Prof. Dr. Ir. J.-P. Katoen
N. Jansen & B. Kaminiski

# Modeling and Verification of Probabilistic Systems
# Summer term 2014

## – Series 3 –

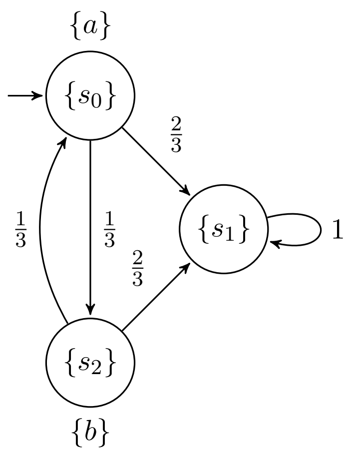
Hand in on May 15 before the exercise class.

### Exercise 1 (4 points)

Non-deterministic Büchi automata are strictly more expressive than deterministic ones. Recall the "powerset construction", which is used to compute a deterministic finite automaton from a non-deterministic one.

a) Construct a non-deterministic Büchi automaton for the language $(a + b)^*a^\omega$, apply the powerset construction to determinize this automaton, and compare the languages of the resulting automaton and the original one by either showing their equivalence or giving a counterexample separating the languages.

b) Consider the following DTMC $D$:




Give a formal definition for the cross-product between a *non-deterministic* Büchi-Automaton and a DTMC. Apply this definition to the DTMC $D$ and the NBA from a). What problems arise?

### Exercise 2 (3 points)

Prove or disprove the following $PCTL$ equivalences:

a) $\mathbb{P}_{=1}(\bigcirc\, \mathbb{P}_{=1}(\Box a)) \equiv \mathbb{P}_{=1}(\Box\, \mathbb{P}_{=1}(\bigcirc\, a))$

b) $\mathbb{P}_{>0.5}(\bigcirc\, \mathbb{P}_{>0.5}(\Diamond a)) \equiv \mathbb{P}_{>0.5}(\Diamond\, \mathbb{P}_{>0.5}(\bigcirc\, a))$

c) $\mathbb{P}_{=1}(\bigcirc\, \mathbb{P}_{=1}(\Diamond a)) \equiv \mathbb{P}_{=1}(\Diamond\, \mathbb{P}_{=1}(\bigcirc\, a))$

### Exercise 3 (3 points)

Consider a DTMC $D$, the PCTL property $\mathbb{P}_{\leq\lambda}(\Phi \mathsf{U} \Psi)$ and the following two sets: $S_{yes} = Sat(\mathbb{P}_{\geq 1}(\Phi \mathsf{U} \Psi))$ and $S_{no} = Sat(\mathbb{P}_{\leq 0}(\Phi \mathsf{U} \Psi))$. Give algorithms to compute these sets and sketch their complexity.