Model checking

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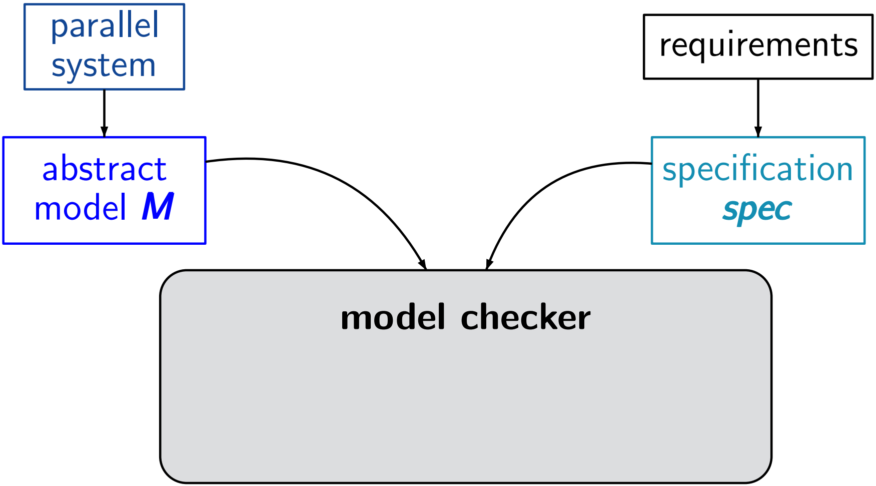

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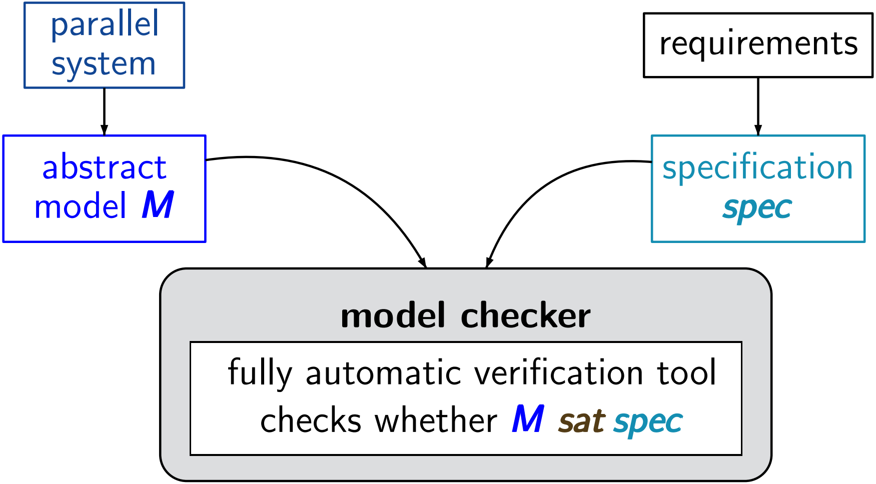

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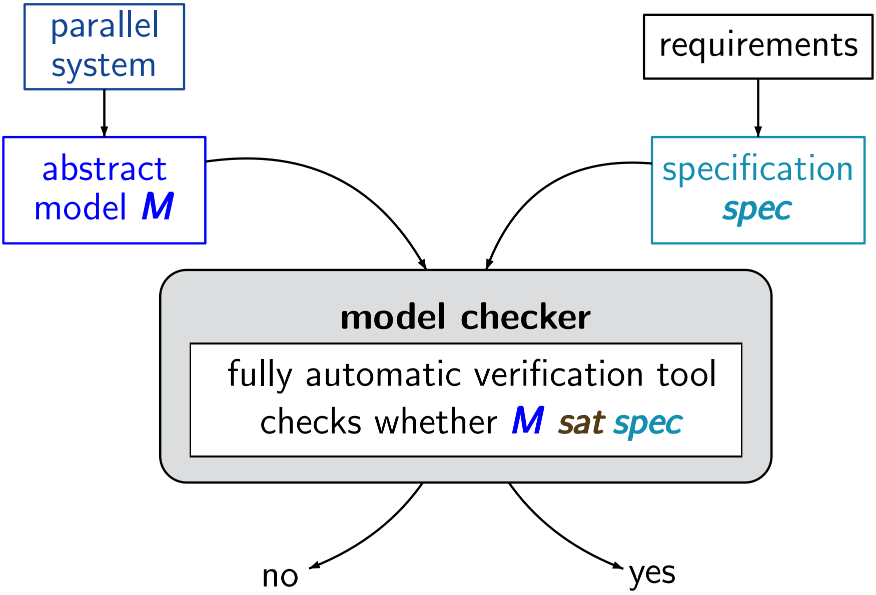

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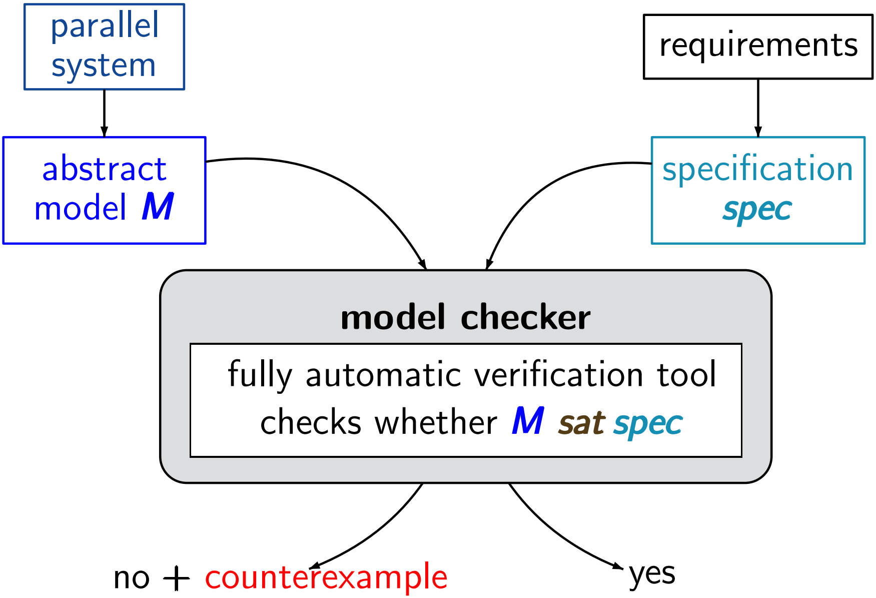

Decidability of the model checking problem?

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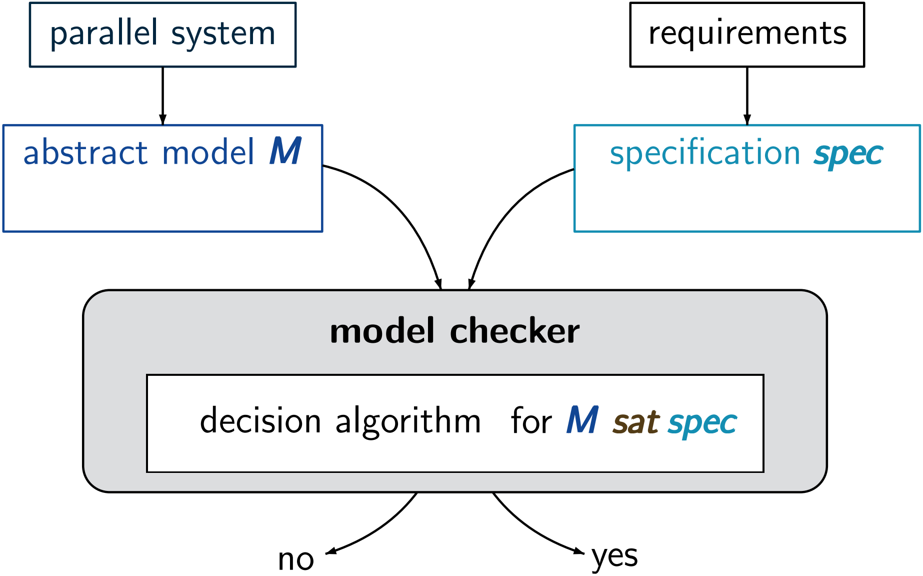

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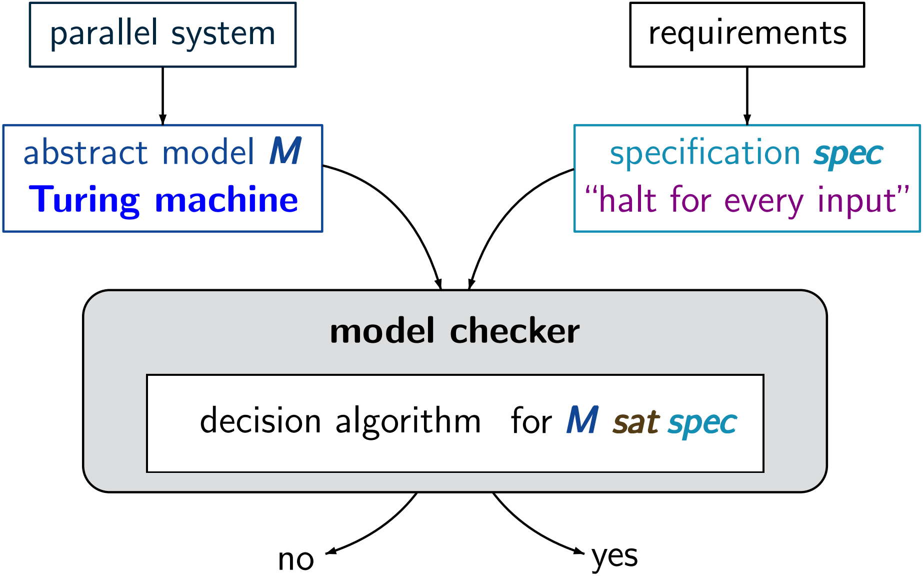
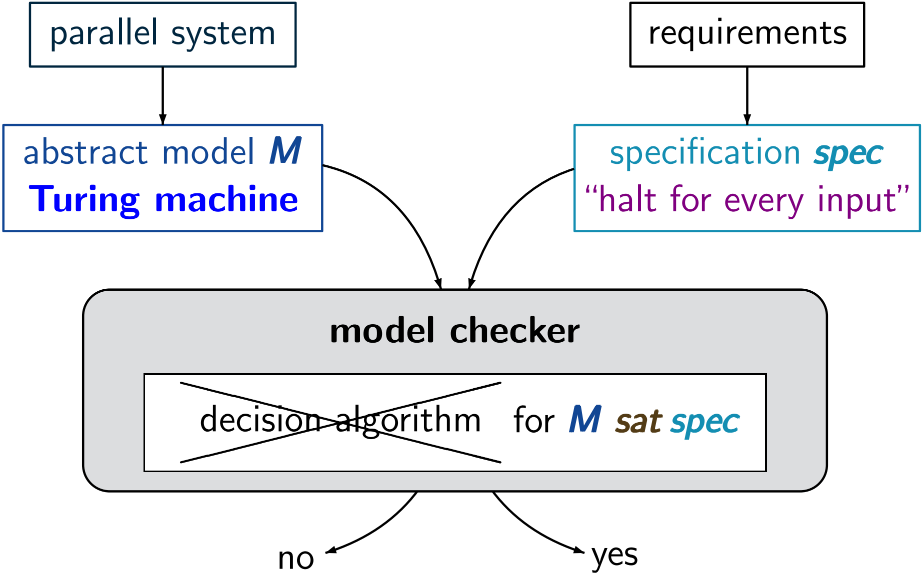


General model checking problem is undecidable



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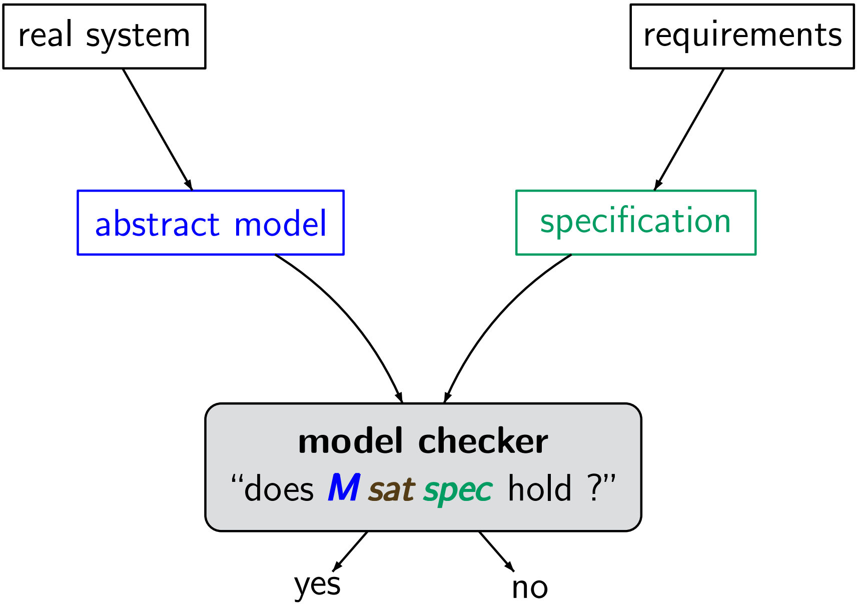

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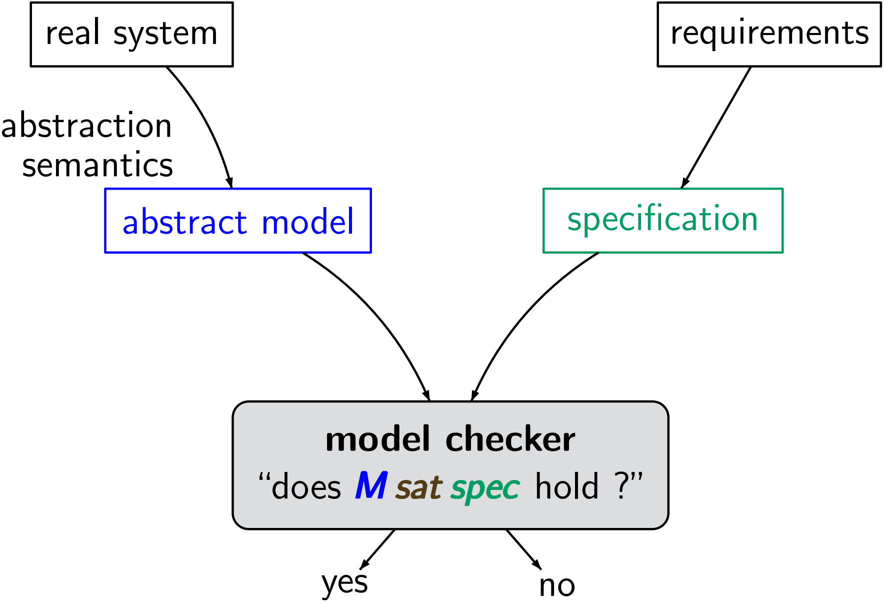

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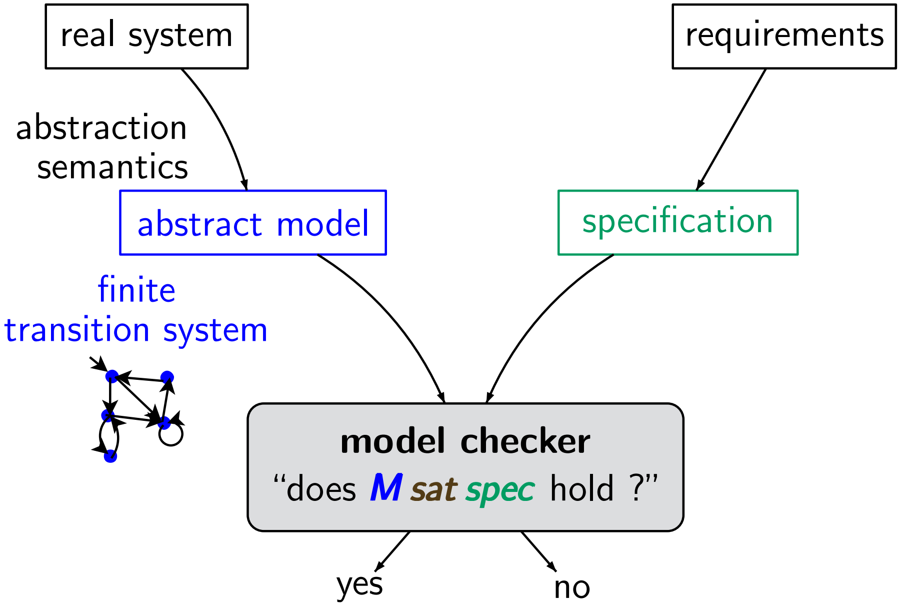

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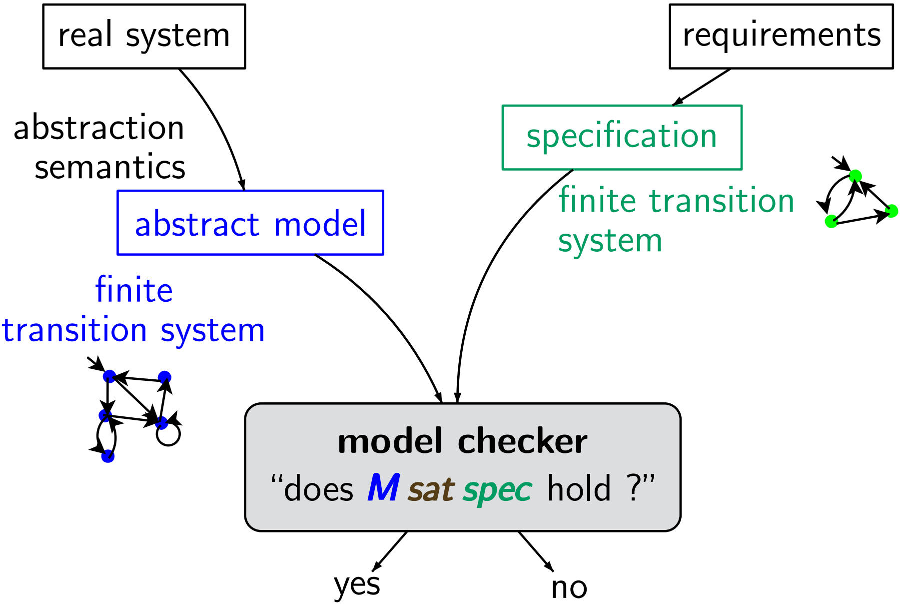

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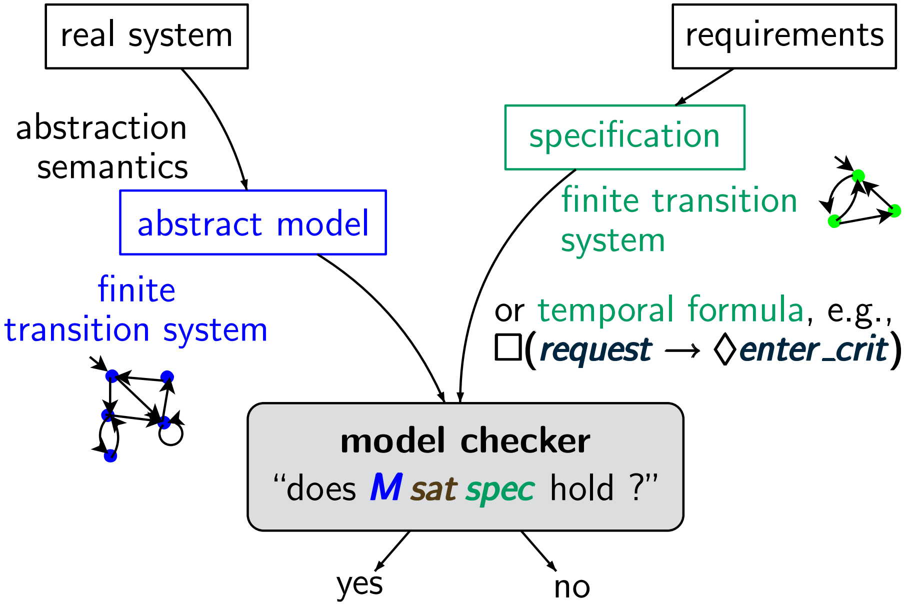

Model checking vs other validation techniques

The validation techniques (testing, simulation, deductive verification, model checking) are complementary to each other.

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model checking

- most efficient validation technique, fully automatic
- but mostly only applicable for finite models with "small" (or "sufficiently structured") state space
- industrial applications:
  - hardware systems
  - communication protocols
  - coordination protocols for distributed systems
  - ⋮

Historical notes

1976	Keller	transition systems (TS) to model parallel systems
1977	Pnueli	temporal logic to specify parallel systems
1981	Clarke/Emerson Queille/Sifakis	first model checker

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1983	Kanellakis/Smolka	model checking for homogeneous TS-based specifications
1985	Lichtenstein/Pnueli	model checking for **LTL**
1986	Vardi/Wolper	model checking for **LTL**

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⋮	⋮	⋮
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state explosion problem

state space of industrial systems too large to be handled by naïve implementations of model checking algorithms

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ca. since 1990
"advanced techniques"

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symbolic model checking with BDDs
partial order reduction
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model checking for infinite systems, quantitative analysis, e.g., real-time systems, probabilistic systems

Transition system (TS)

A transition system is a tuple

$$\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$$

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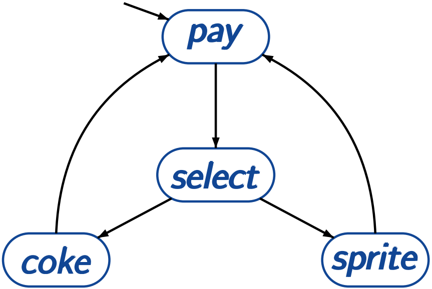
- $S_0 \subseteq S$ the set of initial states,
- AP a set of atomic propositions,
- $L : S \rightarrow 2^{AP}$ the labeling function

Transition system for beverage machine



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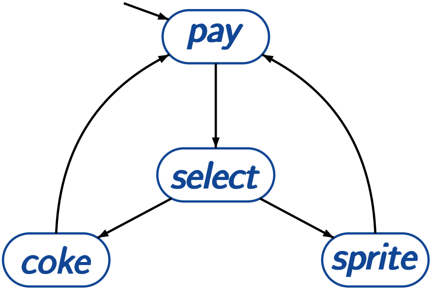

state space $S = \{pay, select, coke, sprite\}$

set of initial states: $S_0 = \{pay\}$

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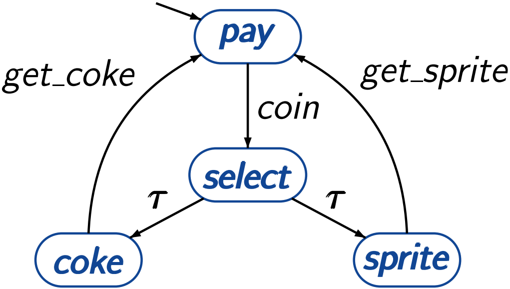

actions:

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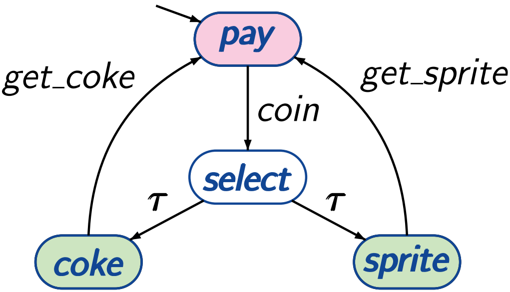

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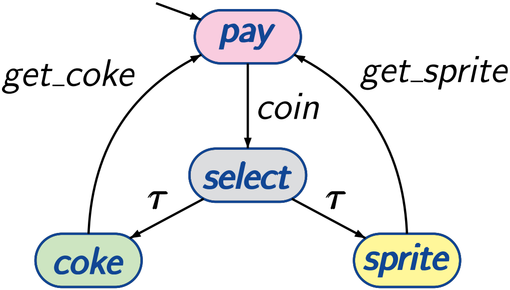
set of atomic propositions: $AP = \{pay, drink\}$

labeling function: $L(coke) = L(sprite) = \{drink\}$

$L(pay) = \{pay\}, \; L(select) = \emptyset$

Transition system for beverage machine



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set of atomic propositions: $AP = S$

labeling function: $L(s) = \{s\}$ for each state s

"Behaviour" of transition systems

possible behaviours of a TS result from:

```
select nondeterministically an initial state s ∈ S0
WHILE  s is non-terminal DO
      select nondeterministically a transition s --α--> s'
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executions: maximal "transition sequences"

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reachable fragment:

$Reach(\mathcal{T})$ = set of all states that are reachable from an initial state through some execution

Linear-time vs branching-time

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transition system

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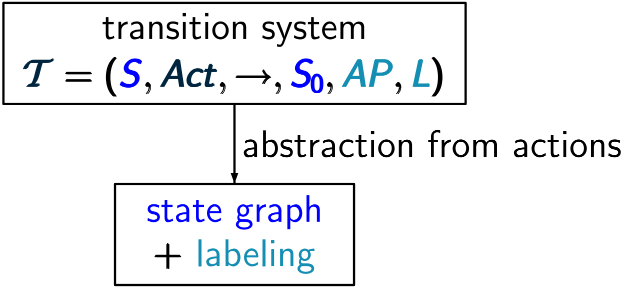

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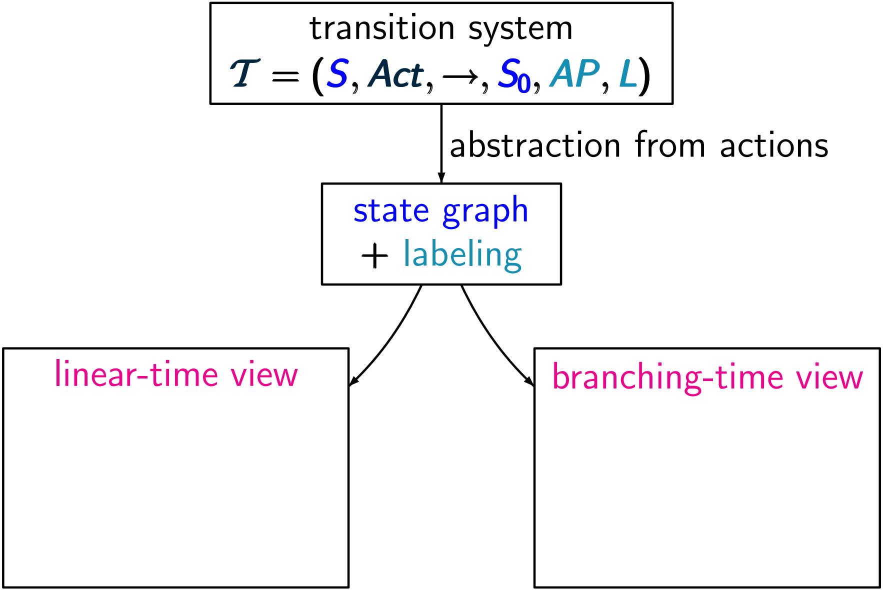

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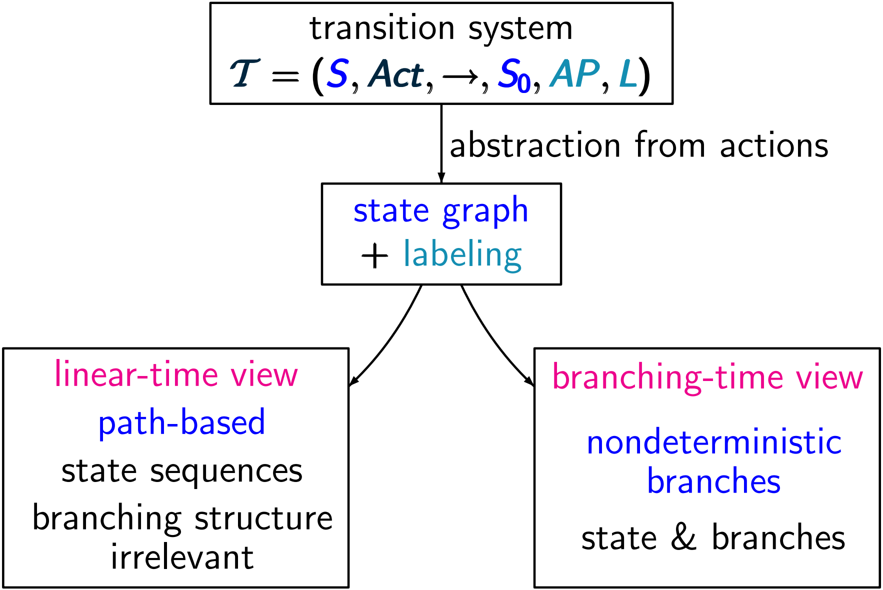

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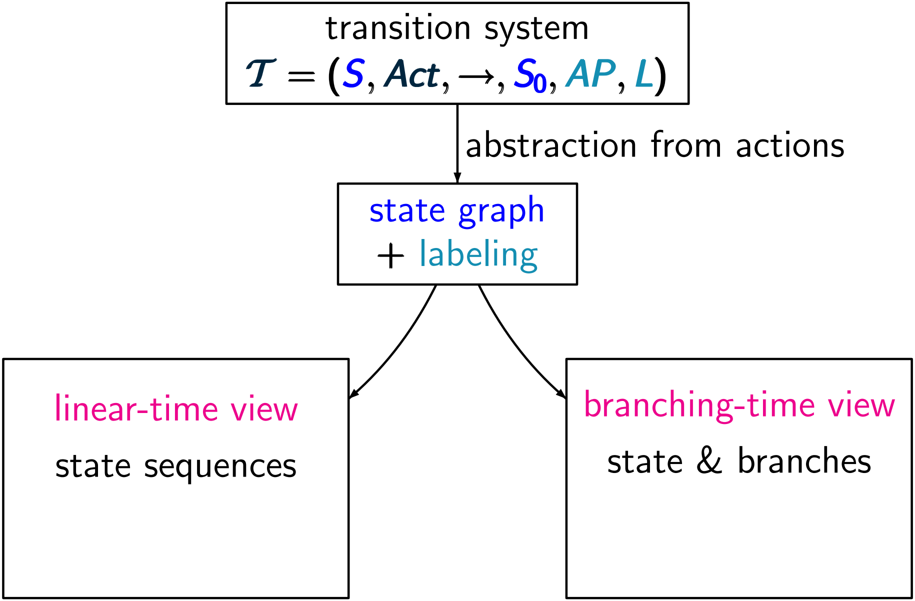

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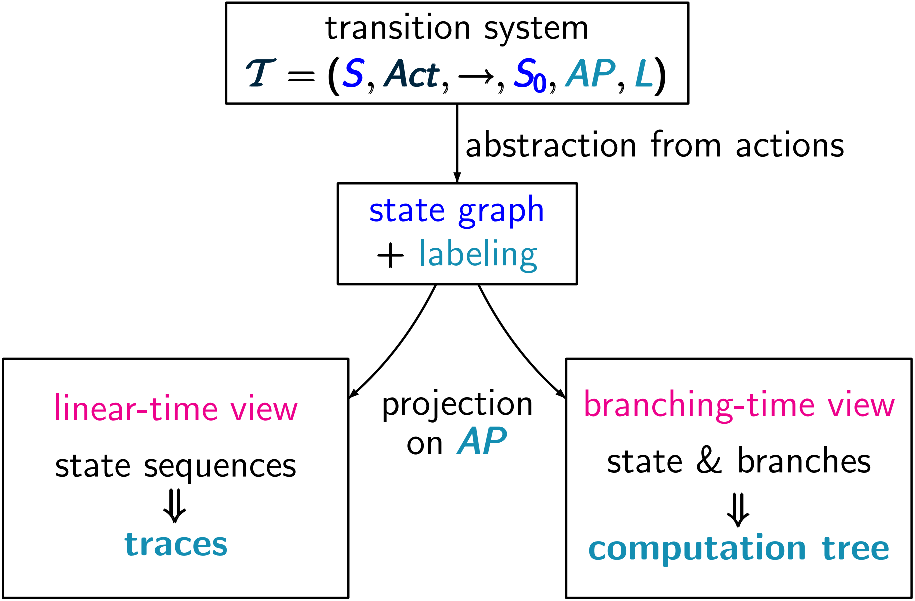

Traces

LTB2.4-4

for TS with labeling function $L : S \to 2^{AP}$

execution: states + actions

$s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ infinite or finite

↓

paths: sequences of states

$s_0\, s_1\, s_2 \ldots$ infinite or $s_0\, s_1 \ldots s_n$ finite

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Traces of a transition system

Let $\mathcal{T}$ be a TS

$$Traces(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\pi) : \pi \in Paths(\mathcal{T})\}$$

$$Traces_{fin}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\widehat{\pi}) : \widehat{\pi} \in Paths_{fin}(\mathcal{T})\}$$

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↑ initial, maximal path fragment

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Let $\mathcal{T}$ be a TS ⟵ *without* terminal states

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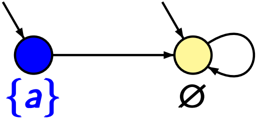

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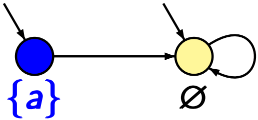

TS $\mathcal{T}$ with a single atomic proposition a

$$Traces(\mathcal{T}) = \{\, \{a\}\varnothing^{\omega}, \varnothing^{\omega} \,\}$$

$$Traces_{fin}(\mathcal{T}) = \{\{a\}\varnothing^{n} : n \geq 0\} \cup \{\varnothing^{m} : m \geq 1\}$$

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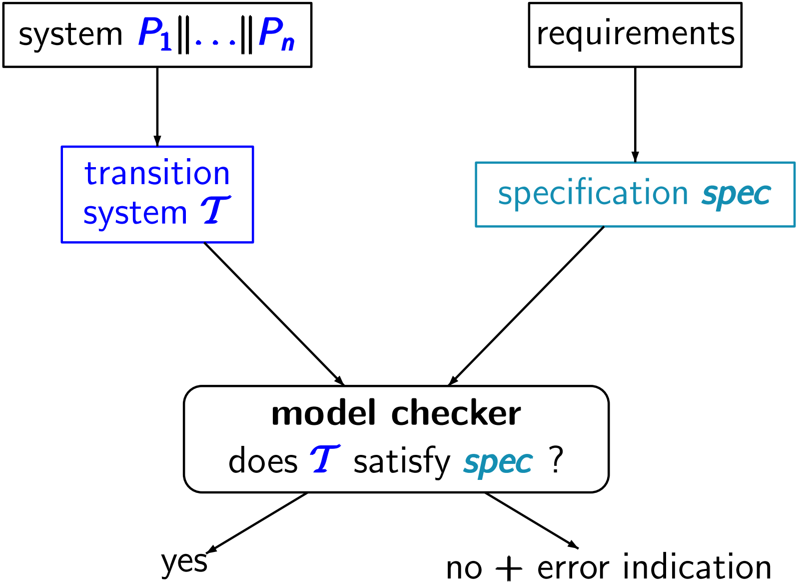

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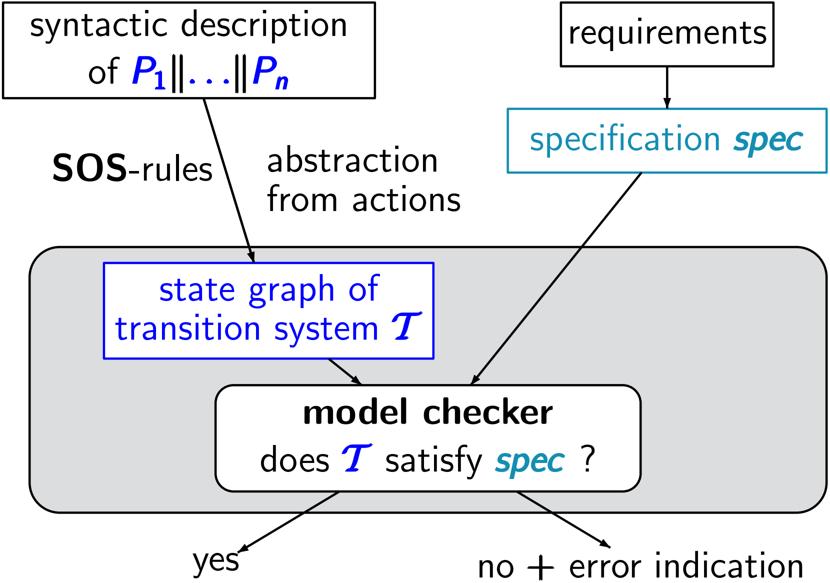

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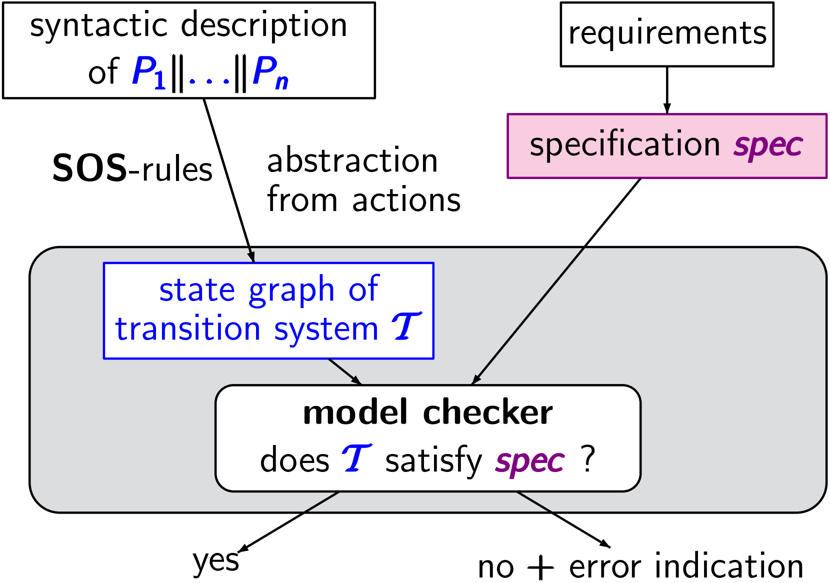

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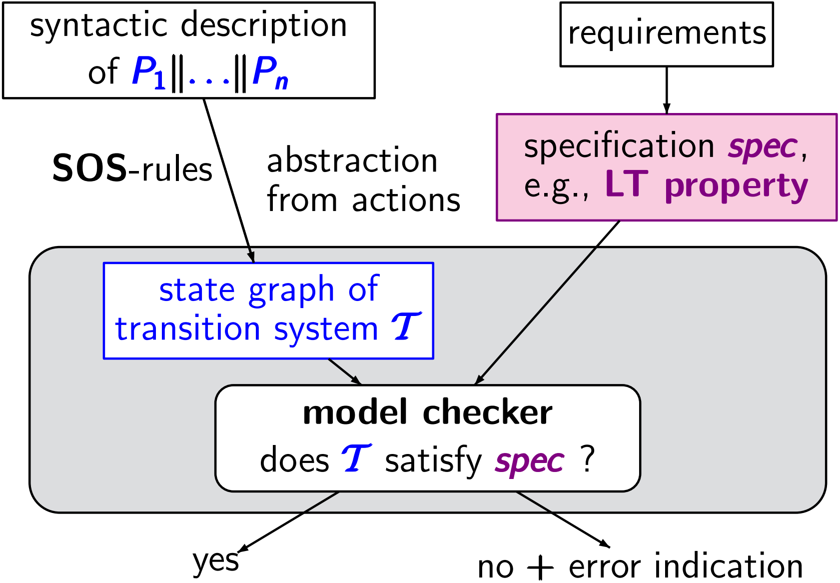

Linear-time properties (LT properties)

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for TS over AP without terminal states

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E.g., for mutual exclusion problems and
$AP = \{crit_1, crit_2, \ldots\}$

safety:

$MUTEX =$ set of all infinite words $A_0\, A_1\, A_2 \ldots$ over 2^{AP} such that for all $i \in \mathbb{N}$:
$crit_1 \notin A_i$ or $crit_2 \notin A_i$

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If s is a state in $\mathcal{T}$ then

$$s \models E \quad \text{iff} \quad Traces(s) \subseteq E$$

LT properties and trace inclusion

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Consequence of these definitions:

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then for all LT properties E over AP:

$$Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2) \wedge \mathcal{T}_2 \models E \implies \mathcal{T}_1 \models E$$

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note: $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2) \subseteq E$

LT properties and trace inclusion

An LT property over AP is a language E of infinite words over the alphabet $\Sigma = 2^{AP}$, i.e., $E \subseteq \left(2^{AP}\right)^{\omega}$.

If $\mathcal{T}$ is a TS over AP then $\mathcal{T} \models E$ iff $Traces(\mathcal{T}) \subseteq E$.

If $\mathcal{T}_1$ and $\mathcal{T}_2$ are TS over AP then the following statements are equivalent:

(1) $Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$

(2) for all LT-properties E over AP: whenever $\mathcal{T}_2 \models E$ then $\mathcal{T}_1 \models E$

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$(1) \Longrightarrow (2)$: $\checkmark$

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(2) $\Longrightarrow$ (1): consider $E = Traces(\mathcal{T}_2)$

Trace equivalence

LTB2.4-21A

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Trace equivalent TS satisfy the **same LT properties**

LT properties and trace relations

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Linear Temporal Logic (LTL)

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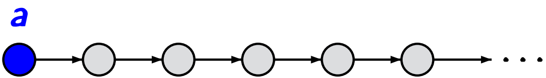

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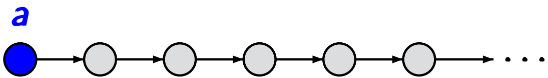
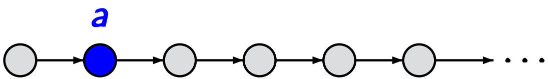


next operator $\bigcirc a$



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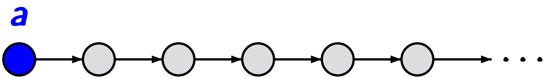

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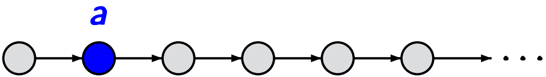
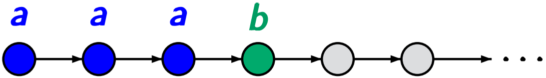


until operator $a \, \mathsf{U} \, b$



Derived operators in LTL

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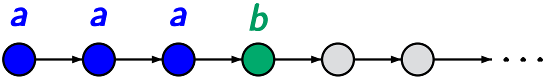
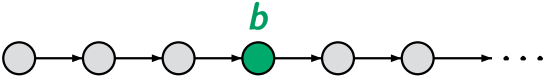


eventually
$\Diamond b$



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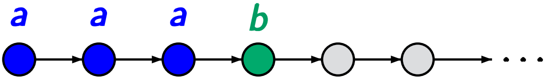

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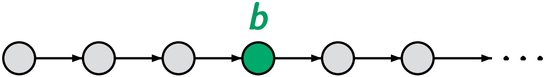

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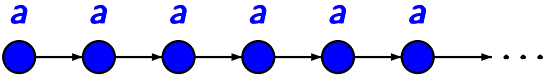
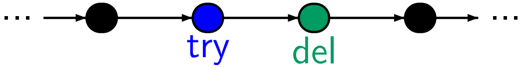
$\Box a$



Next $\bigcirc$, until U and eventually $\Diamond$

$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\, \text{delivered})$



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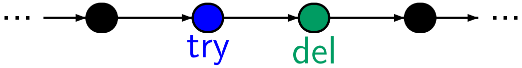

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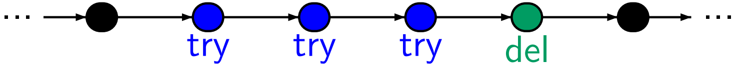

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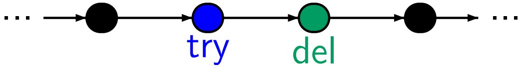

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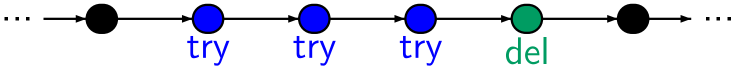

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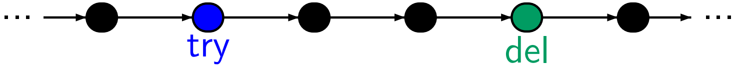

Examples for LTL formulas

$$\varphi \;::=\; \textit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc \varphi \;\big|\; \varphi_1 \,\mathsf{U}\, \varphi_2$$

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traffic light: $\Box(yellow \vee \bigcirc\neg red)$

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Infinitely often and eventually forever

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LTL-semantics

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interpretation of **LTL** formulas over traces, i.e.,
infinite words over 2^{AP}

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formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$

Semantics of LTL over infinite words

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$$
\begin{array}{lll}
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$\sigma \models \varphi_1 \, \mathsf{U} \, \varphi_2$ iff there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j \, A_{j+1} \, A_{j+2} \, \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i \, A_{i+1} \, A_{i+2} \, \ldots \models \varphi_1$ for $0 \leq i < j$

LTL semantics over TS

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given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

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assumption: $\mathcal{T}$ has no terminal states, i.e.,
all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

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LTL formula φ over AP

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interpretation of φ over infinite path fragments

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remind: LT property of an LTL formula:

$$Words(\varphi) = \left\{\sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi\right\}$$

LTL semantics over the states of a TS

LTLSF3.1-SEM-STATES

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↑

satisfaction relation for LT properties

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LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$\begin{aligned} s \models \varphi \quad & \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s) \\ & \text{iff} \quad s \models Words(\varphi) \\ & \text{iff} \quad Traces(s) \subseteq Words(\varphi) \end{aligned}$$

Interpretation of LTL formulas over TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
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$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

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& \text{iff} \quad Traces(\mathcal{T}) \subseteq Words(\varphi) \\
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$$

↑ satisfaction relation for LT properties

Linear-time implementation relations

Linear-time implementation relations

finite trace inclusion and equivalence:

e.g., $\mathit{Tracesfin}(\mathcal{T}_1) \subseteq \mathit{Tracesfin}(\mathcal{T}_2)$

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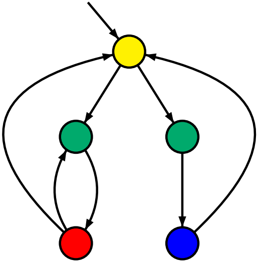
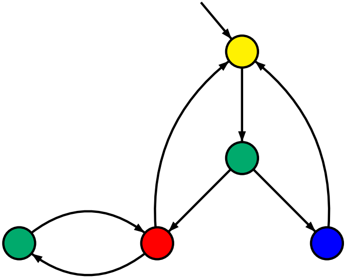
* **minimization ???**

Minimization w.r.t. trace equivalence?

$\mathcal{T}_1$:



$\mathcal{T}_2$:



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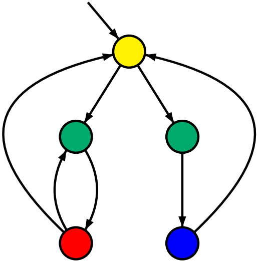

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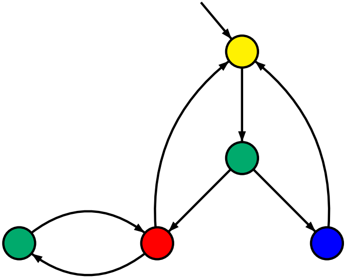

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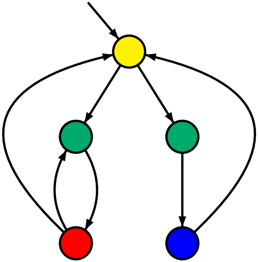

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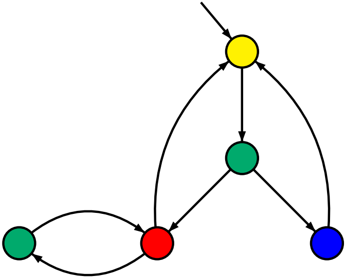

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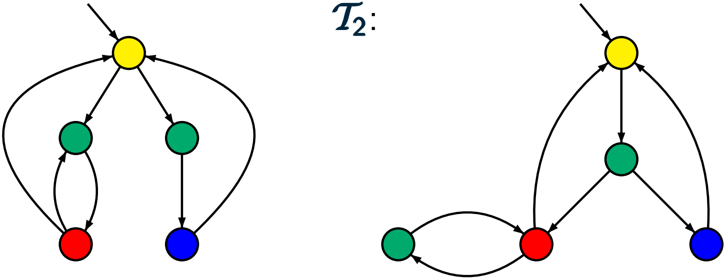

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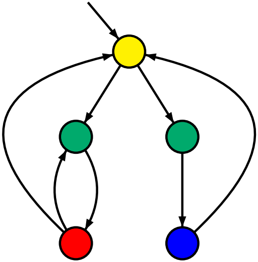

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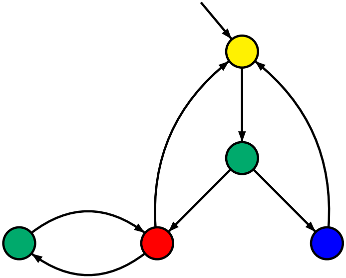

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- $\mathcal{T}_1$, $\mathcal{T}_2$ have **5** states and **7** transitions each
- there is no smaller TS that is trace-equivalent to $\mathcal{T}_i$

Classification of implementation relations BSEQOR5.1-6

Classification of implementation relations

- linear vs. branching time
  - linear time: trace relations
  - branching time: (bi)simulation relations

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  - linear time: trace relations
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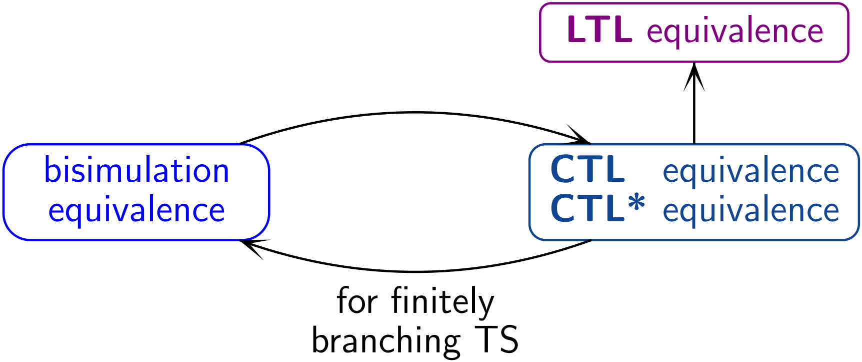
- (nonsymmetric) preorders vs. equivalences:
  - preorders: trace inclusion, simulation
  - equivalences: trace equivalence, bisimulation
- strong vs. weak relations
  - strong: reasoning about all transitions
  - weak: abstraction from stutter steps

Summary: equivalences



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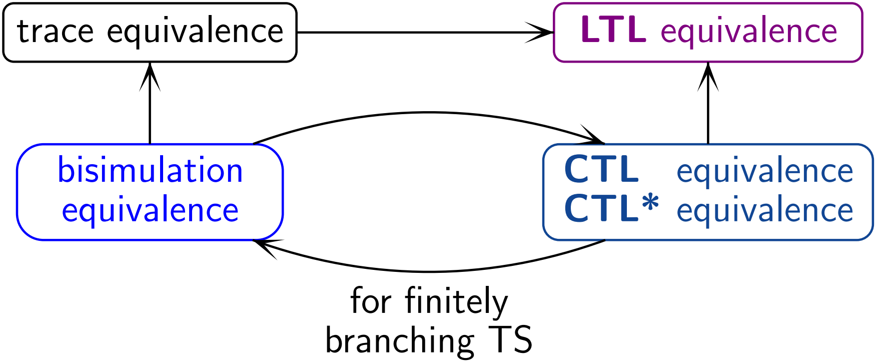

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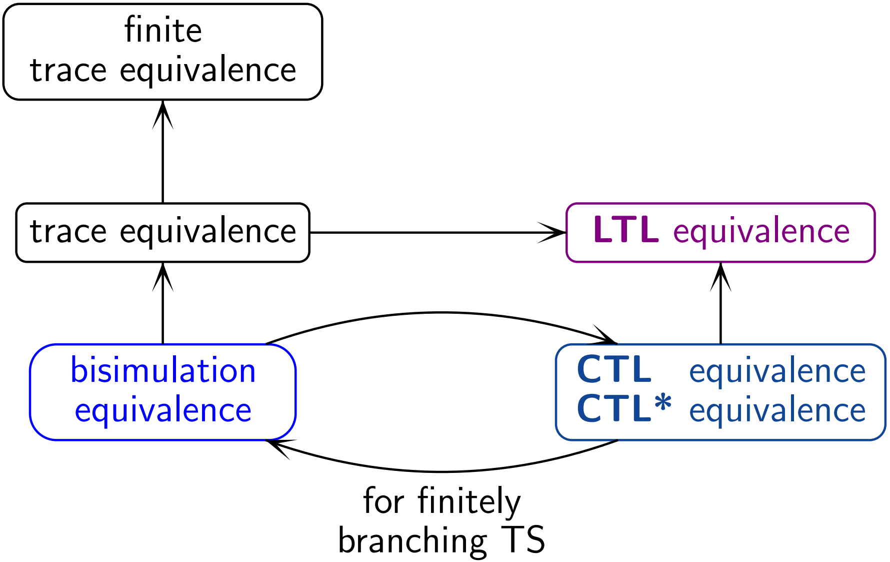

Summary: equivalences