

# Compiler Construction

## Lecture 9: Syntax Analysis V ( $LR(k)$ Grammars)

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Summer Semester 2014

- 1 Recap: Nondeterministic Bottom-Up Parsing
- 2 Resolving Termination Nondeterminism
- 3  $LR(k)$  Grammars
- 4  $LR(0)$  Grammars
- 5 Examples of  $LR(0)$  Conflicts
- 6  $LR(0)$  Parsing

## Approach:

- 1 Given  $G \in CFG_{\Sigma}$ , construct a **nondeterministic bottom-up parsing automaton** (NBA) which accepts  $L(G)$  and which additionally computes corresponding (reversed) rightmost analyses
  - input alphabet:  $\Sigma$
  - pushdown alphabet:  $X$
  - output alphabet:  $[p]$  (where  $p := |P|$ )
  - state set: omitted
  - transitions:
    - shift: shifting input symbols onto the pushdown
    - reduce: replacing the right-hand side of a production by its left-hand side (= inverse expansion steps)
- 2 Remove nondeterminism by allowing **lookahead** on the input:  
 $G \in LR(k)$  iff  $L(G)$  recognizable by deterministic bottom-up parsing automaton with lookahead of  $k$  symbols

# Nondeterministic Bottom-Up Automaton I

## Definition (Nondeterministic bottom-up parsing automaton)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ . The **nondeterministic bottom-up parsing automaton** of  $G$ ,  $NBA(G)$ , is defined by the following components.

- **Input alphabet:**  $\Sigma$
- **Pushdown alphabet:**  $X$
- **Output alphabet:**  $[p]$
- **Configurations:**  $\Sigma^* \times X^* \times [p]^*$  (top of pushdown to the right)
- **Transitions** for  $w \in \Sigma^*$ ,  $\alpha \in X^*$ , and  $z \in [p]^*$ :  
shifting steps:  $(aw, \alpha, z) \vdash (w, \alpha a, z)$  if  $a \in \Sigma$   
reduction steps:  $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$  if  $\pi_i = A \rightarrow \beta$
- **Initial configuration** for  $w \in \Sigma^*$ :  $(w, \varepsilon, \varepsilon)$
- **Final configurations:**  $\{\varepsilon\} \times \{S\} \times [p]^*$

# Nondeterminism in $NBA(G)$

**Observation:**  $NBA(G)$  is generally **nondeterministic**

- **Shift or reduce?** Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \text{ if } \pi_i = A \rightarrow a$$

- If reduce: **which "handle"  $\beta$ ?** Example:

$$(w, \alpha ab, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha aB, zj) \end{cases} \text{ if } \pi_i = A \rightarrow ab \text{ and } \pi_j = B \rightarrow b$$

- If reduce  $\beta$ : **which left-hand side  $A$ ?** Example:

$$(w, \alpha a, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha B, zj) \end{cases} \text{ if } \pi_i = A \rightarrow a \text{ and } \pi_j = B \rightarrow a$$

- **When to terminate parsing?** Example:

$$\underbrace{(\varepsilon, S, z)}_{\text{final}} \vdash (\varepsilon, A, zi) \text{ if } \pi_i = A \rightarrow S$$

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**General assumption** to avoid nondeterminism of last type:  
every grammar is start separated

## Definition 9.1 (Start separation)

A grammar  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  is called **start separated** if  $S$  only occurs in productions of the form  $S \rightarrow A$  where  $A \neq S$ .

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### Remarks:

- Start separation always possible by adding  $S' \rightarrow S$  with **new start symbol**  $S'$
- From now on consider only **reduced** grammars of this form  
( $\pi_0 := S' \rightarrow S$ )



# Resolving Termination Nondeterminism II

Start separation removes last form of nondeterminism (“When to terminate parsing?”):

## Lemma 9.2

*If  $G \in CFG_{\Sigma}$  is start separated, then no successor of a final configuration  $(\varepsilon, S', z)$  in  $NBA(G)$  is again a final configuration.  
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## Proof.

- To  $(\varepsilon, S', z)$ , only reductions by  $\varepsilon$ -productions can be applied:

$$(\varepsilon, S', z) \vdash (\varepsilon, S'A, zi) \quad \text{if } \pi_i = A \rightarrow \varepsilon$$

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- Every resulting configuration is of the (non-final) form

$$(\varepsilon, S'B_1 \dots B_k, z) \quad \text{where } k \geq 1$$



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**Goal:** resolve remaining nondeterminism of  $NBA(G)$  by supporting lookahead of  $k \in \mathbb{N}$  symbols on the input

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## Definition 9.3 ( $LR(k)$ grammar)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  be start separated and  $k \in \mathbb{N}$ . Then  $G$  has the  **$LR(k)$  property** (notation:  $G \in LR(k)$ ) if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha \beta y \end{cases}$$

such that  $\text{first}_k(w) = \text{first}_k(y)$ , it follows that  $\alpha = \gamma$ ,  $A = B$ , and  $x = y$ .

## Remarks:

- If  $G \in LR(k)$ , then the reduction of  $\alpha\beta w$  to  $\alpha Aw$  is already determined by  $\text{first}_k(w)$ .
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- **Computation of  $\text{NBA}(G)$**  for  $S \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta w$ :

$$(w'w, \varepsilon, \varepsilon) \vdash^* (w, \alpha\beta, z) \stackrel{\text{red } i}{\vdash} (w, \alpha A, zi) \vdash \dots$$

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- **Computation of  $\text{NBA}(G)$  for  $S \Rightarrow_r^* \gamma Bx \Rightarrow_r \alpha\beta y$ :**
  - with direct reduction ( $y = x, \alpha\beta = \gamma\delta, \pi_j = B \rightarrow \delta$ ):

$$(y'y, \varepsilon, \varepsilon) \vdash^* (y, \alpha\beta, z') \stackrel{\text{red } j}{\vdash} (x, \gamma B, z'j) \vdash \dots$$

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- with previous shifts ( $y = x'x, \alpha\beta x' = \gamma\delta, \pi_j = B \rightarrow \delta$ ):

$$\begin{aligned} (y'y, \varepsilon, \varepsilon) &\vdash^* (y, \alpha\beta, z') = (x'x, \alpha\beta, z') \\ &\stackrel{\text{shift}^*}{\vdash} (x, \alpha\beta x', z') = (x, \gamma\delta, z') \\ &\stackrel{\text{red } j}{\vdash} (x, \gamma B, z'j) \vdash \dots \end{aligned}$$

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## Corollary 9.4 (LR(0) grammar)

$G \in CFG_\Sigma$  has the **LR(0) property** if for all rightmost derivations of the form

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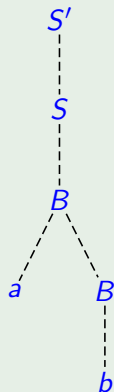
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**Goal:** derive a **finite information** from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by fo-sets)

## Example 9.5

$G$  :  $S' \rightarrow S$  (0)  
 $S \rightarrow B \mid C$  (1,2)  
 $B \rightarrow aB \mid b$  (3,4)  
 $C \rightarrow aC \mid c$  (5,6)

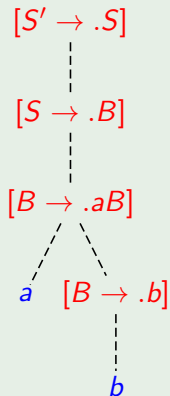




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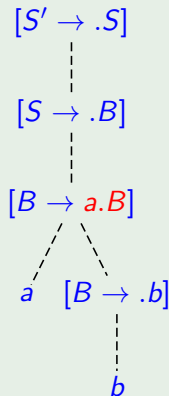
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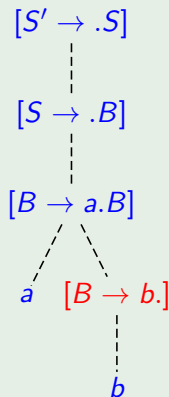
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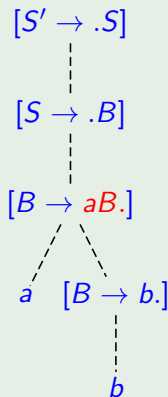
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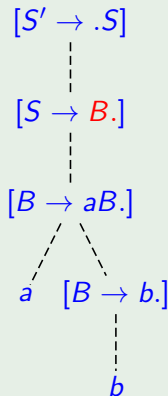


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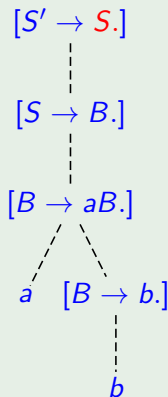


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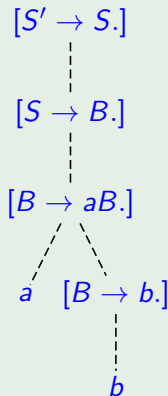


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## Definition 9.6 (LR(0) items and sets)

Let  $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$  be start separated by  $S' \rightarrow S$  and  $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$  (i.e.,  $A \rightarrow \beta_1 \beta_2 \in P$ ).

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- 4 The item  $[A \rightarrow \beta_1 \cdot Y \beta_2] \in LR(0)(\gamma)$  indicates an incomplete handle  $\beta_1$  (to be completed by shift operations or  $\epsilon$ -reductions).

## Definition 9.8 (LR(0) conflicts)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  and  $I \in LR(0)(G)$ .

- $I$  has a **shift/reduce conflict** if there exist  $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$  such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

## Definition 9.8 (LR(0) conflicts)

Let  $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$  and  $I \in \text{LR}(0)(G)$ .

- $I$  has a **shift/reduce conflict** if there exist  $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$  such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

- $I$  has a **reduce/reduce conflict** if there exist  $A \rightarrow \alpha, B \rightarrow \beta \in P$  with  $A \neq B$  or  $\alpha \neq \beta$  such that

$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$



## Definition 9.8 (LR(0) conflicts)

Let  $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$  and  $I \in \text{LR}(0)(G)$ .

- $I$  has a **shift/reduce conflict** if there exist  $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$  such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

- $I$  has a **reduce/reduce conflict** if there exist  $A \rightarrow \alpha, B \rightarrow \beta \in P$  with  $A \neq B$  or  $\alpha \neq \beta$  such that

$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

## Lemma 9.9

$G \in \text{LR}(0)$  iff no  $I \in \text{LR}(0)(G)$  contains conflicting items.

## Proof.

omitted □

## Theorem 9.10 (Computing $LR(0)$ sets)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  be start separated by  $S' \rightarrow S$  and reduced.

- ①  $LR(0)(\varepsilon)$  is the least set such that
- $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$  and
  - if  $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$  and  $B \rightarrow \beta \in P$ ,  
then  $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$ .

## Theorem 9.10 (Computing $LR(0)$ sets)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  be start separated by  $S' \rightarrow S$  and reduced.

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  - if  $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$  and  $B \rightarrow \beta \in P$ , then  $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$ .
- ②  $LR(0)(\alpha Y)$  ( $\alpha \in X^*$ ,  $Y \in X$ ) is the least set such that
  - if  $[A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$ , then  $[A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$  and
  - if  $[A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y)$  and  $B \rightarrow \beta \in P$ , then  $[B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$ .

## Example 9.11 (cf. Example 9.5)

$G$  :  $S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S]$

## Example 9.11 (cf. Example 9.5)

$G$  :  $S' \rightarrow S$   
 $S \rightarrow B \mid C$        $[A \rightarrow \cdot B\gamma] \in LR(0)(\epsilon), B \rightarrow \beta \in P$   
 $B \rightarrow aB \mid b$        $\implies [B \rightarrow \cdot \beta] \in LR(0)(\epsilon)$   
 $C \rightarrow aC \mid c$

$I_0 := LR(0)(\epsilon) : \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B]$

## Example 9.11 (cf. Example 9.5)

$G$  :  $S' \rightarrow S$   
 $S \rightarrow B \mid C$        $[A \rightarrow \cdot B\gamma] \in LR(0)(\epsilon), B \rightarrow \beta \in P$   
 $B \rightarrow aB \mid b$        $\implies [B \rightarrow \cdot \beta] \in LR(0)(\epsilon)$   
 $C \rightarrow aC \mid c$

$I_0 := LR(0)(\epsilon) :$      $[S' \rightarrow \cdot S]$      $[S \rightarrow \cdot B]$      $[S \rightarrow \cdot C]$

## Example 9.11 (cf. Example 9.5)

$G$  :  $S' \rightarrow S$   
 $S \rightarrow B \mid C$        $[A \rightarrow \cdot B\gamma] \in LR(0)(\epsilon), B \rightarrow \beta \in P$   
 $B \rightarrow aB \mid b$        $\implies [B \rightarrow \cdot \beta] \in LR(0)(\epsilon)$   
 $C \rightarrow aC \mid c$

$I_0 := LR(0)(\epsilon)$  :       $[S' \rightarrow \cdot S]$        $[S \rightarrow \cdot B]$        $[S \rightarrow \cdot C]$        $[B \rightarrow \cdot aB]$   
                                  $[B \rightarrow \cdot b]$



## Example 9.11 (cf. Example 9.5)

$G$  :  $S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$[A \rightarrow \cdot B\gamma] \in LR(0)(\epsilon), B \rightarrow \beta \in P$   
 $\implies [B \rightarrow \cdot \beta] \in LR(0)(\epsilon)$

$I_0 := LR(0)(\epsilon) :$

$[S' \rightarrow \cdot S]$	$[S \rightarrow \cdot B]$	$[S \rightarrow \cdot C]$	$[B \rightarrow \cdot aB]$
$[B \rightarrow \cdot b]$	$[C \rightarrow \cdot aC]$	$[C \rightarrow \cdot c]$	

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$$\begin{aligned} I_0 := LR(0)(\varepsilon) : & \quad [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\ I_1 := LR(0)(S) : & [S' \rightarrow S \cdot] & & & \end{aligned}$$

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$$\begin{aligned} [A \rightarrow \gamma_1 \cdot Y \gamma_2] &\in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] &\in LR(0)(\alpha Y) \end{aligned}$$

$$\begin{aligned} I_0 := LR(0)(\varepsilon) : & \quad [S' \rightarrow \cdot S] & \quad [S \rightarrow \cdot B] & \quad [S \rightarrow \cdot C] & \quad [B \rightarrow \cdot aB] \\ & \quad [B \rightarrow \cdot b] & \quad [C \rightarrow \cdot aC] & \quad [C \rightarrow \cdot c] & \\ I_1 := LR(0)(S) : & \quad [S' \rightarrow S \cdot] & & & \\ I_2 := LR(0)(B) : & \quad [S \rightarrow B \cdot] & & & \end{aligned}$$

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$   
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon) : \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB]$   
 $\quad \quad \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$   
 $I_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$   
 $I_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$   
 $I_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
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$[A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$   
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$I_0 := LR(0)(\varepsilon) : \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB]$   
 $\quad \quad \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$I_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$

$I_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$

$I_4 := LR(0)(a) : \quad [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C]$

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$[A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$   
 $\implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon) :$   $[S' \rightarrow \cdot S]$   $[S \rightarrow \cdot B]$   $[S \rightarrow \cdot C]$   $[B \rightarrow \cdot aB]$   
 $[B \rightarrow \cdot b]$   $[C \rightarrow \cdot aC]$   $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S) :$   $[S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) :$   $[S \rightarrow B \cdot]$

$I_3 := LR(0)(C) :$   $[S \rightarrow C \cdot]$

$I_4 := LR(0)(a) :$   $[B \rightarrow a \cdot B]$   $[C \rightarrow a \cdot C]$   $[B \rightarrow \cdot aB]$   $[B \rightarrow \cdot b]$

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$[A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$   
 $\implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon) :$ 

$[S' \rightarrow \cdot S]$	$[S \rightarrow \cdot B]$	$[S \rightarrow \cdot C]$	$[B \rightarrow \cdot aB]$
$[B \rightarrow \cdot b]$	$[C \rightarrow \cdot aC]$	$[C \rightarrow \cdot c]$	

$I_1 := LR(0)(S) :$ 

$[S' \rightarrow S \cdot]$			
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$I_2 := LR(0)(B) :$ 

$[S \rightarrow B \cdot]$			
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$I_3 := LR(0)(C) :$ 

$[S \rightarrow C \cdot]$			
---------------------------	--	--	--

$I_4 := LR(0)(a) :$ 

$[B \rightarrow a \cdot B]$	$[C \rightarrow a \cdot C]$	$[B \rightarrow \cdot aB]$	$[B \rightarrow \cdot b]$
$[C \rightarrow \cdot aC]$	$[C \rightarrow \cdot c]$		

# Computing $LR(0)$ Sets II

## Example 9.11 (cf. Example 9.5)

$$\begin{aligned} G : \quad & S' \rightarrow S \\ & S \rightarrow B \mid C \\ & B \rightarrow aB \mid b \\ & C \rightarrow aC \mid c \end{aligned}$$
$$\begin{aligned} & [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ \implies & [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \end{aligned}$$
$$\begin{aligned} I_0 := LR(0)(\varepsilon) : & \quad [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\ I_1 := LR(0)(S) : & [S' \rightarrow S \cdot] & & & \\ I_2 := LR(0)(B) : & [S \rightarrow B \cdot] & & & \\ I_3 := LR(0)(C) : & [S \rightarrow C \cdot] & & & \\ I_4 := LR(0)(a) : & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\ & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \\ I_5 := LR(0)(b) : & [B \rightarrow b \cdot] & & & \end{aligned}$$



## Example 9.11 (cf. Example 9.5)

$$\begin{aligned} G : \quad & S' \rightarrow S \\ & S \rightarrow B \mid C \\ & B \rightarrow aB \mid b \\ & C \rightarrow aC \mid c \end{aligned}$$
$$\begin{aligned} & [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ \implies & [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \end{aligned}$$
$$\begin{aligned} I_0 := LR(0)(\varepsilon) : & \quad [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ & [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \\ I_1 := LR(0)(S) : & [S' \rightarrow S \cdot] & & & \\ I_2 := LR(0)(B) : & [S \rightarrow B \cdot] & & & \\ I_3 := LR(0)(C) : & [S \rightarrow C \cdot] & & & \\ I_4 := LR(0)(a) : & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\ & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \\ I_5 := LR(0)(b) : & [B \rightarrow b \cdot] & & & \\ I_6 := LR(0)(c) : & [C \rightarrow c \cdot] & & & \end{aligned}$$

## Example 9.11 (cf. Example 9.5)

$$G : \begin{array}{l} S' \rightarrow S \\ S \rightarrow B \mid C \\ B \rightarrow aB \mid b \\ C \rightarrow aC \mid c \end{array}$$

$$\begin{array}{l} [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \end{array}$$

$$l_0 := LR(0)(\varepsilon) : \begin{array}{llll} [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \end{array}$$

$$l_1 := LR(0)(S) : [S' \rightarrow S \cdot]$$

$$l_2 := LR(0)(B) : [S \rightarrow B \cdot]$$

$$l_3 := LR(0)(C) : [S \rightarrow C \cdot]$$

$$l_4 := LR(0)(a) : \begin{array}{llll} [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & & \end{array}$$

$$l_5 := LR(0)(b) : [B \rightarrow b \cdot]$$

$$l_6 := LR(0)(c) : [C \rightarrow c \cdot]$$

$$l_7 := LR(0)(aB) : [B \rightarrow aB \cdot]$$

# Computing $LR(0)$ Sets II

## Example 9.11 (cf. Example 9.5)

$G : S' \rightarrow S$   
 $S \rightarrow B \mid C$   
 $B \rightarrow aB \mid b$   
 $C \rightarrow aC \mid c$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$   
 $\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\epsilon) :$   $[S' \rightarrow \cdot S]$      $[S \rightarrow \cdot B]$      $[S \rightarrow \cdot C]$      $[B \rightarrow \cdot aB]$   
 $[B \rightarrow \cdot b]$      $[C \rightarrow \cdot aC]$      $[C \rightarrow \cdot c]$

$I_1 := LR(0)(S) :$   $[S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) :$   $[S \rightarrow B \cdot]$

$I_3 := LR(0)(C) :$   $[S \rightarrow C \cdot]$

$I_4 := LR(0)(a) :$   $[B \rightarrow a \cdot B]$      $[C \rightarrow a \cdot C]$      $[B \rightarrow \cdot aB]$      $[B \rightarrow \cdot b]$   
 $[C \rightarrow \cdot aC]$      $[C \rightarrow \cdot c]$

$I_5 := LR(0)(b) :$   $[B \rightarrow b \cdot]$

$I_6 := LR(0)(c) :$   $[C \rightarrow c \cdot]$

$I_7 := LR(0)(aB) :$   $[B \rightarrow aB \cdot]$

$I_8 := LR(0)(aC) :$   $[C \rightarrow aC \cdot]$

## Example 9.11 (cf. Example 9.5)

$$\begin{aligned} G : \quad & S' \rightarrow S \\ & S \rightarrow B \mid C \\ & B \rightarrow aB \mid b \\ & C \rightarrow aC \mid c \end{aligned}$$
$$I_0 := LR(0)(\varepsilon) : \quad \begin{array}{llll} [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] & [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] & \end{array}$$
$$I_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$$
$$I_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$$
$$I_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$$
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$$I_5 := LR(0)(b) : \quad [B \rightarrow b \cdot]$$
$$I_6 := LR(0)(c) : \quad [C \rightarrow c \cdot]$$
$$I_7 := LR(0)(aB) : \quad [B \rightarrow aB \cdot]$$
$$I_8 := LR(0)(aC) : \quad [C \rightarrow aC \cdot]$$

$(LR(0)(aa) = LR(0)(a) = I_4, LR(0)(ab) = LR(0)(b) = I_5,$   
 $LR(0)(ac) = LR(0)(c) = I_6, \dots, I_9 := LR(0)(\gamma) = \emptyset$  in all remaining cases)

## Example 9.11 (cf. Example 9.5)

$$\begin{aligned}G : \quad & S' \rightarrow S \\ & S \rightarrow B \mid C \\ & B \rightarrow aB \mid b \\ & C \rightarrow aC \mid c\end{aligned}$$

$$l_0 := LR(0)(\varepsilon) : \quad \begin{array}{l} [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array}$$

$$l_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$$

$$l_2 := LR(0)(B) : \quad [S \rightarrow B \cdot]$$

$$l_3 := LR(0)(C) : \quad [S \rightarrow C \cdot]$$

$$l_4 := LR(0)(a) : \quad \begin{array}{l} [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array}$$

$$l_5 := LR(0)(b) : \quad [B \rightarrow b \cdot]$$

$$l_6 := LR(0)(c) : \quad [C \rightarrow c \cdot]$$

$$l_7 := LR(0)(aB) : \quad [B \rightarrow aB \cdot]$$

$$l_8 := LR(0)(aC) : \quad [C \rightarrow aC \cdot]$$

$(LR(0)(aa) = LR(0)(a) = l_4, LR(0)(ab) = LR(0)(b) = l_5,$   
 $LR(0)(ac) = LR(0)(c) = l_6, \dots, l_9 := LR(0)(\gamma) = \emptyset$  in all remaining cases)

no conflicts  $\implies G \in LR(0)$  (but  $G \notin LL(1)$ )

- 1 Recap: Nondeterministic Bottom-Up Parsing
- 2 Resolving Termination Nondeterminism
- 3  $LR(k)$  Grammars
- 4  $LR(0)$  Grammars
- 5 Examples of  $LR(0)$  Conflicts
- 6  $LR(0)$  Parsing

## Example 9.12

$G : S' \rightarrow S$   
 $S \rightarrow Aa \mid Bb$   
 $A \rightarrow a$   
 $B \rightarrow a$

# Reduce/Reduce Conflicts

## Example 9.12

$G : S' \rightarrow S$   
 $S \rightarrow Aa \mid Bb$   
 $A \rightarrow a$   
 $B \rightarrow a$

$LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$   
 $LR(0)(S) : [S' \rightarrow S \cdot]$   
 $LR(0)(A) : [S \rightarrow A \cdot a]$   
 $LR(0)(B) : [S \rightarrow B \cdot a]$   
 $LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$   
 $LR(0)(Aa) : [S \rightarrow Aa \cdot]$   
 $LR(0)(Ba) : [S \rightarrow Ba \cdot]$



# Reduce/Reduce Conflicts

## Example 9.12

$G : S' \rightarrow S$   
 $S \rightarrow Aa \mid Bb$   
 $A \rightarrow a$   
 $B \rightarrow a$

$LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$   
 $LR(0)(S) : [S' \rightarrow S \cdot]$   
 $LR(0)(A) : [S \rightarrow A \cdot a]$   
 $LR(0)(B) : [S \rightarrow B \cdot a]$   
 $LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$   
 $LR(0)(Aa) : [S \rightarrow Aa \cdot]$   
 $LR(0)(Ba) : [S \rightarrow Ba \cdot]$

**Note:**  $G$  is unambiguous

## Example 9.13

$$G : \begin{array}{l} S' \rightarrow S \\ S \rightarrow aS \mid a \end{array}$$

## Example 9.13

$G : S' \rightarrow S$   
 $S \rightarrow aS \mid a$

$LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(a) : [S \rightarrow a \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a] \quad [S \rightarrow a \cdot]$

$LR(0)(aS) : [S \rightarrow aS \cdot]$

## Example 9.13

$G : S' \rightarrow S$   
 $S \rightarrow aS \mid a$

$LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(a) : [S \rightarrow a \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a] \quad [S \rightarrow a \cdot]$

$LR(0)(aS) : [S \rightarrow aS \cdot]$

**Note:**  $G$  is unambiguous

- 1 Recap: Nondeterministic Bottom-Up Parsing
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# The goto Function I

**Observation:** if  $G \in LR(0)$ , then  $LR(0)(\gamma)$  yields **deterministic shift/reduce decision** for  $NBA(G)$  in a configuration with pushdown  $\gamma$   
 $\implies$  **new pushdown alphabet:**  $LR(0)(G)$  in place of  $X$

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 $\implies$  **new pushdown alphabet:**  $LR(0)(G)$  in place of  $X$

Moreover  $LR(0)(\gamma Y)$  is determined by  $LR(0)(\gamma)$  and  $Y$  but **independent from**  $\gamma$  in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

# The goto Function I

**Observation:** if  $G \in LR(0)$ , then  $LR(0)(\gamma)$  yields **deterministic shift/reduce decision** for  $NBA(G)$  in a configuration with pushdown  $\gamma$   
 $\implies$  **new pushdown alphabet:**  $LR(0)(G)$  in place of  $X$

Moreover  $LR(0)(\gamma Y)$  is determined by  $LR(0)(\gamma)$  and  $Y$  but **independent from**  $\gamma$  in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

## Definition 9.14 ( $LR(0)$ goto function)

The function **goto** :  $LR(0)(G) \times X \rightarrow LR(0)(G)$  is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).$$



## Example 9.15 (cf. Example 9.11)

$$\begin{aligned}l_0 &:= LR(0)(\varepsilon) : & [S' \rightarrow \cdot S] \\ & & [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \\ & & [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \\ & & [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\l_1 &:= LR(0)(S) : & [S' \rightarrow S \cdot] \\l_2 &:= LR(0)(B) : & [S \rightarrow B \cdot] \\l_3 &:= LR(0)(C) : & [S \rightarrow C \cdot] \\l_4 &:= LR(0)(a) : & [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \\ & & [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \\ & & [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\l_5 &:= LR(0)(b) : & [B \rightarrow b \cdot] \\l_6 &:= LR(0)(c) : & [C \rightarrow c \cdot] \\l_7 &:= LR(0)(aB) : & [B \rightarrow aB \cdot] \\l_8 &:= LR(0)(aC) : & [C \rightarrow aC \cdot] \\l_9 &:= \emptyset\end{aligned}$$

# The goto Function II

## Example 9.15 (cf. Example 9.11)

$l_0 := LR(0)(\epsilon) :$   $[S' \rightarrow \cdot S]$   
 $[S \rightarrow \cdot B]$   $[S \rightarrow \cdot C]$   
 $[B \rightarrow \cdot aB]$   $[B \rightarrow \cdot b]$   
 $[C \rightarrow \cdot aC]$   $[C \rightarrow \cdot c]$

$l_1 := LR(0)(S) :$   $[S' \rightarrow S \cdot]$

$l_2 := LR(0)(B) :$   $[S \rightarrow B \cdot]$

$l_3 := LR(0)(C) :$   $[S \rightarrow C \cdot]$

$l_4 := LR(0)(a) :$   $[B \rightarrow a \cdot B]$   $[C \rightarrow a \cdot C]$   
 $[B \rightarrow \cdot aB]$   $[B \rightarrow \cdot b]$   
 $[C \rightarrow \cdot aC]$   $[C \rightarrow \cdot c]$

$l_5 := LR(0)(b) :$   $[B \rightarrow b \cdot]$

$l_6 := LR(0)(c) :$   $[C \rightarrow c \cdot]$

$l_7 := LR(0)(aB) :$   $[B \rightarrow aB \cdot]$

$l_8 := LR(0)(aC) :$   $[C \rightarrow aC \cdot]$

$l_9 := \emptyset$

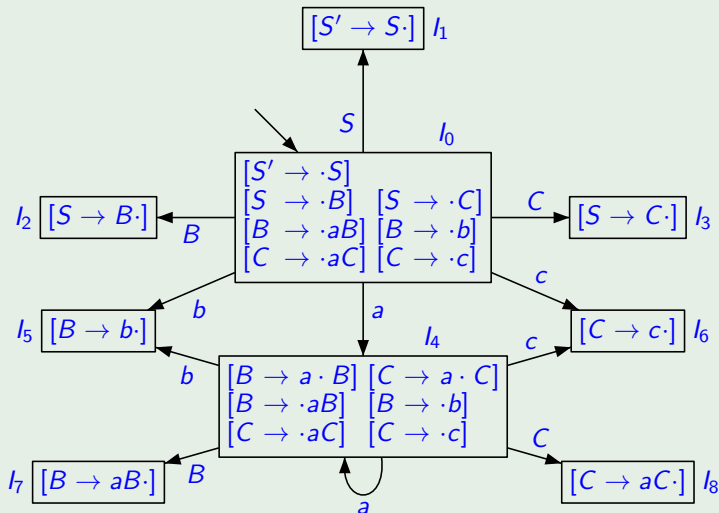
goto	S	B	C	a	b	c
$l_0$	$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$
$l_1$						
$l_2$						
$l_3$						
$l_4$		$l_7$	$l_8$	$l_4$	$l_5$	$l_6$
$l_5$						
$l_6$						
$l_7$						
$l_8$						
$l_9$						

(empty =  $l_9$ )

# The goto Function III

## Example 9.15 (continued)

Representation of `goto` function as finite automaton:



# The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision.

(Reminder:  $\pi_0 = S' \rightarrow S$ )

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## Definition 9.16 ( $LR(0)$ action function)

The  $LR(0)$  action function

$$\text{act} : LR(0)(G) \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot a\alpha_2] \in I \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

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## Corollary 9.17

For every  $G \in CFG_\Sigma$ ,  $G \in LR(0)$  iff  $\text{act}$  is well defined.

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