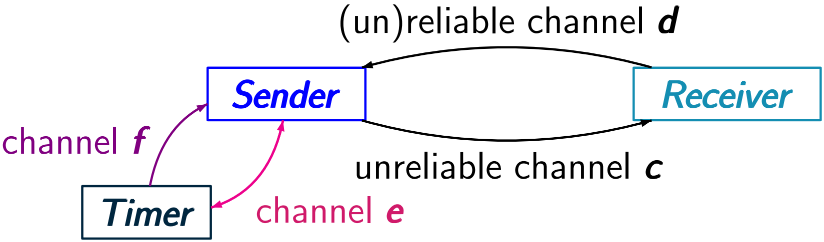


channel system: [*Sender* | *Timer* | *Receiver*]

synchronous message passing between
Timer and *Sender*

asynchronous message passing between
Receiver and *Sender*

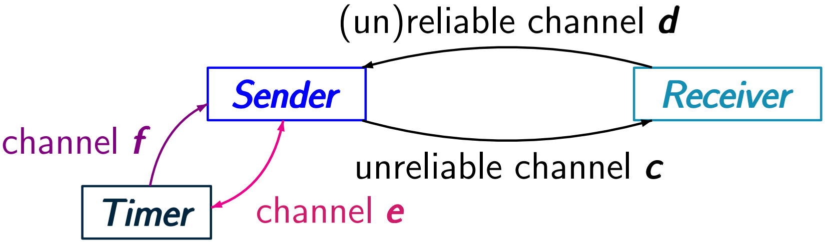
Alternating bit protocol (ABP)



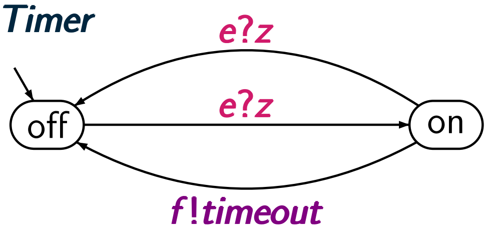
channel system: [*Sender* | *Timer* | *Receiver*]

synchronous message passing between *Timer* and *Sender* $\longleftarrow$ channels *e* and *f* asynchronous message passing between *Receiver* and *Sender* $\longleftarrow$ channels *c*, *d*

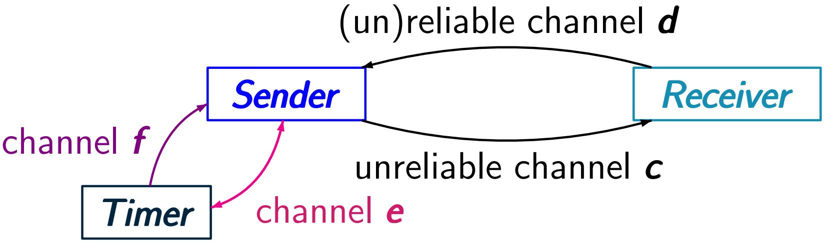
Alternating bit protocol (ABP)



channel system: $[\,\mathit{Sender} \mid \mathit{Timer} \mid \mathit{Receiver}\,]$

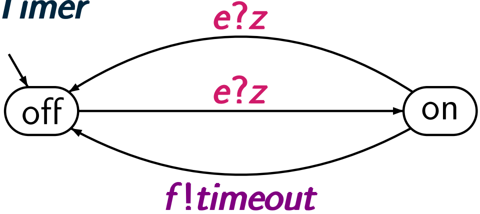


Alternating bit protocol (ABP)



channel system: [*Sender* | *Timer* | *Receiver*]

Timer

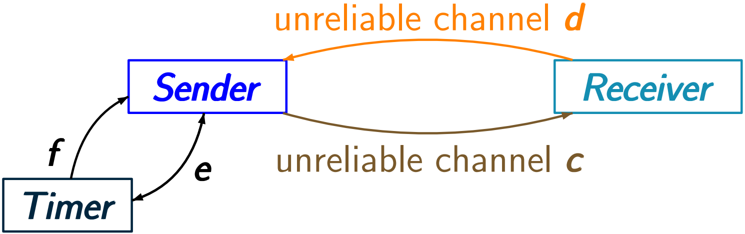


actions of *Sender*:

⋮

e!timer_on
e!timer_off
$f?z'$

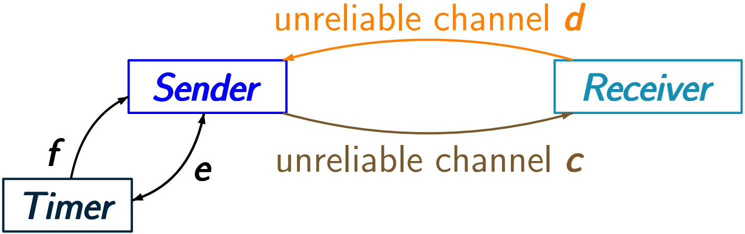
⋮



specify the sender by a program graph using

- asynchronous channels *c* and *d*
- synchronous channels *e* and *f*

Program graph for the sender

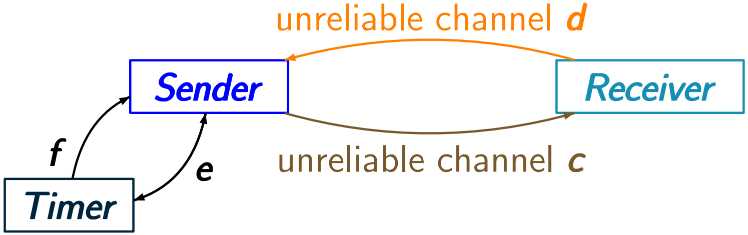


specify the sender by a program graph using

- asynchronous channels *c* and *d*
- synchronous channels *e* and *f*

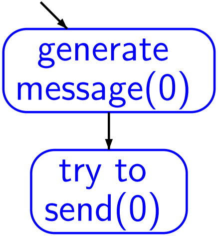
simply write **!*timeout***
?*timer_on*
?*timer_off*

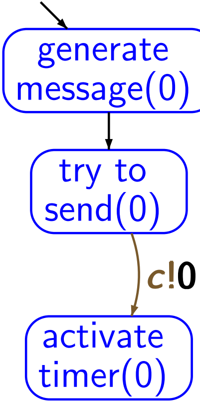
Program graph for the sender

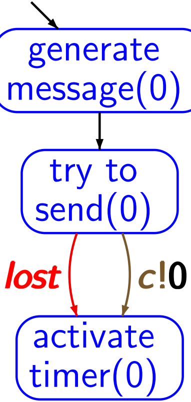


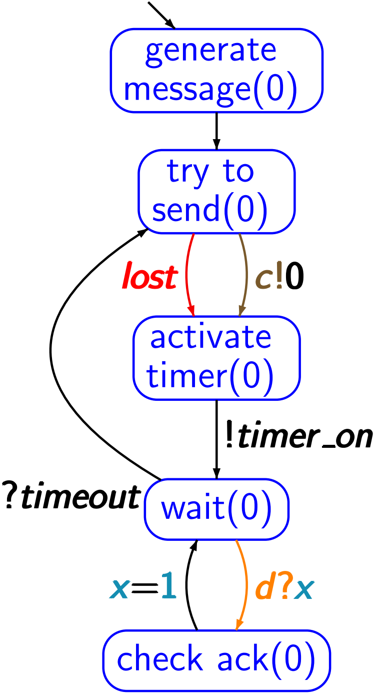
specify the sender by a program graph using

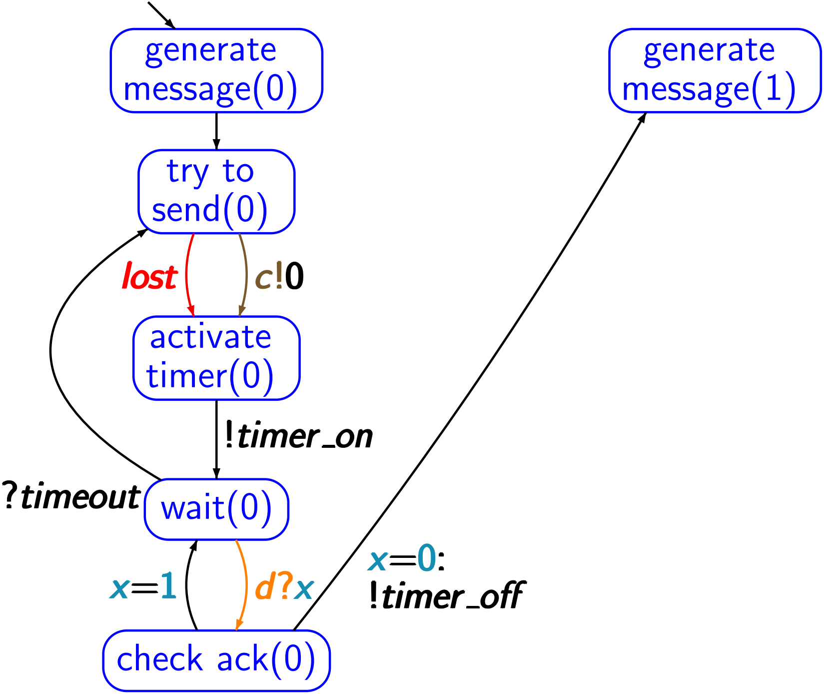
- asynchronous channels *c* and *d*
- synchronous channels *e* and *f*
- Boolean variable *x* for the acknowledgement bit sent by the receiver

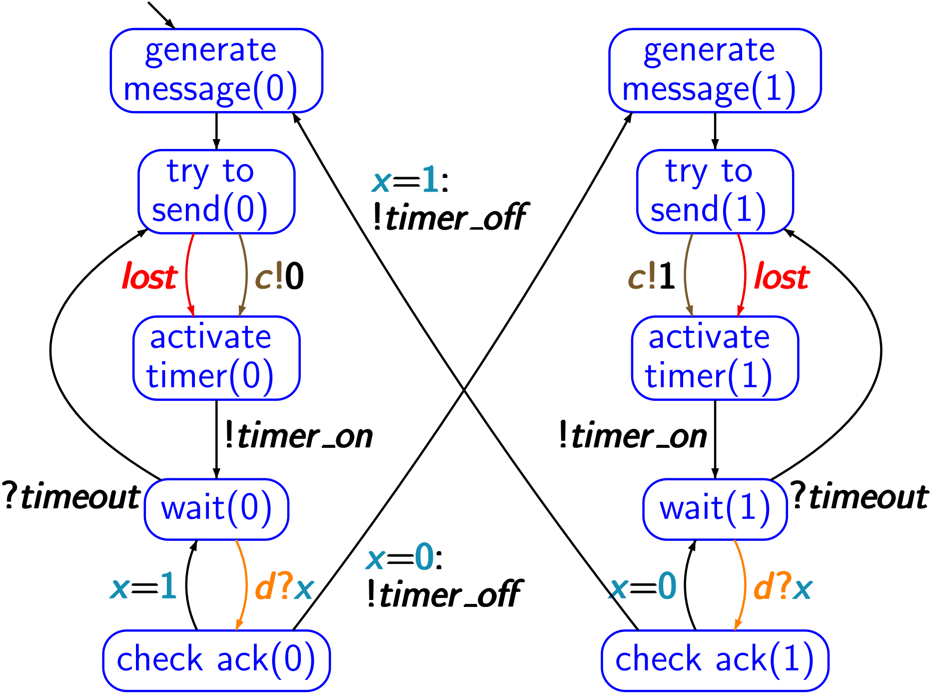

generate message(0)
try to send(0)


generate message(0)
try to send(0)
c!0
activate timer(0)

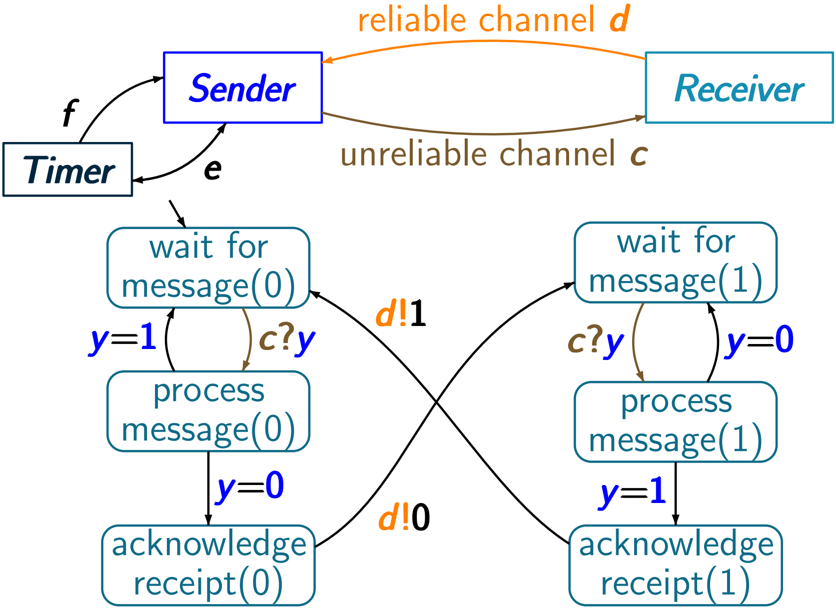

generate message(0)
try to send(0)
lost
c!0
activate timer(0)

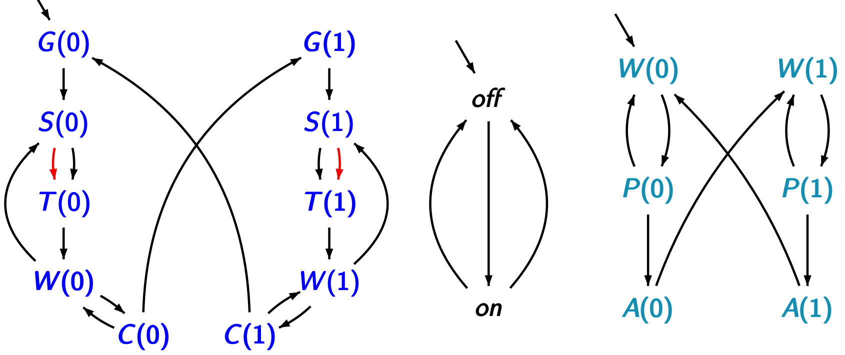

generate message(0)
try to send(0)
lost
c!0
activate timer(0)
!timer_on
?timeout
wait(0)
x=1
d?x
check ack(0)

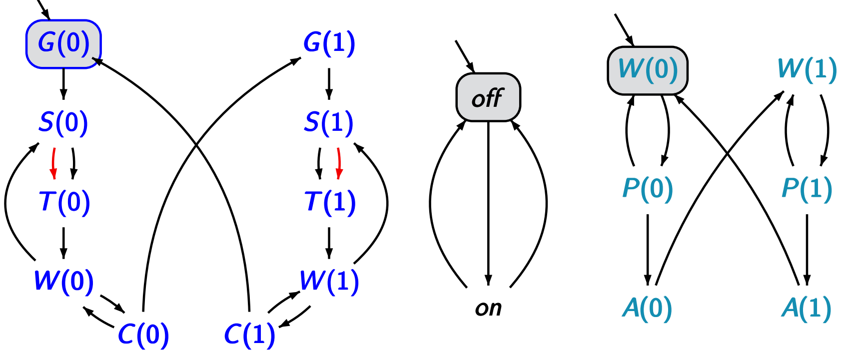

generate message(0)
generate message(1)
try to send(0)
lost
c!0
activate timer(0)
!timer_on
?timeout
wait(0)
x=1
d?x
x=0:
!timer_off
check ack(0)


generate message(0)
try to send(0)
lost
c!0
activate timer(0)
!timer_on
?timeout
wait(0)
x=1
d?x
check ack(0)
x=1:
!timer_off
x=0:
!timer_off
generate message(1)
try to send(1)
c!1
lost
activate timer(1)
!timer_on
wait(1)
?timeout
x=0
d?x
check ack(1)

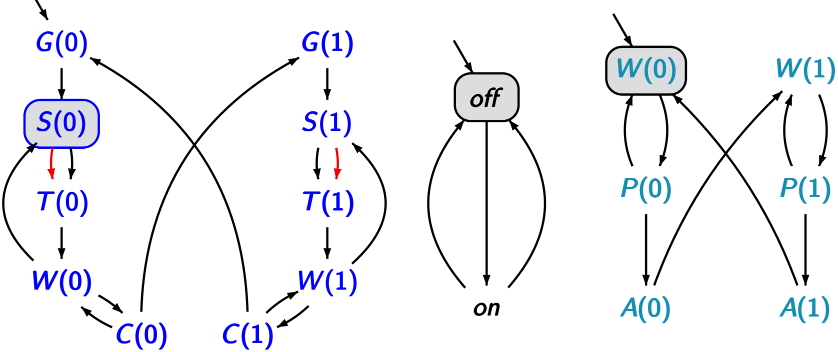
Program graph for the receiver



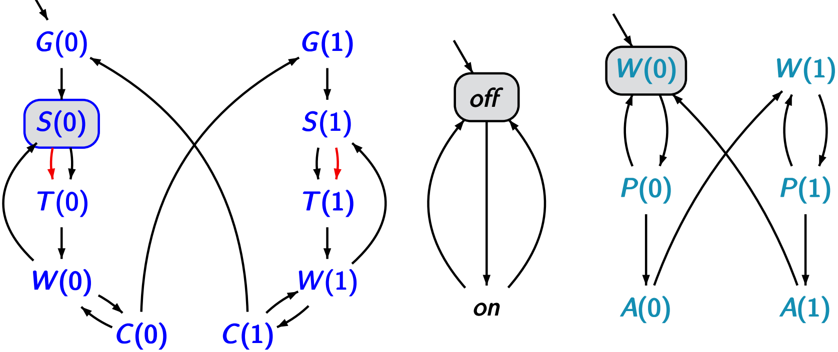

G(0)
G(1)
S(0)
S(1)
T(0)
T(1)
W(0)
W(1)
C(0)
C(1)
off
on
W(0)
W(1)
P(0)
P(1)
A(0)
A(1)



Generate(0) off Wait(0) $c = \varepsilon$ $d = \varepsilon$

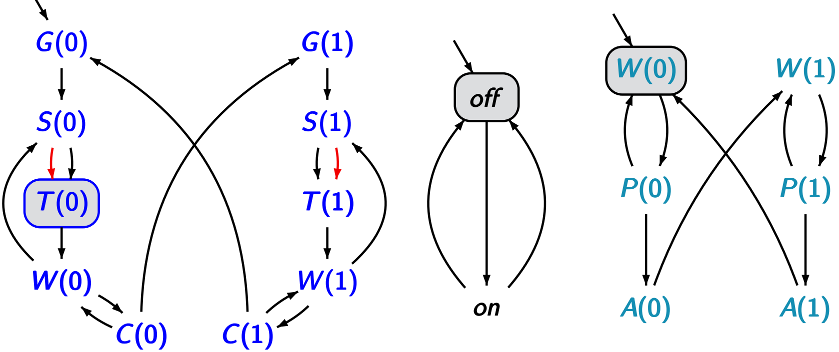


Generate(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$
Send(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$

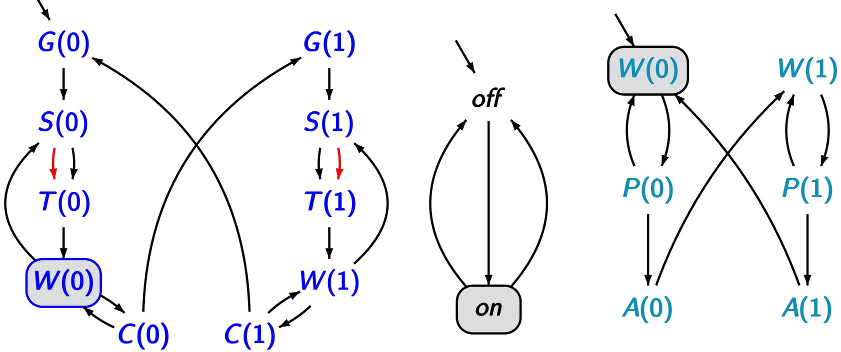


Generate(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$

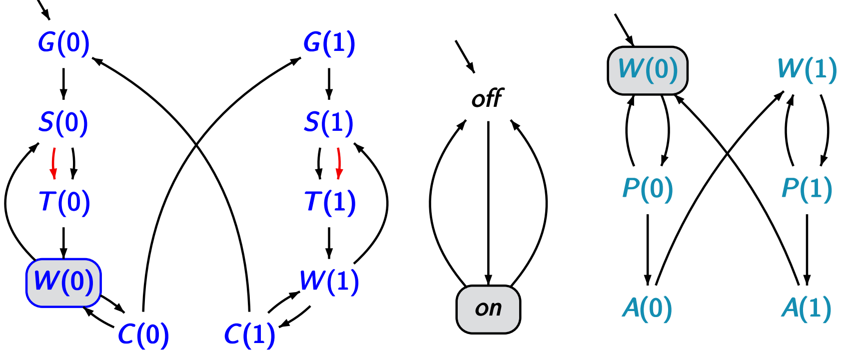
Send(0) off Wait(0) $c{=}\varepsilon$ $d{=}\varepsilon$ message lost



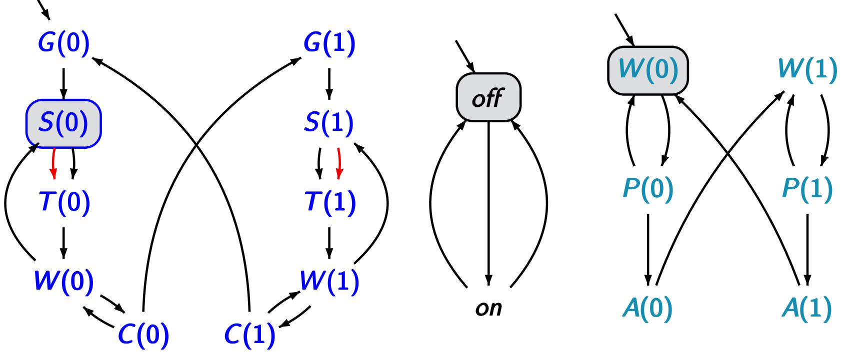
Generate(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	
Send(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	message lost
Timer_on(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	



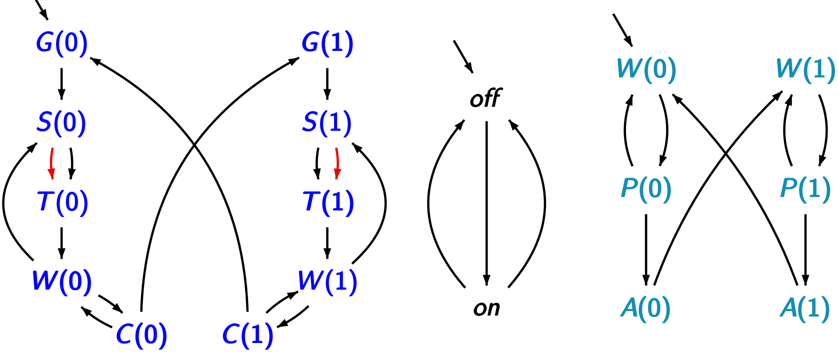
Generate(0)	off	Wait(0)	c=ε	d=ε	
Send(0)	off	Wait(0)	c=ε	d=ε	message lost
Timer_on(0)	off	Wait(0)	c=ε	d=ε	
Wait(0)	on	Wait(0)	c=ε	d=ε	

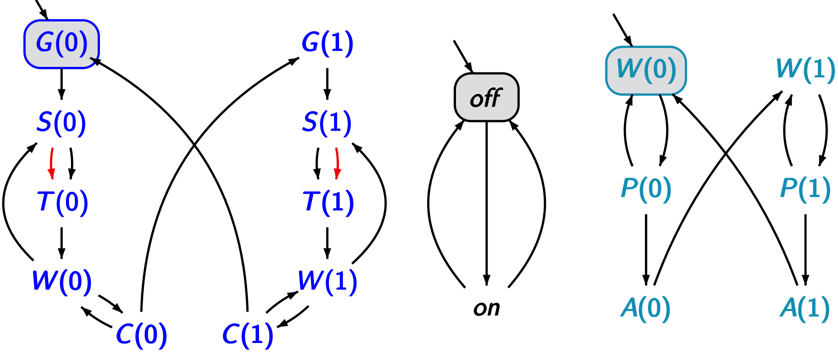


Generate(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	
Send(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	message lost
Timer_on(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	
Wait(0)	on	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	timeout

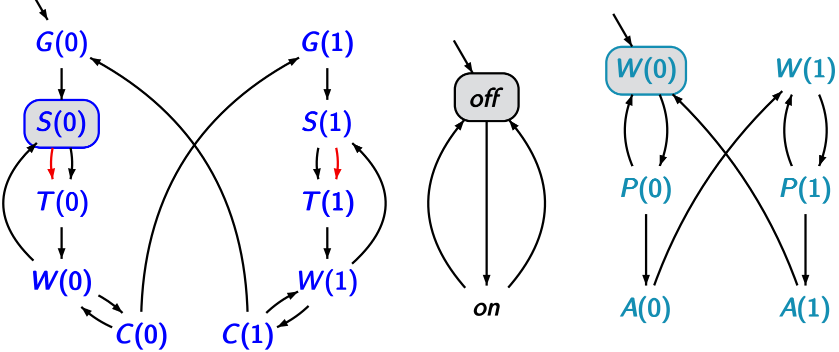


Generate(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	
Send(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	message lost
Timer_on(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	
Wait(0)	on	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	timeout
Send(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	
		⋮			try again

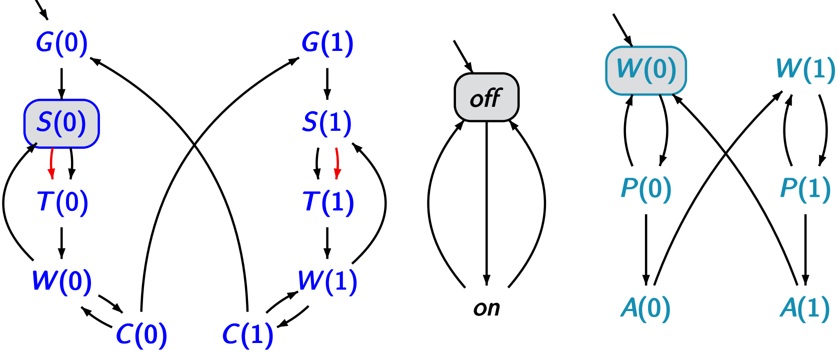

G(0)
S(0)
T(0)
W(0)
C(0)
G(1)
S(1)
T(1)
W(1)
C(1)
off
on
W(0)
P(0)
A(0)
W(1)
P(1)
A(1)



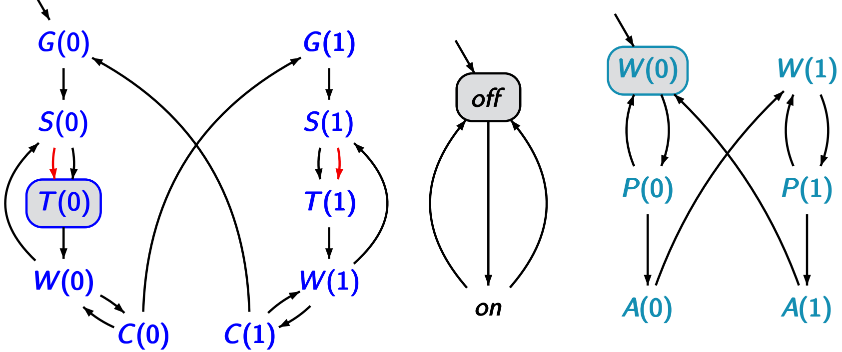
Generate(0) off Wait(0) $c=\varepsilon$ $d=\varepsilon$



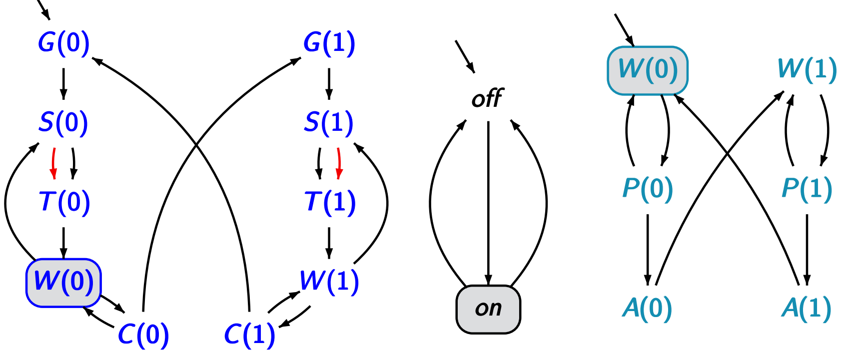
Generate(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$
Send(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$



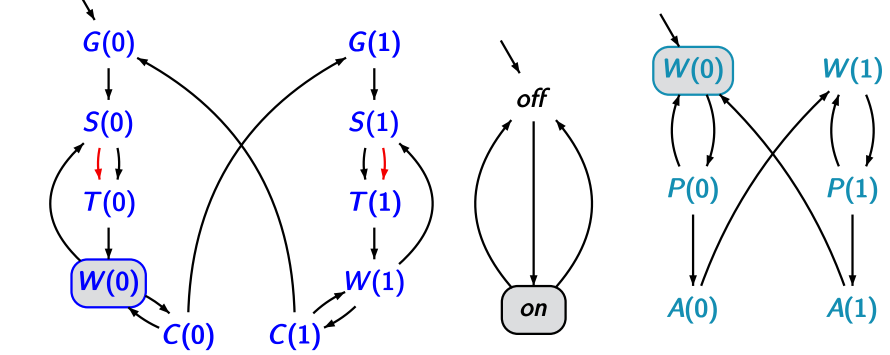
Generate(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	
Send(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	message **0** sent



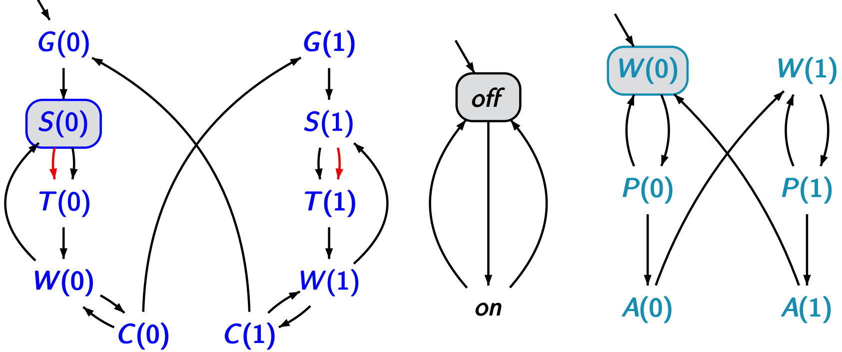
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	



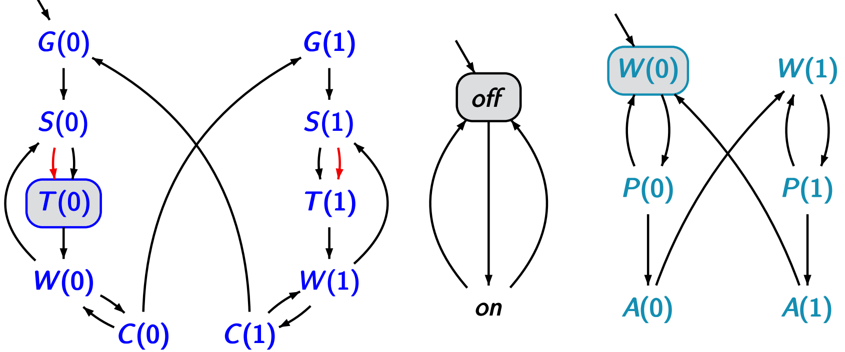
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	



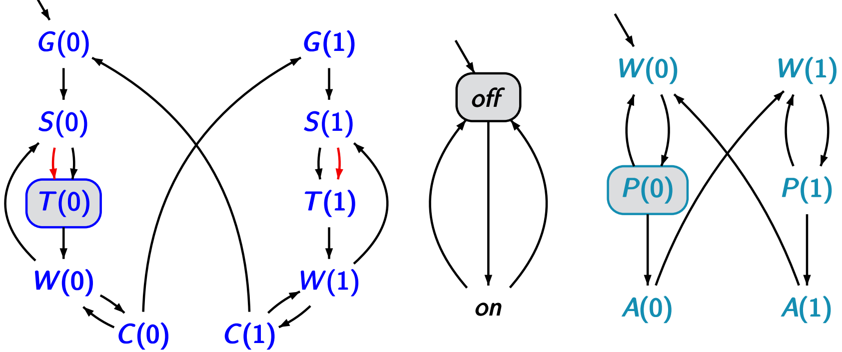
Generate(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	
Send(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c=0$	$d=\varepsilon$	
Wait(0)	on	Wait(0)	$c=0$	$d=\varepsilon$	timeout



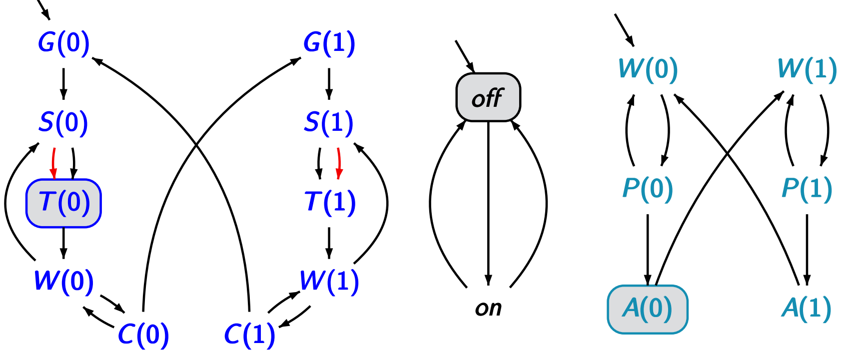
Generate(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	
Send(0)	off	Wait(0)	$c = \varepsilon$	$d = \varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c = 0$	$d = \varepsilon$	
Wait(0)	on	Wait(0)	$c = 0$	$d = \varepsilon$	timeout
Send(0)	off	Wait(0)	$c = 0$	$d = \varepsilon$	



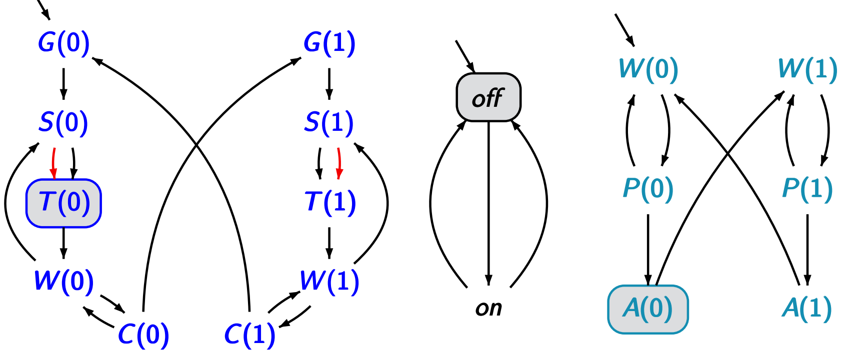
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	



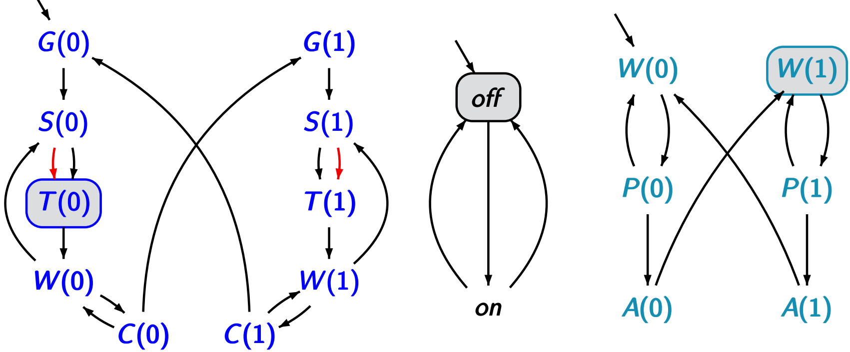
Generate(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	
Send(0)	off	Wait(0)	$c=\varepsilon$	$d=\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c=0$	$d=\varepsilon$	
Wait(0)	on	Wait(0)	$c=0$	$d=\varepsilon$	timeout
Send(0)	off	Wait(0)	$c=0$	$d=\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c=00$	$d=\varepsilon$	
Timer_on(0)	off	Proc(0)	$c=0$	$d=\varepsilon$	message received



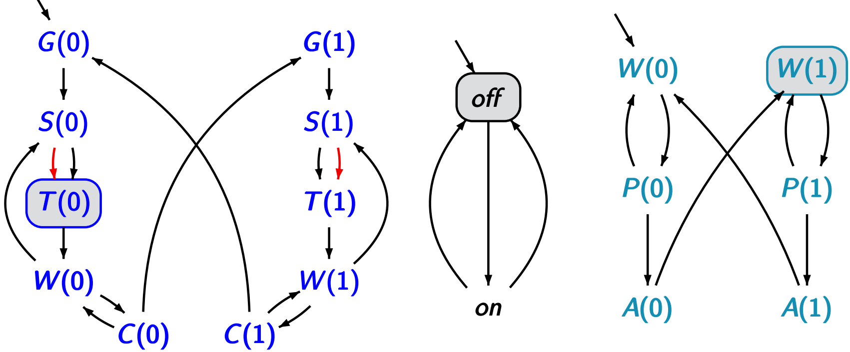
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	
Timer_on(0)	off	Proc(0)	$c{=}0$	$d{=}\varepsilon$	message received
Timer_on(0)	off	Ack(0)	$c{=}0$	$d{=}\varepsilon$	



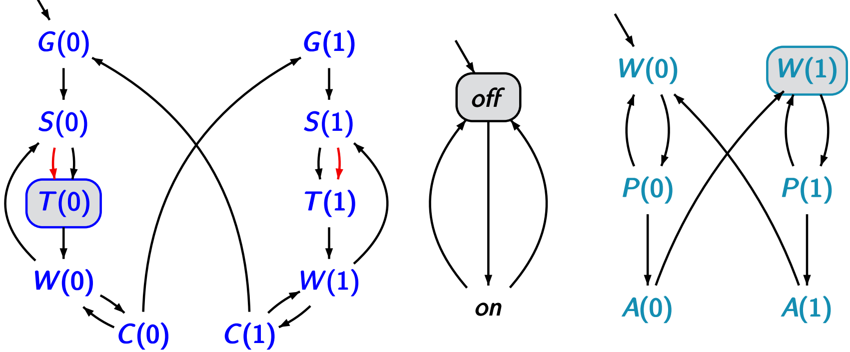
Generate(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	
Send(0)	off	Wait(0)	$c{=}\varepsilon$	$d{=}\varepsilon$	message 0 sent
Timer_on(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	
Wait(0)	on	Wait(0)	$c{=}0$	$d{=}\varepsilon$	timeout
Send(0)	off	Wait(0)	$c{=}0$	$d{=}\varepsilon$	0 sent again
Timer_on(0)	off	Wait(0)	$c{=}00$	$d{=}\varepsilon$	
Timer_on(0)	off	Proc(0)	$c{=}0$	$d{=}\varepsilon$	message received
Timer_on(0)	off	Ack(0)	$c{=}0$	$d{=}\varepsilon$	send ack via d



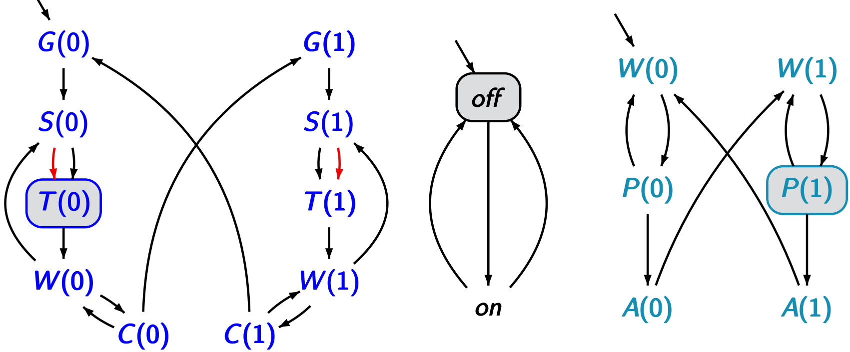
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
Wait(0)	on	Wait(0)	c=0	d=ε	timeout
Send(0)	off	Wait(0)	c=0	d=ε	0 sent again
Timer_on(0)	off	Wait(0)	c=00	d=ε	
Timer_on(0)	off	Proc(0)	c=0	d=ε	message received
Timer_on(0)	off	Ack(0)	c=0	d=ε	send ack via d
Timer_on(0)	off	Wait(1)	c=0	d=0	receiver changes its mode



$\vdots$ Timer_on(0) $\vdots$ off $\vdots$ Wait(1) $\vdots$ $c{=}0$ $\vdots$ $d{=}0$



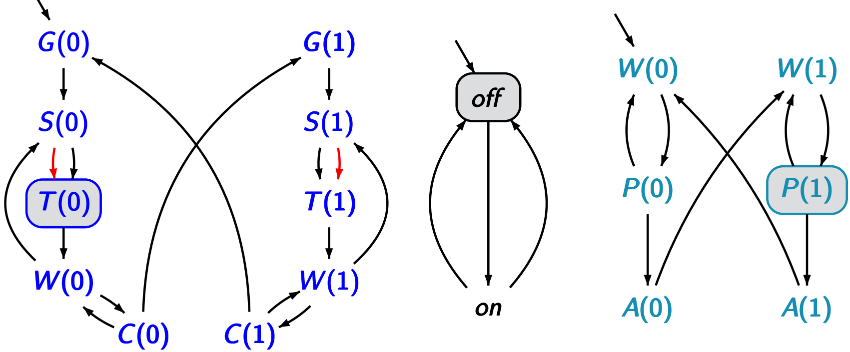
Timer_on(0) off Wait(1) c=0 d=0 receiver reads the same message again



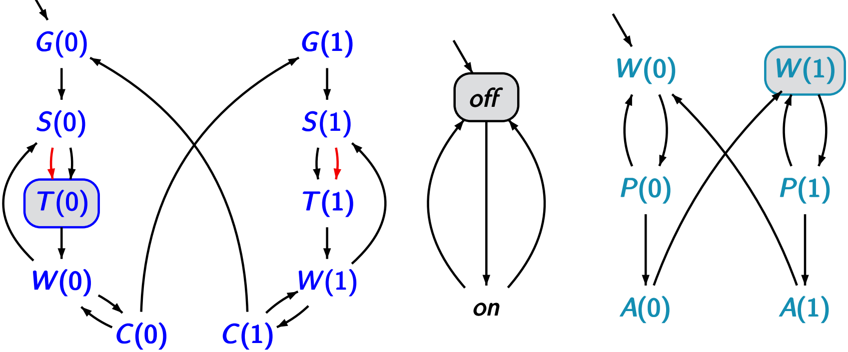
⋮ ⋮ ⋮ ⋮ ⋮

Timer_on(0) off Wait(1) $c{=}0$ $d{=}0$ receiver reads the same message again

Timer_on(0) off Proc(1) $c{=}\varepsilon$ $d{=}0$



⋮	⋮	⋮	⋮	⋮	
Timer_on(0)	off	Wait(1)	$c = 0$	$d = 0$	receiver reads the same message again
Timer_on(0)	off	Proc(1)	$c = \varepsilon$	$d = 0$	receiver discards the message

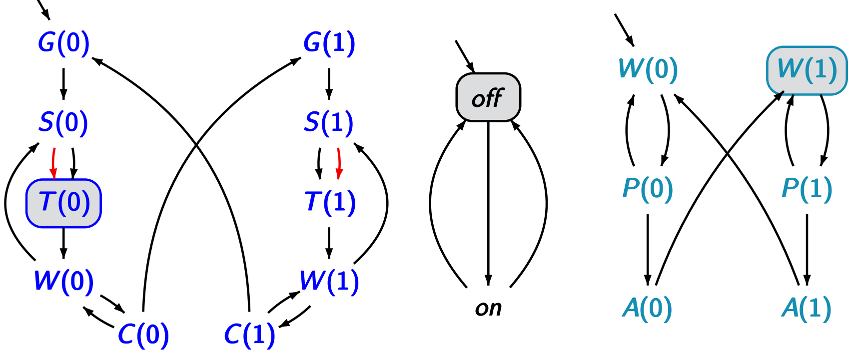


⋮ ⋮ ⋮ ⋮ ⋮

Timer_on(0) off Wait(1) $c{=}0$ $d{=}0$ receiver reads the same message again

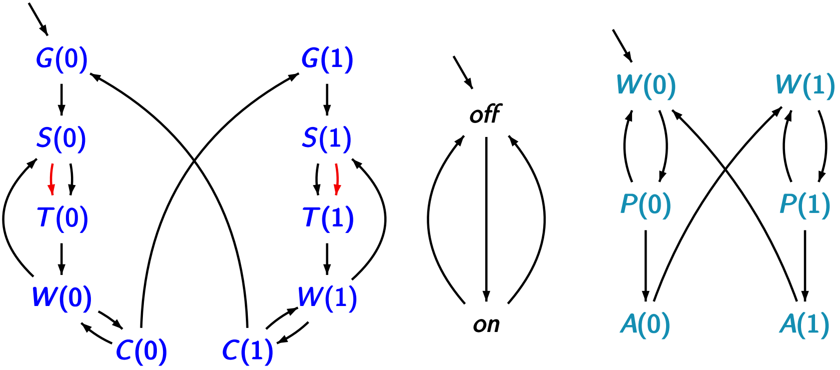
Timer_on(0) off Proc(1) $c{=}\varepsilon$ $d{=}0$ receiver discards the message

Timer_on(0) off Wait(1) $c{=}\varepsilon$ $d{=}0$



⋮	⋮	⋮	⋮	⋮	
Timer_on(0)	off	Wait(1)	c=0	d=0	receiver reads the same message again
Timer_on(0)	off	Proc(1)	$c=\varepsilon$	d=0	receiver discards the message
Timer_on(0)	off	Wait(1)	$c=\varepsilon$	d=0	
⋮	⋮	⋮	⋮	⋮	

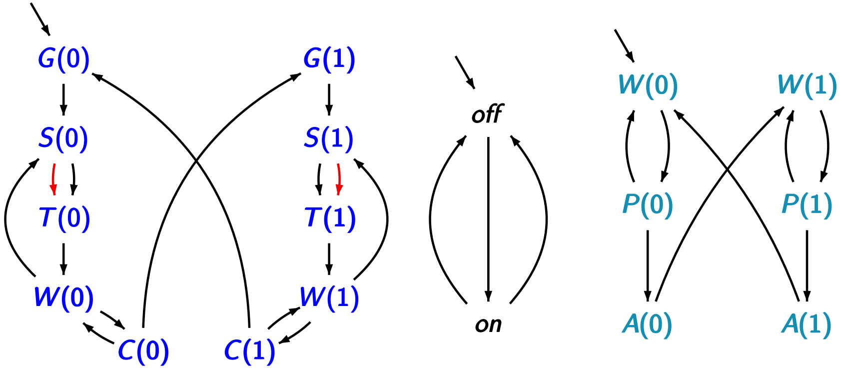
Alternating bit protocol (ABP)



number of states in the TS:

$\mathbf{10} \cdot 2 \cdot \mathbf{6} \cdot \#$channel evaluations

Alternating bit protocol (ABP)



number of states in the TS:

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$> \mathbf{10^8}$ for FIFOs with capacity **10**

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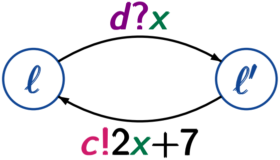

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$$\begin{tikzcd} \ell \arrow[r, bend left, "d?x"] & \ell' \arrow[l, bend left, "c!2x+7"] \end{tikzcd}$$

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PC2.2-38

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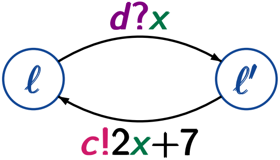

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$\rightsquigarrow$ more compact TS-representations

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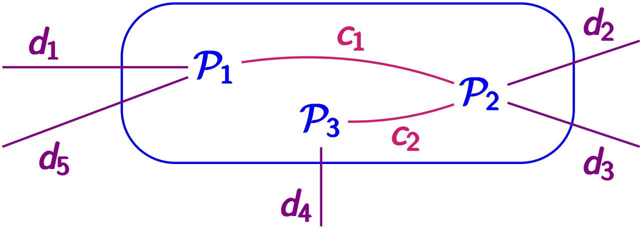

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- interleaving, shared variables
- synchronous and asynchronous message passing

synchronous product for TS $\mathcal{T}_1 \otimes \mathcal{T}_2$

- no interleaving, "pure" synchronization

Synchronous product

for parallel systems with fully synchronized processes

$$\left.\begin{array}{l} \mathcal{T}_1 = (S_1, Act_1, \longrightarrow_1, \ldots) \\ \mathcal{T}_2 = (S_2, Act_2, \longrightarrow_2, \ldots) \end{array}\right\} \text{ two TS}$$

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if action names are irrelevant: $Act_1 = Act_2 = Act = \{\tau\}$

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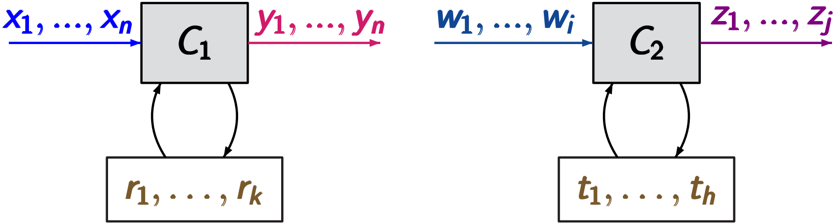
transition relation $\longrightarrow$:

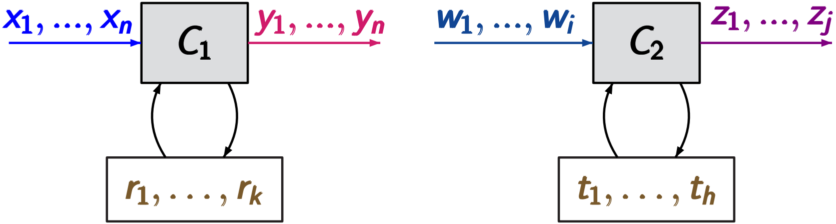
$$\frac{s_1 \xrightarrow{\alpha}_1 s_1' \;\wedge\; s_2 \xrightarrow{\beta}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha * \beta} \langle s_1', s_2' \rangle}$$

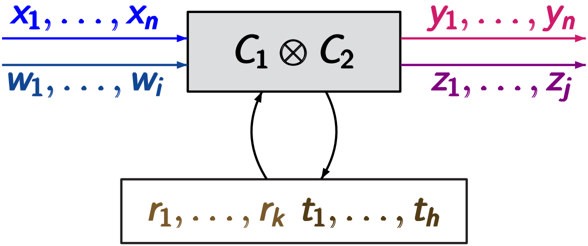
Synchronous product for composing circuits



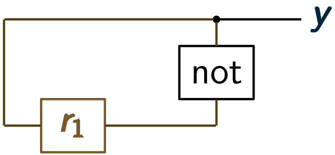
2 sequential circuits

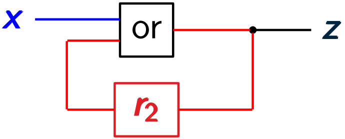
Synchronous product for composing circuits





Synchronous product: example





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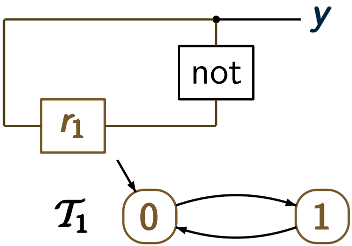

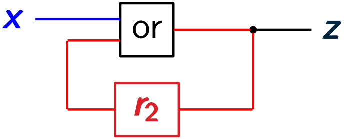

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transition function:
$\delta_{r_1} = \neg r_1$

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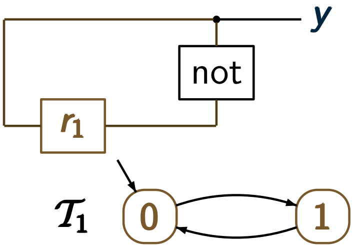

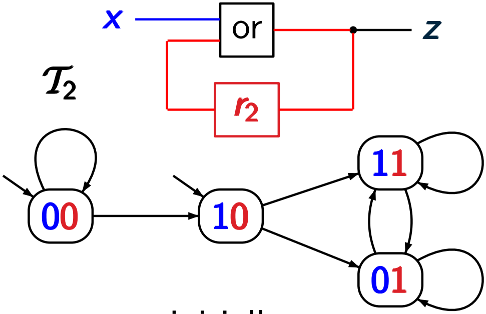

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transition function:
$\delta_{r_2} = r_2 \vee x$

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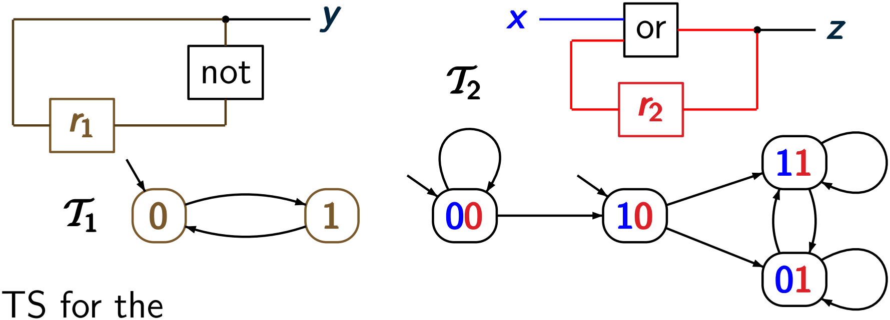
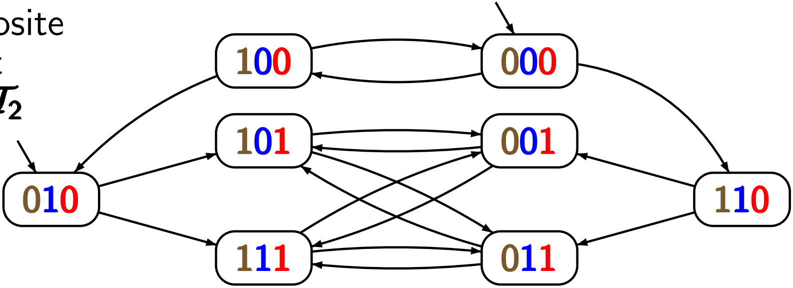


TS for the composite circuit $\mathcal{T}_1 \otimes \mathcal{T}_2$



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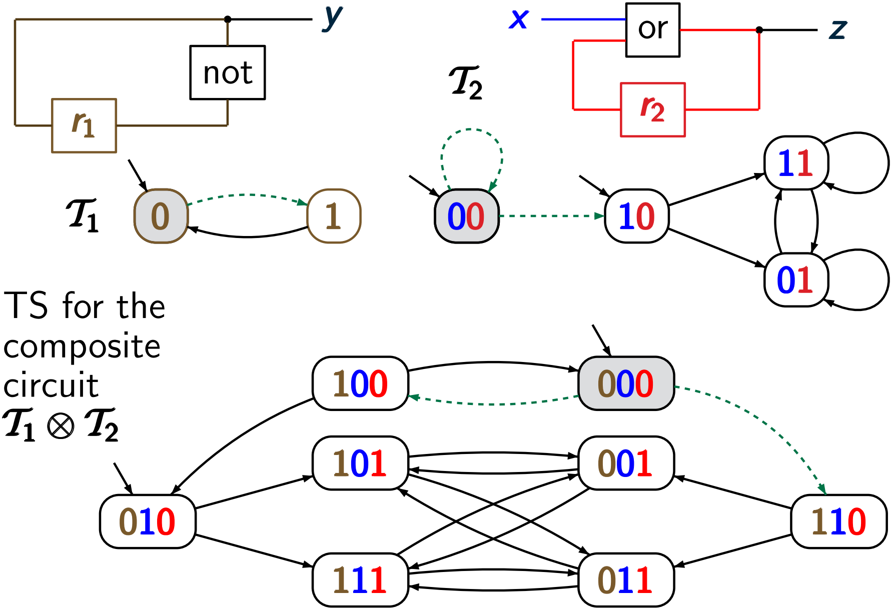

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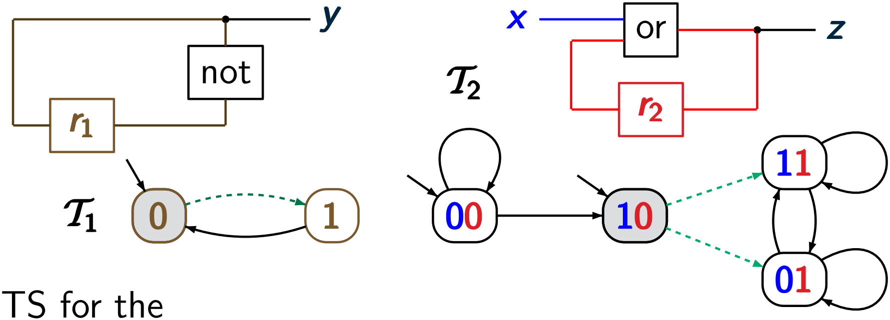

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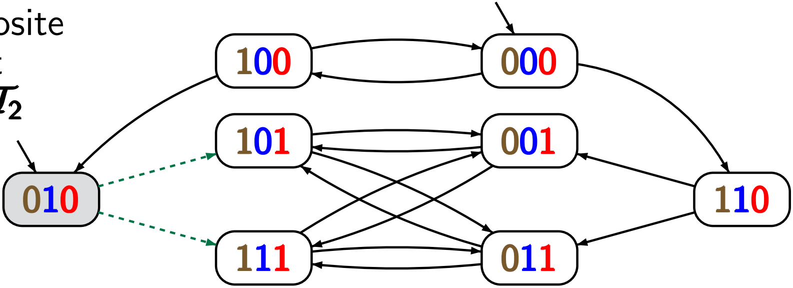

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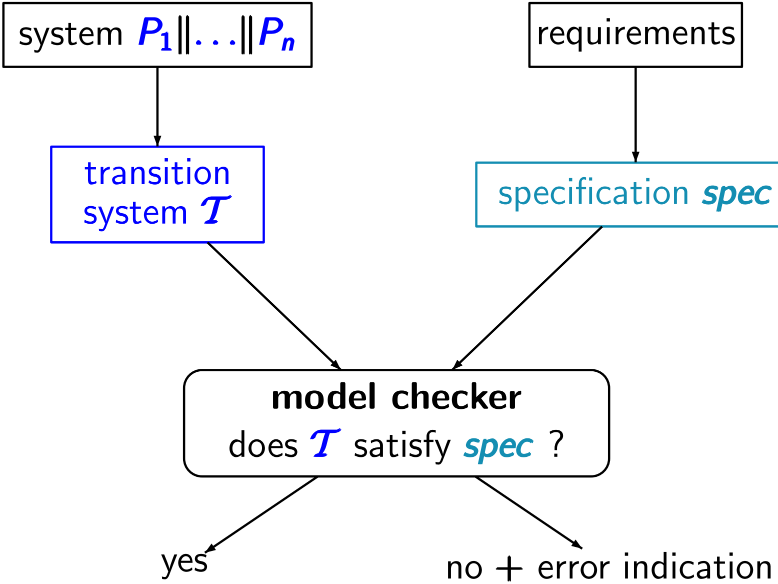
e.g., for channel systems: size of the state space is

$$|Loc_1| \cdot \ldots \cdot |Loc_n| \cdot \prod_{x \in Var} |Dom(x)| \cdot \prod_{c \in Chan} |Dom(c)|^{cap(c)}$$

Model checking



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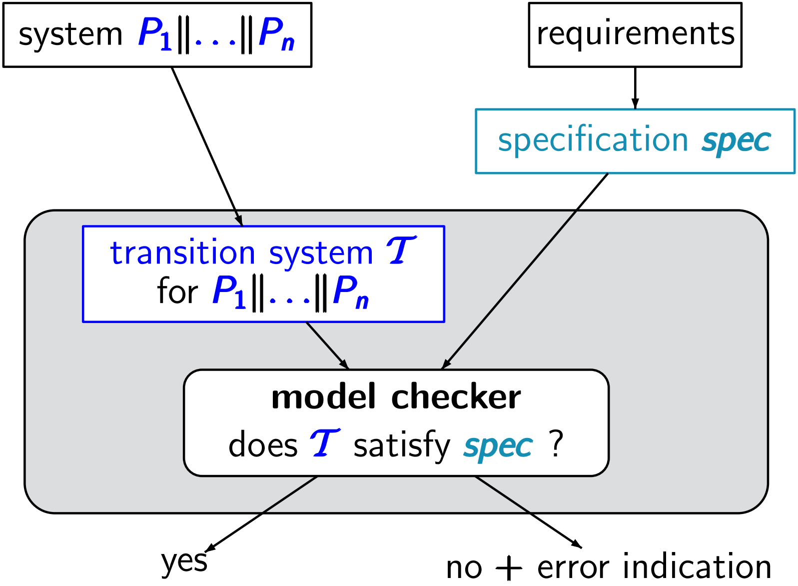

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