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Modeling and Verification of Probabilistic Systems
Summer term 2014

– Series 1 –

Hand in on April 24 before the exercise class.

Exercise 1 (4 points)

Recall the definition of a *geometric distribution* as given in the lecture:

Definition 1 *Let X be a discrete random variable, natural $k > 0$ and $0 < p \leq 1$. The mass function of a* geometric distribution *is given by:*

$$\Pr\{X = k\} = (1-p)^{k-1} \cdot p$$

Please give a formal proof of the following theorem concerning the memoryless property of geometric distributions:

Theorem 1 *For any random variable X with a geometric distribution over T it holds that*

$$\Pr\{X = k + m \mid X > m\} \quad = \quad \Pr\{X = k\} \quad \textit{for any} \quad m \in T, k \geq 1 \ .$$

Hint: Use properties of probability measures and the geometric distribution as presented in the lecture.

Exercise 2 (3 points)

Given a DTMC D, a state s is called *transient*, if when starting in s there is a non-zero probability that s is not visited again. A state is called *recurrent*, if it is not transient.

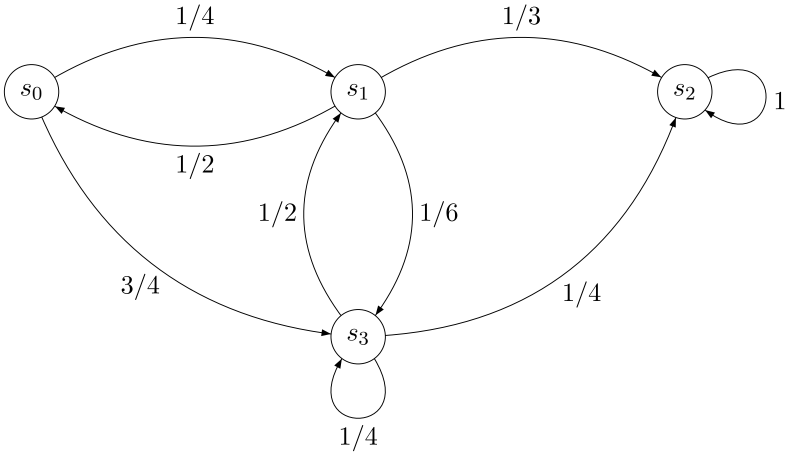
a) Give a formal definition of transient and recurrent states of DTMCs.

b) Give an informal algorithm for computing the set of recurrent states of a DTMC. What is the complexity?

c) Give an example DTMC containing both transient and recurrent states and compute the limiting probabilities.

Exercise 3 **(3 points)**

Consider the following DTMC:



a) Compute the probability of going from s_0 to s_3 in *exactly* 3 steps;
(*Hints: by the end of the* 3rd *step the system is in state* 3.)

b) Compute the probability of going from s_0 to s_3 in *at most* 3 steps;
(*Hints: by the end of the* 3rd *step the system has been in state* 3.)

c) Compute the probability of being in state s_2 after 3 steps when the initial distribution is uniform over all states.