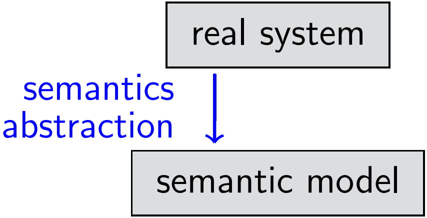
Overview

- Introduction
- **Modelling parallel systems**
    - Transition systems ⟵
    - Modeling hard- and software systems
    - Parallelism and communication
- Linear Time Properties
- Regular Properties
- Linear Temporal Logic
- Computation-Tree Logic
- Equivalences and Abstraction

Transition systems

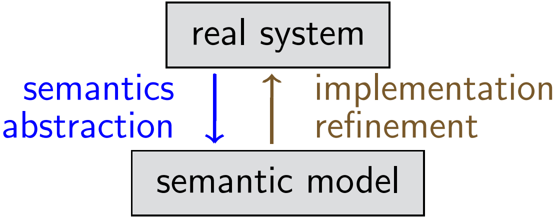


real system

semantics
abstraction

semantic model

Transition systems



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implementation
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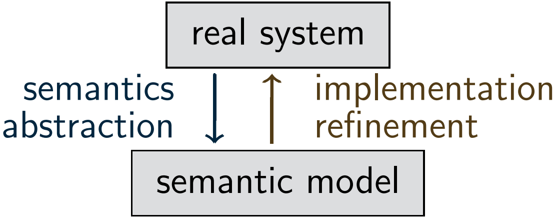
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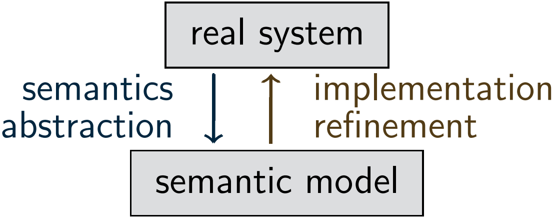
The semantic model yields a formal representation of:

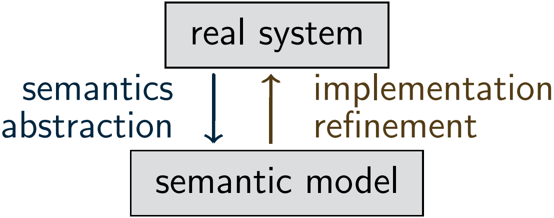
Transition systems



The semantic model yields a formal representation of:

- the states of the system
- the stepwise behaviour
- the initial states

Transition systems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The semantic model yields a formal representation of:

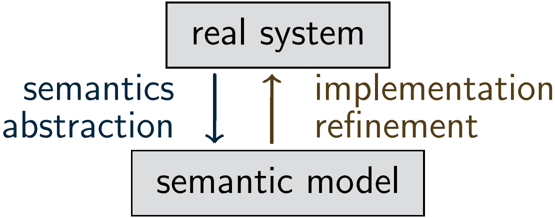
- the states of the system

↑

control component + information on "relevant" data

- the stepwise behaviour
- the initial states

Transition systems $\widehat{=}$ extended digraphs



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The semantic model yields a formal representation of:

- the states of the system ⟵ **nodes**

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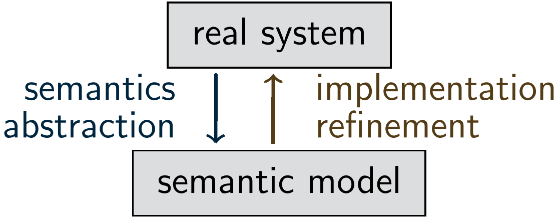
- the stepwise behaviour ⟵ **edges**
- the initial states

Transition systems $\widehat{=}$ extended digraphs



real system

semantics abstraction ↓

↑ implementation refinement

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The semantic model yields a formal representation of:

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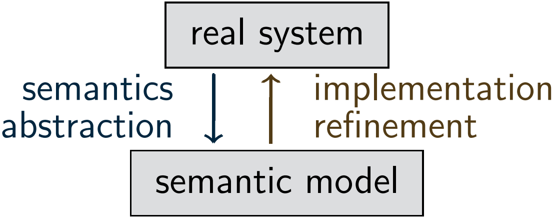

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The semantic model yields a formal representation of:

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- the initial states
- additional information on
  communication
  state properties

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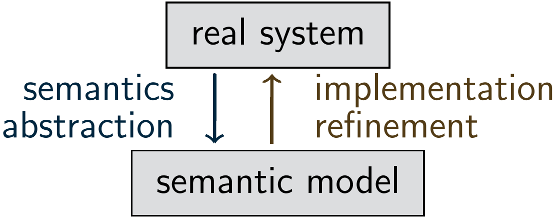

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- the states of the system ⟵ **nodes**
- the stepwise behaviour ⟵ **transitions**
- the initial states
- additional information on
  communication ⟵ **actions**
  state properties ⟵ **atomic proposition**

Transition system (TS)

A transition system is a tuple

$$\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$$

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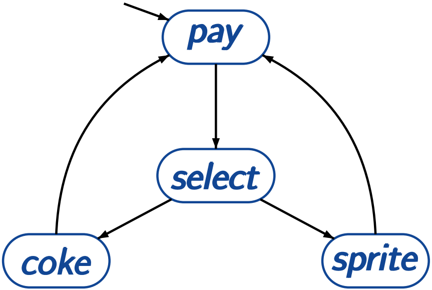
- $S_0 \subseteq S$ the set of initial states,
- AP a set of atomic propositions,
- $L : S \to 2^{AP}$ the labeling function

Transition system for beverage machine



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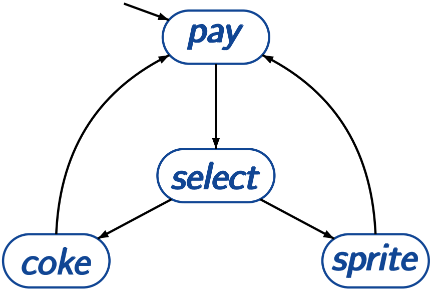
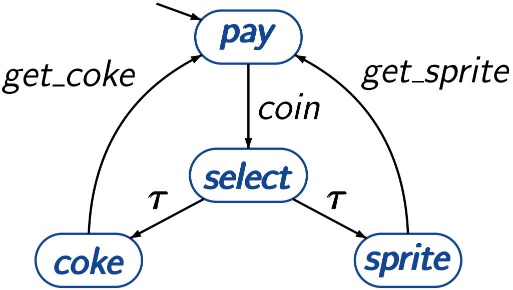


state space $S = \{pay, select, coke, sprite\}$

set of initial states: $S_0 = \{pay\}$

Transition system for beverage machine



actions:

coin

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get_sprite

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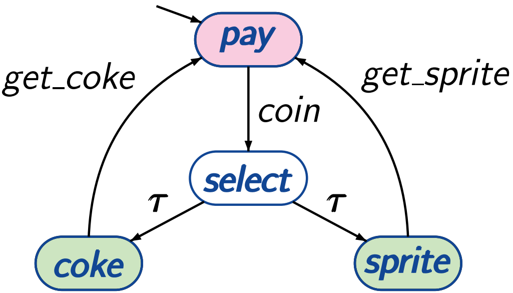


actions:
- *coin*
- τ
- *get_sprite*
- *get_coke*

state space $S = \{pay, select, coke, sprite\}$

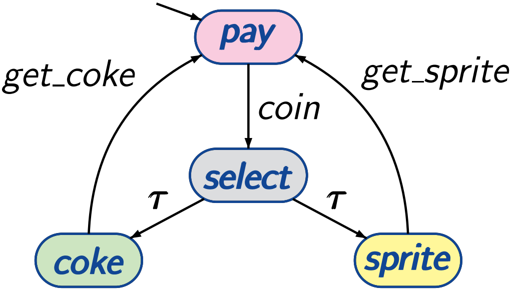
set of initial states: $S_0 = \{pay\}$

set of atomic propositions: $AP = \{pay, drink\}$

labeling function: $L(coke) = L(sprite) = \{drink\}$

$L(pay) = \{pay\}$, $L(select) = \emptyset$

Transition system for beverage machine



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set of atomic propositions: $AP = S$

labeling function: $L(s) = \{s\}$ for each state s

“Behaviour” of transition systems

possible behaviours of a TS result from:

```
select nondeterministically an initial state $s \in S_0$
WHILE  $s$ is non-terminal DO
        select nondeterministically a transition $s \xrightarrow{\alpha} s'$
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“Behaviour” of transition systems

TS1.4-3

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executions: maximal “transition sequences”

$$s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots \text{ with } s_0 \in S_0$$

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reachable fragment:

$\mathit{Reach}(\mathcal{T})$ = set of all states that are reachable from an initial state through some execution

Possible meanings of nondeterminism in TS

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- (true) concurrency modeled by interleaving
- competition of parallel dependent actions
- implementational freedom, underspecification
- incomplete information on system environment

Transition system for parallel actions

parallel execution of independent actions

parallel execution of dependent actions

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parallel execution of independent actions

e.g. $\underbrace{x := x+1}_{\text{action } \alpha} \;|||\; \underbrace{y := y-3}_{\text{action } \beta}$ $\quad \alpha, \beta$ independent

parallel execution of dependent actions

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Transition system for parallel actions

parallel execution of independent actions ← $\boxed{\text{interleaving}}$

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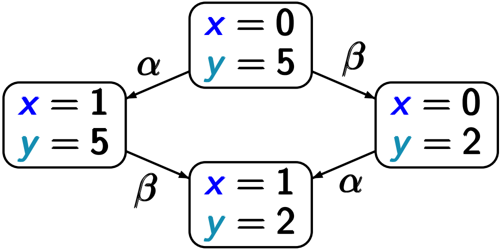

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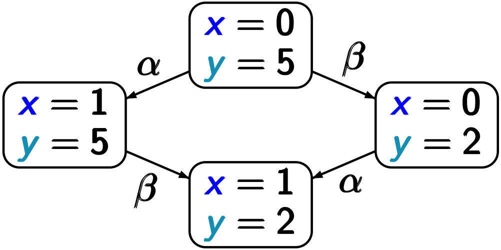
parallel execution of independent actions ← interleaving



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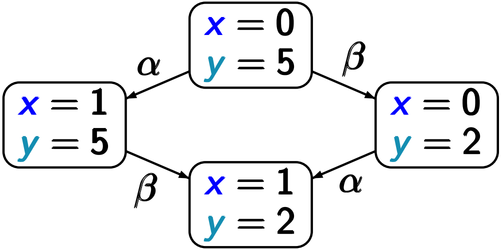
parallel execution of dependent actions ← competition parallel execution of independent actions ← interleaving



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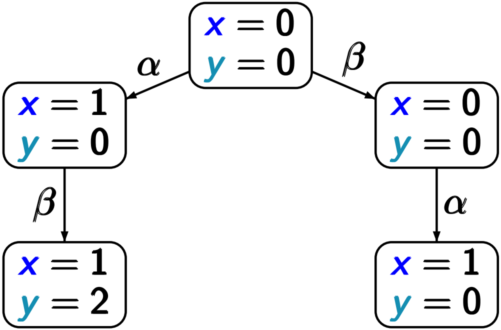

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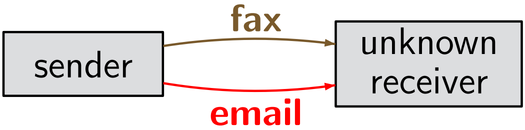
Implementation freedom

TS1.4-5

... modelled by nondeterminism

Implementation freedom



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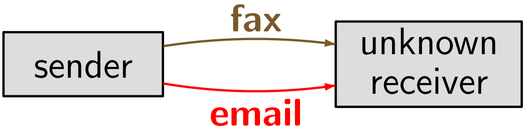
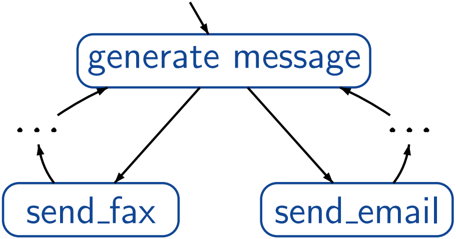
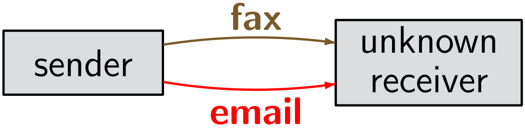


realization by a TS:



Implementation freedom

TS1.4-5



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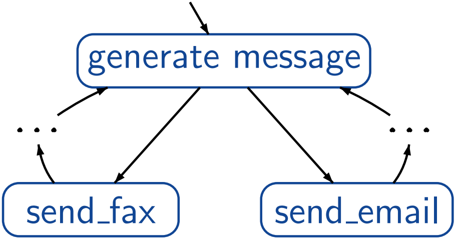
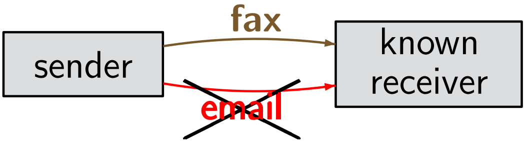


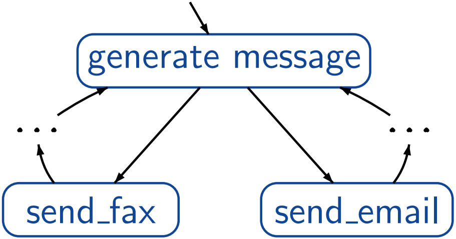
at a future refinement step the nondeterminism is replaced with one of the alternatives

Implementation freedom



without email access

realization by a TS:



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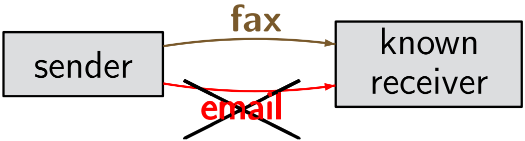

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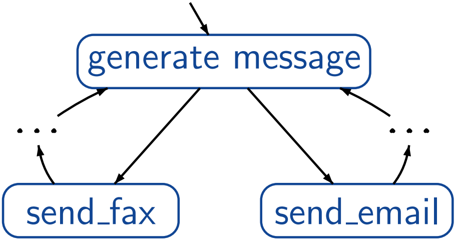
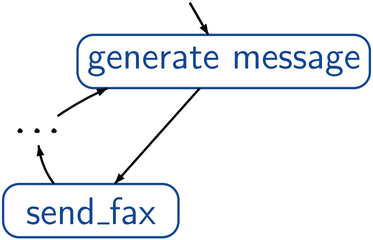


refined TS:



at a future refinement step the nondeterminism
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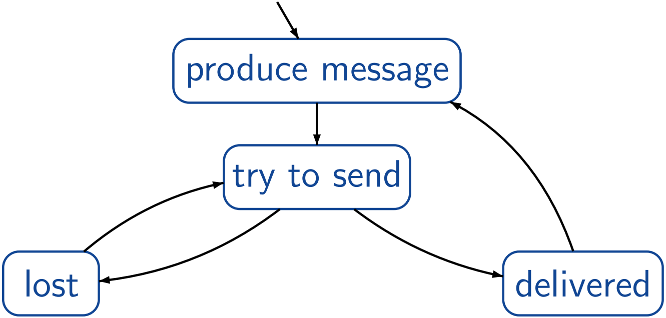
Underspecification

TS1.4-6

Underspecification



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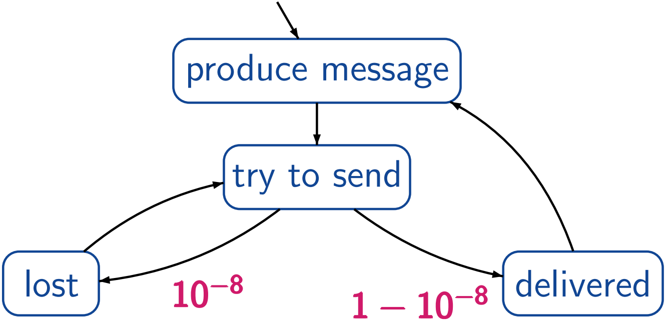

at a future refinement step the nondeterminism is replaced with probabilism

Possible meanings of nondeterminism in TS

- (true) concurrency modeled by interleaving
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- incomplete information on system environment

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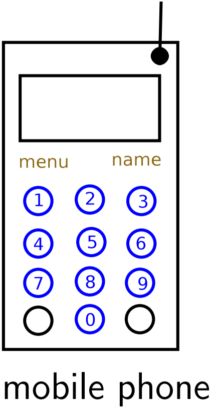
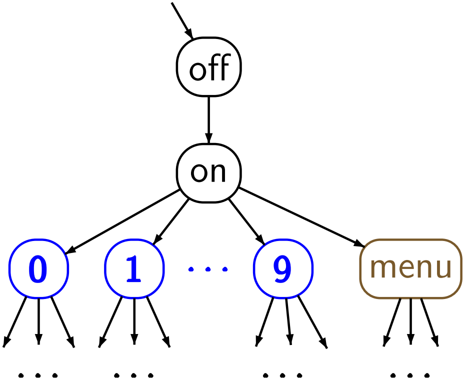
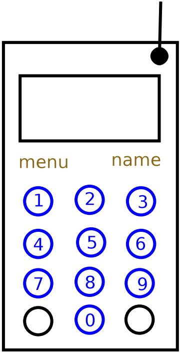
Incomplete information on the environment



mobile phone



Incomplete information on the environment



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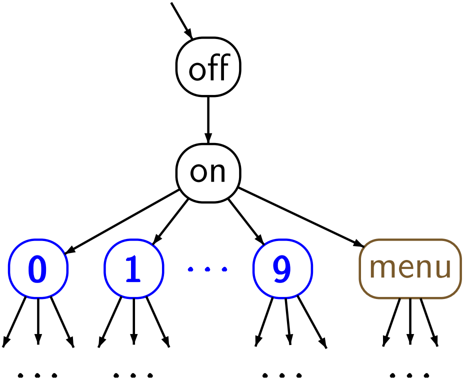
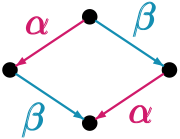


resolution of the nondeterministic choices
by a human user

Possible meanings of nondeterminism in TS

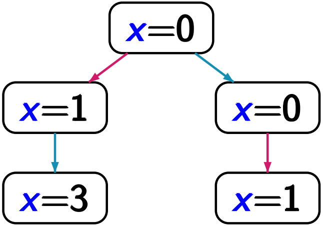
concurrency (interleaving)

$\alpha \,|||\, \beta$ is represented by



competitions

to be resolved by a scheduler
e.g. $x{:=}x{+}1 \parallel x{:=}3x$



underspecification, implementational freedom

incomplete information on system environment, e.g., interfaces with other programs, human users, sensors

Overview

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Modelling parallel systems

- Transition systems
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Linear Time Properties

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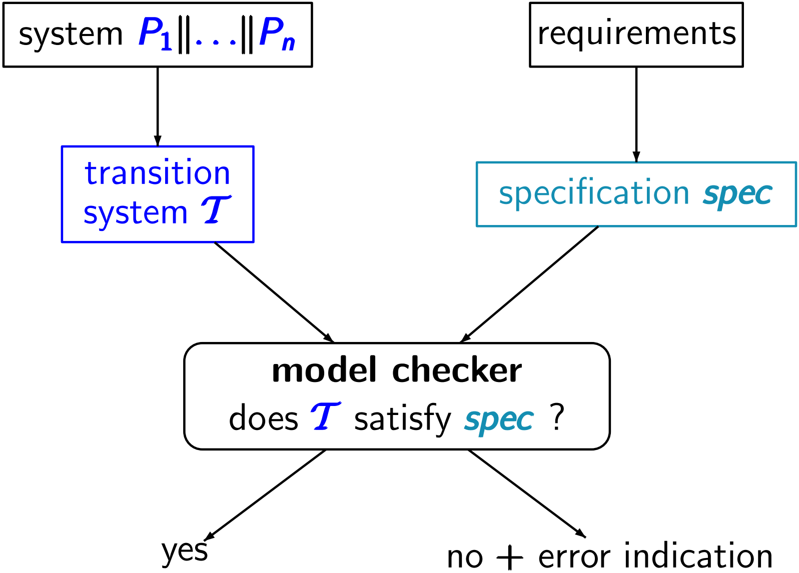
Model checking



system $P_1 \| \ldots \| P_n$

requirements

transition system $\mathcal{T}$

specification *spec*

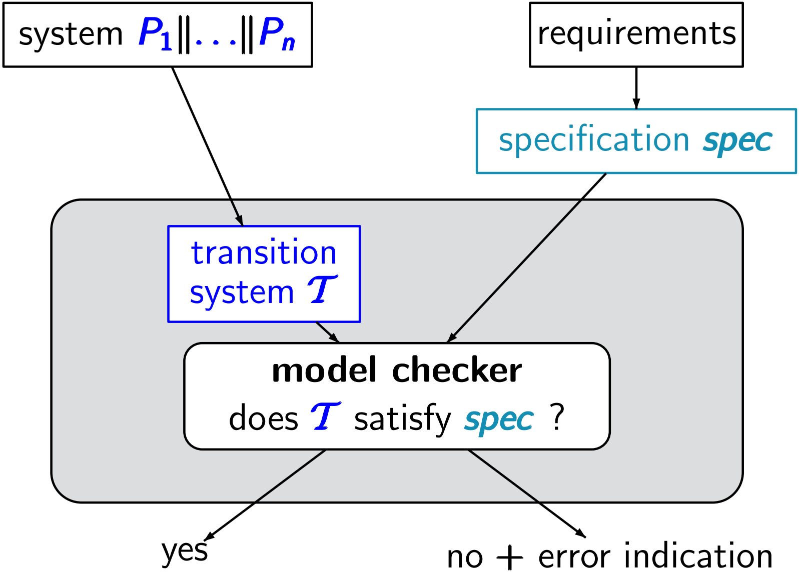
model checker

does $\mathcal{T}$ satisfy *spec* ?

yes

no + error indication

Model checking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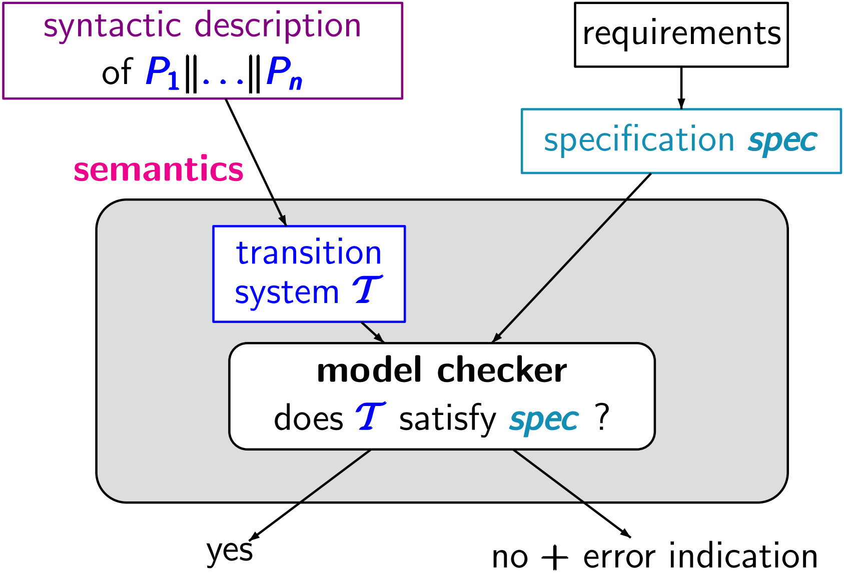
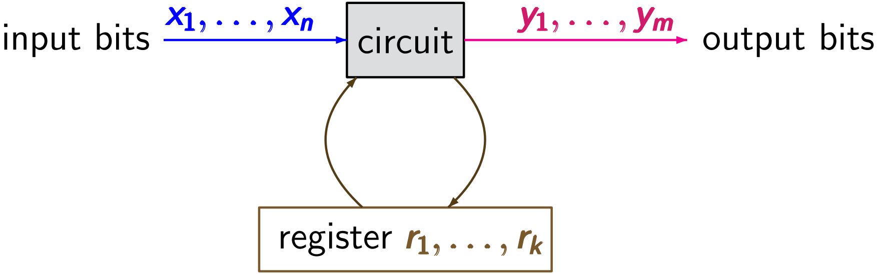


Modelling of sequential circuits by TS



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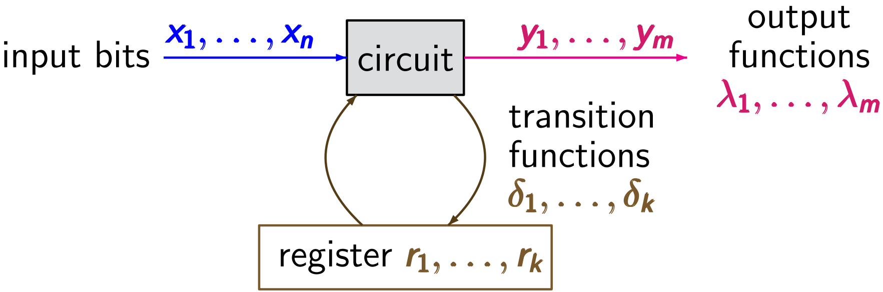

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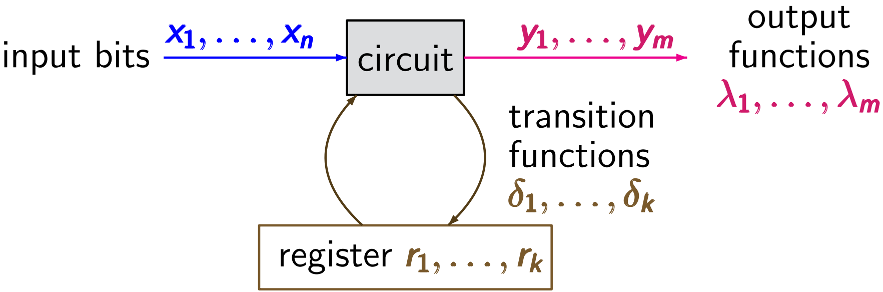

$\delta_j, \lambda_i \mathrel{\widehat{=}}$ switching functions $\{0,1\}^n \times \{0,1\}^k \longrightarrow \{0,1\}$

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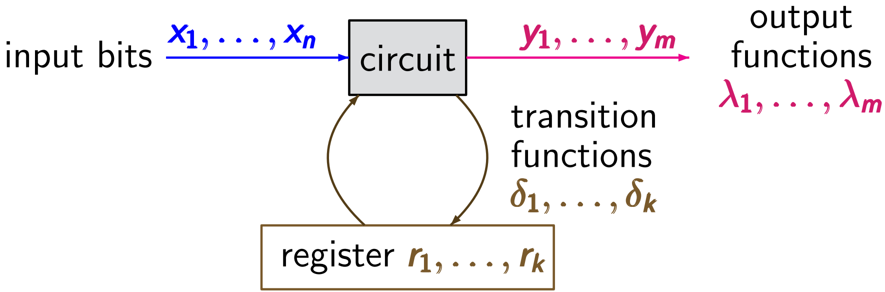
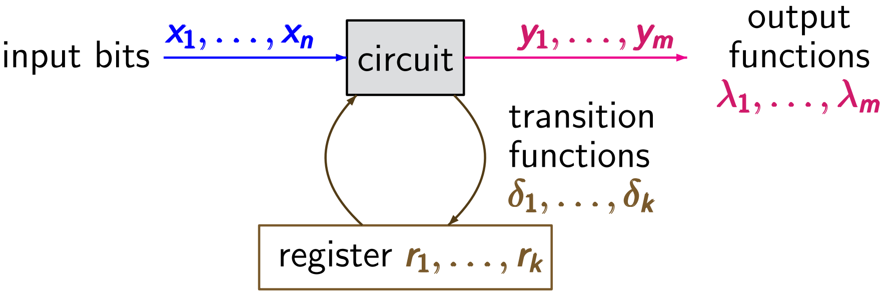


$\delta_j, \lambda_i \mathrel{\widehat{=}}$ switching functions $\{0,1\}^n \times \{0,1\}^k \longrightarrow \{0,1\}$

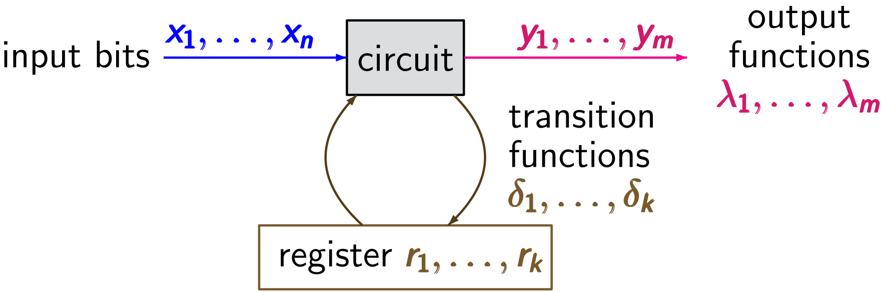
input values $a_1, \ldots, a_n$
for the input variables
+ current values $c_1, \ldots, c_k$
of the registers
	$\mapsto$	output value $\lambda_i(\ldots)$
		for output variable y_i
		next value $\delta_j(\ldots)$
		for register r_j

Modelling of sequential circuits by TS



initial register evaluation $[r_1 = c_{01}, \ldots, r_k = c_{0k}]$

Modelling of sequential circuits by TS



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transition system:

- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$

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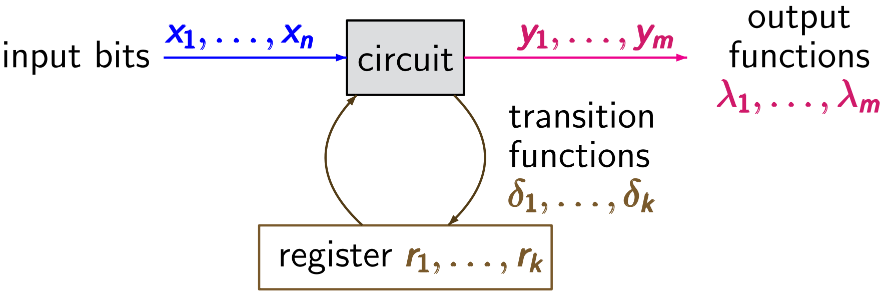

input bits $x_1, \ldots, x_n$

circuit

$y_1, \ldots, y_m$ output functions $\lambda_1, \ldots, \lambda_m$

transition functions $\delta_1, \ldots, \delta_k$

register $r_1, \ldots, r_k$

initial register evaluation $[r_1{=}c_{01}, \ldots, r_k{=}c_{0k}]$

transition system:

- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$
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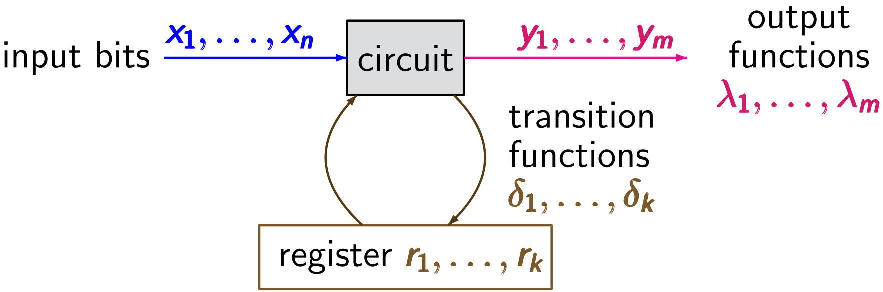
Modelling of sequential circuits by TS



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transition system:

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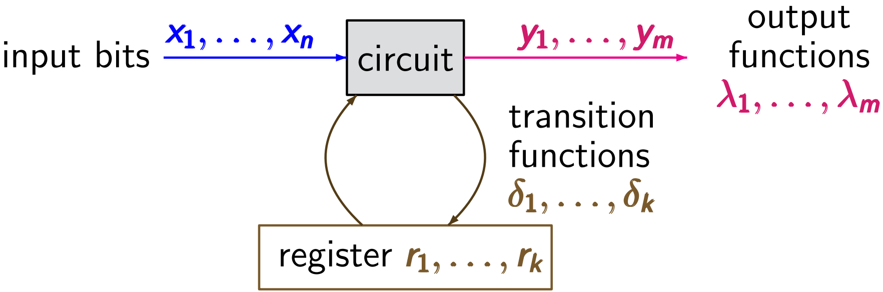
Modelling of sequential circuits by TS

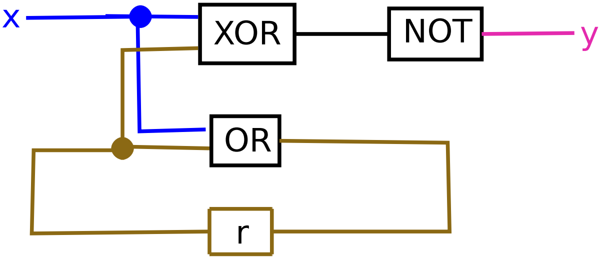


initial register evaluation $[r_1{=}c_{01}, \ldots, r_k{=}c_{0k}]$

transition system:

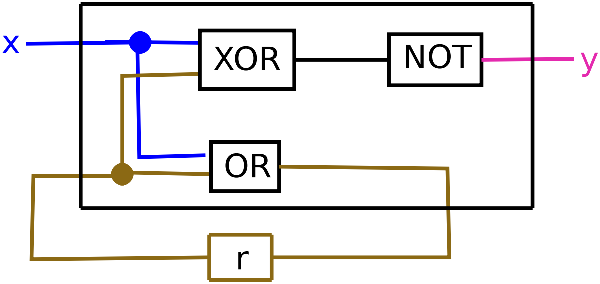
- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$
- transitions represent the stepwise behavior
- values of input bits change nondeterministically
- atomic propositions: $x_1, \ldots, x_n, y_1, \ldots, y_m, r_1, \ldots, r_k$

Example: sequential circuit



Example: sequential circuit

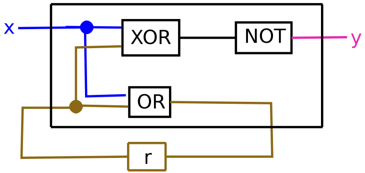
TS1.4-11A



output function: $\lambda_y = \neg(x \oplus r)$

transition function: $\delta_r = x \vee r$

Example: TS for sequential circuit



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$$\delta_r = x \vee r$$

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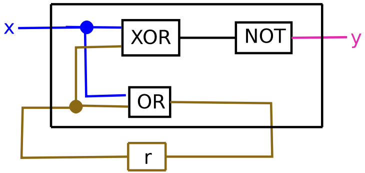

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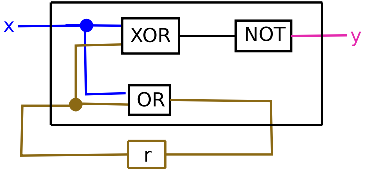

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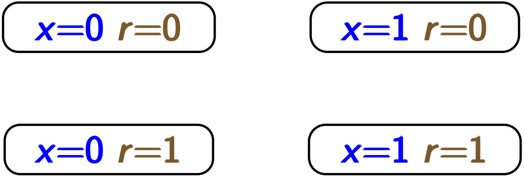

$x{=}0 \;\; r{=}0$

$x{=}1 \;\; r{=}0$

$x{=}0 \;\; r{=}1$

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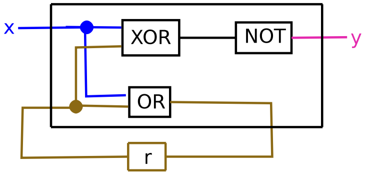

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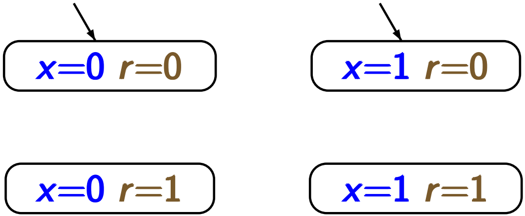
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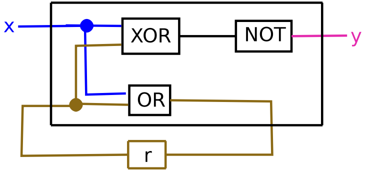

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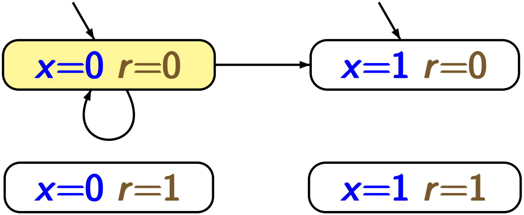
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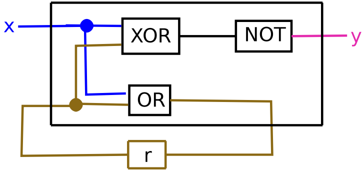

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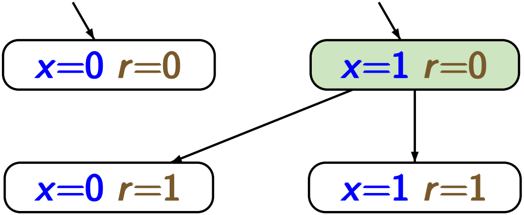

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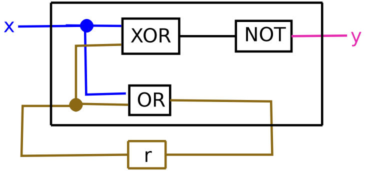

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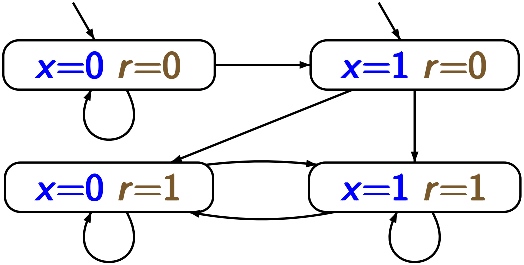

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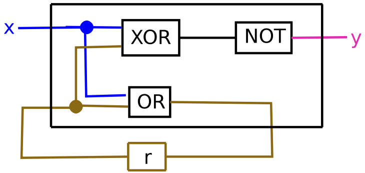

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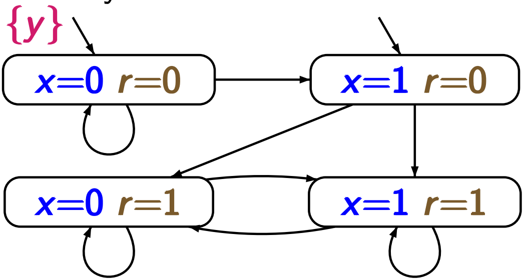

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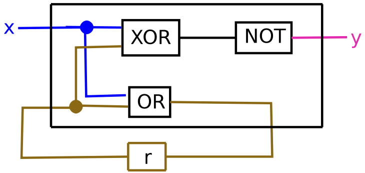

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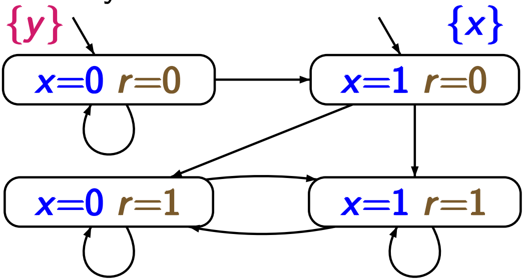

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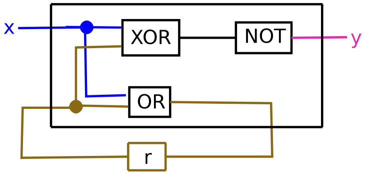

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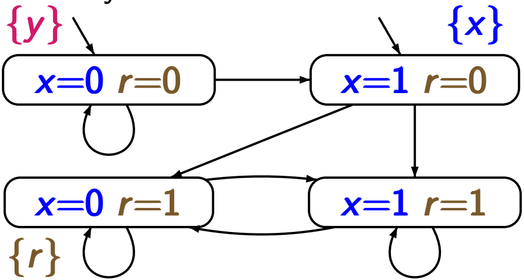

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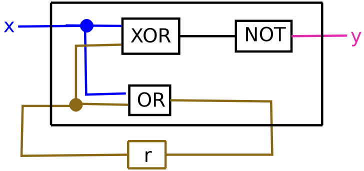

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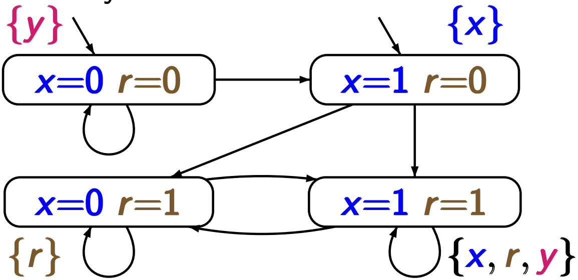
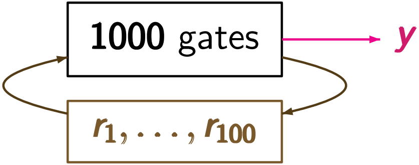


initial register evaluation: r=0

How many states . . .

. . . has the transition system for a circuit of the form?



1 output bit
no input
100 registers

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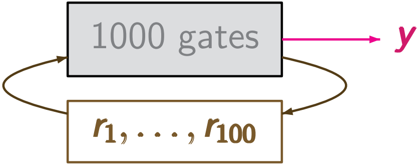

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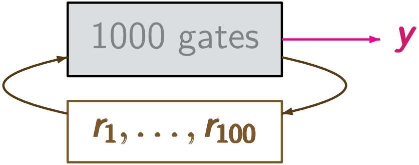

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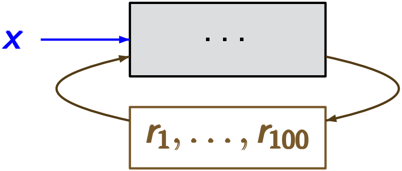

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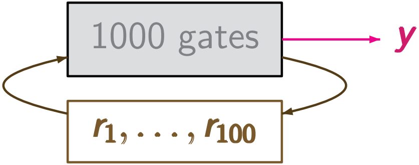

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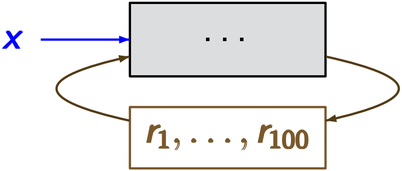

no output
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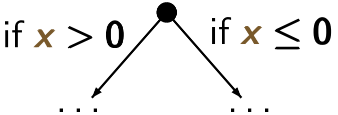
answer: $2^{100} * 2^1 = 2^{101}$

Data-dependent systems

problem: TS-representation of conditional branchings **?**



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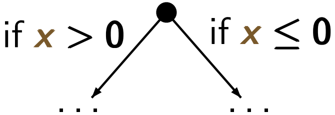

example: sequential program

```
WHILE  $x > 0$ DO
      $x := x - 1$;
      $y := y + 1$
OD
...
```

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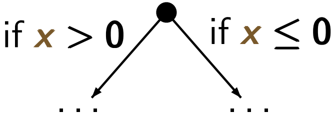

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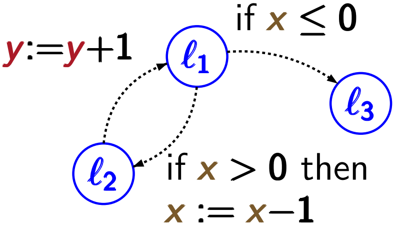

Data-dependent systems

TS1.4-13

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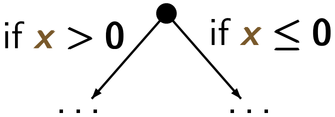

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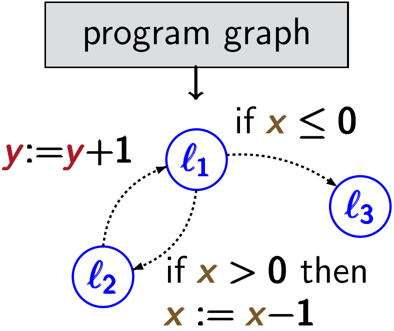

Data-dependent systems

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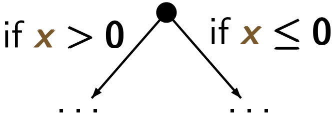

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$\ell_1$→   WHILE  $x > 0$ DO
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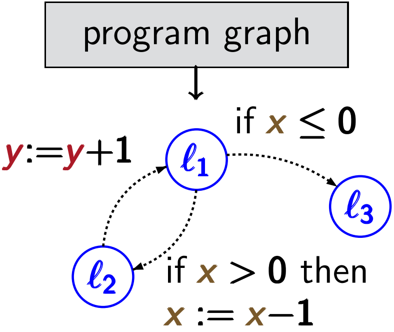
$\ell_3$→   ...
```



ℓ_1, ℓ_2, ℓ_3 are locations,
i.e., control states

Data-dependent systems

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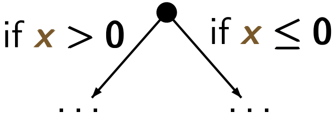

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```
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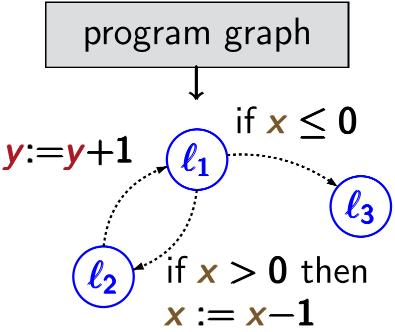

states of the transition system:

locations + relevant data (*here:* values for x and y)

Example: TS for sequential program

initially: $x = 2$, $y = 0$

```
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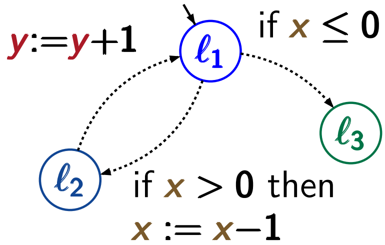
program graph



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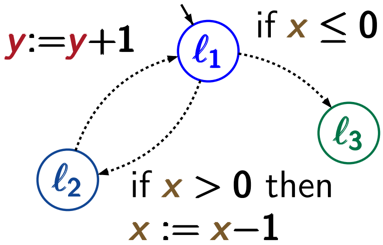

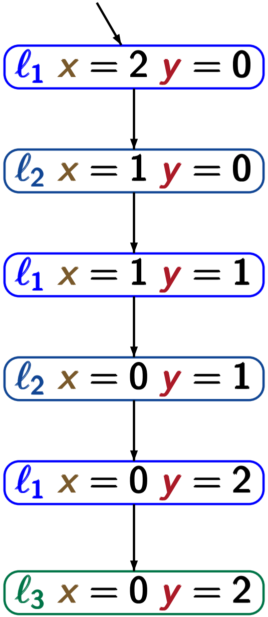

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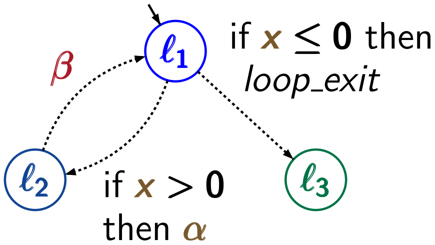
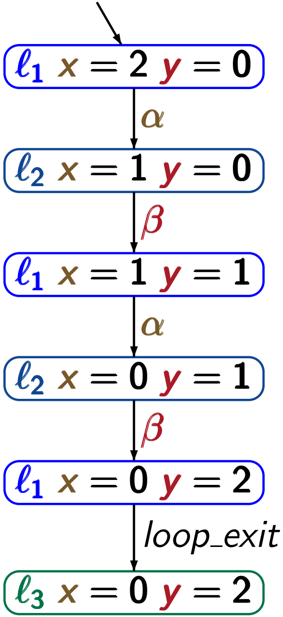
```
$\ell_1 \rightarrow$   WHILE $x > 0$ DO
                 $x := x - 1$   $\leftarrow$ action $\alpha$
$\ell_2 \rightarrow$             $y := y + 1$   $\leftarrow$ action $\beta$
         OD
$\ell_3 \rightarrow$   ...
```

program graph





Typed variables

typed variable: variable $\boldsymbol{x}$ + data domain $\boldsymbol{Dom(x)}$

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- Boolean variable: variable x with $Dom(x) = \{0, 1\}$
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type-consistent function $\eta : Var \rightarrow Values$

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↑ $\eta(x) \in Dom(x)$ for all $x \in Var$

↑ $Values = \bigcup\limits_{x \in Var} Dom(x)$

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$\uparrow$

$\eta(x) \in Dom(x)$
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$\uparrow$

$$Values = \bigcup_{x \in Var} Dom(x)$$

Notation: $Eval(Var)$ = set of evaluations for Var

Conditions on typed variables

If *Var* is a set of typed variables then

$$\boxed{\mathit{Cond}(\mathit{Var}) = \begin{array}{l}\text{set of Boolean conditions} \\ \quad\text{on the variables in } \mathit{Var}\end{array}}$$

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Example: $(\neg x \wedge y{<}z{+}3) \vee w{=}\mathit{red}$

where $\mathit{Dom}(x) = \{0, 1\}$, $\mathit{Dom}(y) = \mathit{Dom}(z) = \mathbb{N}$, $\mathit{Dom}(w) = \{\mathit{yellow}, \mathit{red}, \mathit{blue}\}$

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where $Dom(x) = \{0,1\}$, $Dom(y) = Dom(z) = \mathbb{N}$, $Dom(w) = \{yellow, red, blue\}$

satisfaction relation $\models$ for evaluations and conditions

Conditions on typed variables

If Var is a set of typed variables then

$$\boxed{Cond(Var) = \text{ set of Boolean conditions on the variables in } Var}$$

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satisfaction relation $\models$ for evaluations and conditions

Example:

$$[x{=}0, y{=}3, z{=}6] \models \neg x \wedge y{<}z$$

$$[x{=}0, y{=}3, z{=}6] \not\models x \vee y{=}z$$

Effect-function for actions

Given a set ***Act*** of actions that operate on the variables in ***Var***, the effect of the actions is formalized by:

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$$\mathit{Effect}(\alpha, [x{=}1, y{=}3, \ldots]) \;=\; [x{=}5, y{=}3, \ldots]$$

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if β is "$x{:=}2x{+}y\,;\; y{:=}1{-}x$" then:

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if γ is "$(x, y) := (2x{+}y, 1{-}x)$" then:

$$Effect(\gamma, [x{=}1, y{=}3, \ldots]) \;=\; [x{=}5, y{=}0, \ldots]$$

Program graph (PG)

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Let ***Var*** be a set of typed variables.

A *program graph* over ***Var*** is a tuple

$$\mathcal{P} \;=\; (\mathit{Loc}, \mathit{Act}, \mathit{Effect}, \hookrightarrow, \mathit{Loc}_0, g_0) \text{ where}$$

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↑

function that formalizes the effect of the actions

Program graph (PG)

TRANSSYS/TS-PROGRAM-GRAPH-DEF

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↑

function that formalizes the effect of the actions

example: if α is the assignment $x{:=}x{+}y$ then

$$\mathit{Effect}(\alpha, [x{=}1, y{=}7]) \;=\; [x{=}8, y{=}7]$$

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  specifies conditional transitions of the form $\ell \xhookrightarrow{g:\alpha} \ell'$

ℓ, ℓ' are locations, $g \in \mathit{Cond}(\mathit{Var})$, $\alpha \in \mathit{Act}$

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TS-semantics of a program graph

TS-PG-SEM

TS-semantics of a program graph

$$\boxed{\begin{array}{c}\text{program graph } \mathcal{P} \text{ over } \textbf{\textit{Var}} \\ \Downarrow \\ \text{transition system } \mathcal{T}_{\mathcal{P}}\end{array}}$$

TS-semantics of a program graph

program graph $\mathcal{P}$ over ***Var***

$$\Downarrow$$

transition system $\mathcal{T}_{\mathcal{P}}$

states in $\mathcal{T}_{\mathcal{P}}$ have the form

$$\langle \ell, \eta \rangle$$

location   variable evaluation

TS-semantics of a program graph

Let $\mathcal{P} = (\mathit{Loc}, \mathit{Act}, \mathit{Effect}, \hookrightarrow, \mathit{Loc}_0, g_0)$ be a PG.
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The transition relation $\longrightarrow$ is given by the following rule:

$$\frac{\ell \overset{g:\alpha}{\hookrightarrow} \ell' \;\wedge\; \eta \models g}{\langle \ell, \eta \rangle \xrightarrow{\alpha} \langle \ell', Effect(\alpha, \eta) \rangle}$$

Structured operational semantics (SOS)

The transition system of a program graph $\mathcal{P}$ is

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is a shortform notation in **SOS**-style.

$$\frac{\textit{premise}}{\textit{conclusion}}$$

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is a shortform notation in **SOS**-style.

It means that $\longrightarrow$ is the smallest relation such that:

$$\boxed{\text{if } \ell \xhookrightarrow{g:\alpha} \ell' \ \wedge \ \eta \models g \text{ then } \langle \ell, \eta \rangle \xrightarrow{\alpha} \langle \ell', Effect(\alpha, \eta) \rangle}$$

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$$L(\langle \ell, \eta \rangle) = \{\ell\} \cup \{g \in Cond(Var) : \eta \models g\}$$

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Guarded Command Language (GCL)

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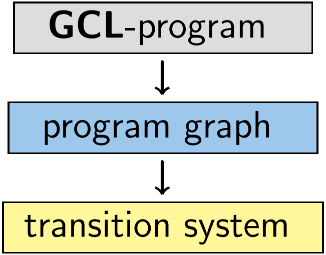
- high-level modeling language that contains features of imperative languages and nondeterministic choice
- semantics:



GCL-program

↓

program graph

↓

transition system

Guarded Command Language (GCL)

guarded command $g \Rightarrow \textit{stmt}$

g	:	guard, i.e., Boolean condition on the program variables
stmt	:	statement

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repetitive command/loop:

DO :: $g \Rightarrow$ ***stmt*** OD

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$\textit{stmt}$: statement

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DO :: $g \Rightarrow \textit{stmt}$ OD ← WHILE g DO $\textit{stmt}$ OD

Guarded Command Language (GCL)

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g : guard, i.e., Boolean condition on the program variables

stmt : statement

repetitive command/loop:

`DO` $:: g \Rightarrow \mathit{stmt}$ `OD` ← `WHILE` g `DO` stmt `OD`

conditional command:

`IF` $:: \; g \Rightarrow \mathit{stmt}_1$

$\quad :: \neg g \Rightarrow \mathit{stmt}_2$

`FI`

Guarded Command Language (GCL)

TS1.4-15

guarded command $g \Rightarrow \mathit{stmt}$ ← enabled if g is true

g	:	guard, i.e., Boolean condition on the program variables
stmt	:	statement

repetitive command/loop:

```
DO :: g ⇒ stmt OD        ←   WHILE g DO stmt OD
```

conditional command:

```
IF  ::  g ⇒ stmt₁              IF g THEN stmt₁
    :: ¬g ⇒ stmt₂        ←          ELSE stmt₂
FI                             FI
```

Guarded Command Language (GCL)

guarded command $g \Rightarrow \mathit{stmt}$ ← enabled if g is true

repetitive command/loop:

$\mathtt{DO} \;::\; g \Rightarrow \mathit{stmt}\; \mathtt{OD}$ ← $\mathtt{WHILE}\; g\; \mathtt{DO}\; \mathit{stmt}\; \mathtt{OD}$

conditional command:

$$\begin{array}{lll} \mathtt{IF} & :: & g \Rightarrow \mathit{stmt}_1 \\ & :: & \neg g \Rightarrow \mathit{stmt}_2 \\ \mathtt{FI} & & \end{array} \quad \longleftarrow \quad \begin{array}{llll} \mathtt{IF} & g & \mathtt{THEN} & \mathit{stmt}_1 \\ & & \mathtt{ELSE} & \mathit{stmt}_2 \\ \mathtt{FI} & & & \end{array}$$

symbol :: stands for the nondeterministic choice between enabled guarded commands

Guarded Command Language (GCL)

modeling language with nondeterministic choice

$$
\begin{array}{lll}
stmt & \overset{\text{def}}{=} & x := expr \quad \mid \quad stmt_1 ; stmt_2 \quad \mid \\
 & & \texttt{DO} :: g_1 \Rightarrow stmt_1 \quad \ldots \quad :: g_n \Rightarrow stmt_n \ \texttt{OD} \\
 & & \texttt{IF} :: g_1 \Rightarrow stmt_1 \quad \ldots \quad :: g_n \Rightarrow stmt_n \ \texttt{FI} \\
 & & \vdots
\end{array}
$$

where x is a typed variable and $expr$ an expression of the same type

Guarded Command Language (GCL)

modeling language with nondeterministic choice

$$
\begin{array}{lcl}
\mathit{stmt} & \stackrel{\text{def}}{=} & x := \mathit{expr} \quad \big| \quad \mathit{stmt}_1 ; \mathit{stmt}_2 \quad \big| \\
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where *x* is a typed variable and *expr* an expression of the same type

semantics of a **GCL**-program: program graph

GCL-program for beverage machine

GCL-program for beverage machine

uses two variables $\#\mathit{sprite}, \#\mathit{coke} \in \{0, 1, \ldots, \mathit{max}\}$
for the number of available drinks (sprite or coke)

GCL-program for beverage machine

pc2.1-GCL-getraenkeautomat

uses two variables $\#\mathit{sprite}, \#\mathit{coke} \in \{0, 1, \ldots, \mathit{max}\}$
for the number of available drinks (sprite or coke)

uses the following actions:

	enabled	effect
get_coke	if $\#\mathit{coke} > 0$	$\#\mathit{coke} := \#\mathit{coke} - 1$
get_sprite	if $\#\mathit{sprite} > 0$	$\#\mathit{sprite} := \#\mathit{sprite} - 1$

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refill	any time	$\#sprite := max$ $\#coke := max$

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refill	any time	$\#sprite := max$ $\#coke := max$
insert_coin	any time	no effect on variables

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get_sprite	if $\#sprite > 0$	$\#sprite := \#sprite - 1$
refill	any time	$\#sprite := max$ $\#coke := max$
insert_coin	any time	no effect on variables
return_coin	if machine is empty and user has entered a coin (no effect on variables)	

GCL-program for beverage machine

```
DO  ::  true $\Rightarrow$ insert_coin;
        IF  ::  $\#sprite = \#coke = 0 \Rightarrow$ return_coin

            ::  $\#coke > 0 \Rightarrow \#coke := \#coke - 1$

            ::  $\#sprite > 0 \Rightarrow \#sprite := \#sprite - 1$

        FI
    ::  true $\Rightarrow \#sprite := max; \#coke := max$

OD
```

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DO  ::  true ⇒ insert_coin; (* user inserts a coin *)
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        FI
   :: true $\Rightarrow \#sprite := max; \#coke := max$
OD
```

GCL-program for beverage machine

```
DO  ::  $\text{true} \Rightarrow \text{insert\_coin}$; (* user inserts a coin *)
        IF  ::  $\#\mathit{sprite} = \#\mathit{coke} = 0 \Rightarrow \text{return\_coin}$
                                (* no beverage available *)
            ::  $\#\mathit{coke} > 0 \Rightarrow \#\mathit{coke} := \#\mathit{coke} - 1$
                                (* user selects coke *)
            ::  $\#\mathit{sprite} > 0 \Rightarrow \#\mathit{sprite} := \#\mathit{sprite} - 1$
                                (* user selects sprite *)
        FI
    ::  $\text{true} \Rightarrow \#\mathit{sprite} := \mathit{max}; \#\mathit{coke} := \mathit{max}$
                                (* refilling of the machine *)
OD
```

GCL-program for beverage machine

```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
        IF :: $\#sprite = \#coke = 0$ ⇒ return_coin
                                  (* no beverage available *)
           :: $\#coke > 0$ ⇒ get_coke
                                  (* user selects coke *)
           :: $\#sprite > 0$ ⇒ get_sprite
                                  (* user selects sprite *)
        FI
   :: true ⇒ refill
                                  (* refilling of the machine *)
OD
```

GCL-program for beverage machine

```
DO :: true ⇒ insert_coin;
      IF :: $\#sprite = \#coke = 0$ ⇒ return_coin

         :: $\#coke > 0$ ⇒ get_coke

         :: $\#sprite > 0$ ⇒ get_sprite

      FI
   :: true ⇒ refill

OD
```

GCL-program for beverage machine

PC2.1-GCL-2

```
DO :: true ⇒ insert_coin;
        IF :: $\#sprite = \#coke = 0$
                                ⇒ return_coin
           :: $\#coke > 0$ ⇒ get_coke
           :: $\#sprite > 0$ ⇒ get_sprite
        FI
   :: true ⇒ refill
OD
```

... yields a program graph with

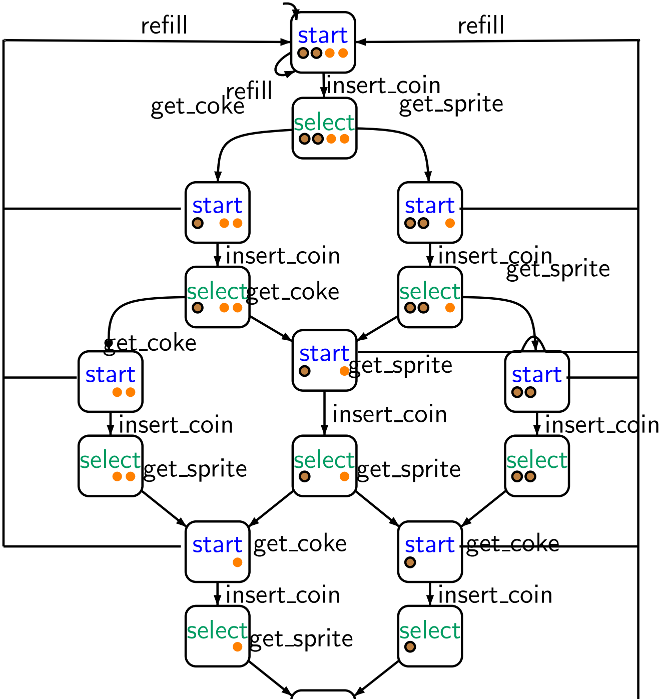
- two variables $\#sprite, \#coke \in \{0, 1, \ldots, max\}$

GCL-program for beverage machine

```
start  → DO :: true ⇒ insert_coin;
select →       IF :: #sprite = #coke = 0
                                 ⇒ return_coin
                  :: #coke  > 0  ⇒ get_coke
                  :: #sprite > 0 ⇒ get_sprite
               FI
            :: true ⇒ refill
         OD
```

... yields a program graph with

- two variables *#sprite*, *#coke* $\in \{0, 1, \ldots, \mathit{max}\}$
- two locations *start* and *select*


refill
start
refill
refill
get_coke
insert_coin
get_sprite
select
start
start
insert_coin
insert_coin
get_sprite
select
get_coke
select
get_coke
start
get_sprite
start
start
insert_coin
insert_coin
insert_coin
select
get_sprite
select
get_sprite
select
start
get_coke
start
get_coke
insert_coin
insert_coin
select
get_sprite
select