4 conditions for ample sets LTL3.4-A4

(A1) $\emptyset \neq \text{ample}(s) \subseteq Act(s)$

(A2) for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from $\text{ample}(s)$ there is some $i < n$ with $\beta_i \in \text{ample}(s)$

(A3) if $\text{ample}(s) \neq Act(s)$ then all actions in $\text{ample}(s)$ are *stutter actions*

(A4) for each *cycle* $s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_n$ in $\mathcal{T}_{red}$ and each action

$$\beta \in \bigcup_{0 \leq i < n} Act(s_i)$$

there is some $i \in \{1, \ldots, n\}$ with $\beta \in \text{ample}(s_i)$

4 conditions for ample sets

(A1) $\emptyset \neq \mathsf{ample}(\mathsf{s}) \subseteq \mathit{Act}(\mathsf{s})$

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Soundness of conditions (A1), (A2), (A3), (A4)

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If the ample sets ample(s) satisfy conditions (A1), (A2), (A3), (A4) then

$$\mathcal{T} \triangleq \mathcal{T}_{\mathsf{red}}$$

remind: $\triangleq$ stutter trace equivalence

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hence: for all $\text{LTL}_{\setminus\bigcirc}$ formulas φ:

$$\mathcal{T} \models \varphi \text{ iff } \mathcal{T}_{\text{red}} \models \varphi$$

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Proof: show that

$$\mathcal{T} \trianglelefteq \mathcal{T}_{\text{red}} \quad \text{and} \quad \mathcal{T}_{\text{red}} \trianglelefteq \mathcal{T}$$

where $\trianglelefteq$ = stutter trace inclusion

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Proof:

- $\mathcal{T}_{\text{red}} \trianglelefteq \mathcal{T}$: $\surd$

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$$\mathcal{T} \triangleq \mathcal{T}_{\text{red}}$$

Proof:

- $\mathcal{T}_{\text{red}} \trianglelefteq \mathcal{T}$: $\surd$
- $\mathcal{T} \trianglelefteq \mathcal{T}_{\text{red}}$:
  show that each execution ρ of $\mathcal{T}$ can be transformed into a stutter equivalent execution ρ' of $\mathcal{T}_{\text{red}}$

Proof of $\mathcal{T} \trianglelefteq \mathcal{T}_{\textsf{red}}$

given: infinite execution fragment ρ of $\mathcal{T}$

goal: construction of a stutter equivalent execution fragment ρ' of $\mathcal{T}_{\textsf{red}}$

Proof of $\mathcal{T} \trianglelefteq \mathcal{T}_{\mathsf{red}}$

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idea: ρ' results from the "limit" of transformations

$$\rho = \rho_0 \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow$$

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Stepwise transformation $\rho_0 \rightsquigarrow \rho_1$

LTL3.4-35A

Stepwise transformation $\rho_0 \rightsquigarrow \rho_1$

case 0: $\rho_0 = \quad s_0 \xrightarrow{\alpha} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\alpha \in \text{ample}(s_0)$

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by (A3): α is a stutter action in cases 1 and 2

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case 2: $\rho_0 = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$

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$\rho_1 \rightsquigarrow \rho_2$:

repeat the same procedure from the 2nd state on

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

idea: the conditions for the ample sets
should ensure that
for each execution ρ in $\mathcal{T}$,
a stutter trace equivalent execution ρ_{red} in $\mathcal{T}_{\text{red}}$
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execution ρ in $\mathcal{T}$	$\rightsquigarrow$	execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \triangleq \rho_{\text{red}}$

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\mathsf{red}}$

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Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\mathsf{red}}$

$$\text{execution } \rho \text{ in } \mathcal{T} \quad \rightsquigarrow \quad \begin{array}{c}\text{execution } \rho_{\mathsf{red}} \text{ in } \mathcal{T}_{\mathsf{red}} \\ \text{s.t. } \rho \overset{\Delta}{=} \rho_{\mathsf{red}}\end{array}$$

by successively applying the following transformations:

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \overset{\Delta}{=} \rho_{\text{red}}$

case 0: $\rho = s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in ample(s_0)$
case 1: $\rho = s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$, $\beta_i \notin ample(s_0)$
case 2: $\rho = s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{red}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{red}$ s.t. $\rho \overset{\Delta}{=} \rho_{red}$

case 0:	$\rho = s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in ample(s_0)$	
	$s_0 \overset{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$	
case 1:	$\rho = s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$, $\beta_i \notin ample(s_0)$	
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Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{red}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{red}$ s.t. $\rho \stackrel{\Delta}{=} \rho_{red}$

case 0:	$\rho =$	$s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in ample(s_0)$ $s_0 \stackrel{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$
case 1:	$\rho =$	$s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$, $\beta_i \notin ample(s_0)$ $s_0 \stackrel{\alpha}{\Longrightarrow} \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$
case 2:	$\rho =$	$s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$ $s_0 \stackrel{\alpha}{\Longrightarrow} \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ for some $\alpha \in ample(s_0)$

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{\text{red}}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{\text{red}}$ s.t. $\rho \overset{\Delta}{=} \rho_{\text{red}}$

ρ_{red} results by an infinite sequence application of *cases 0*, *1* and *2*, i.e.,

$$\rho \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow \ldots$$

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where for $i < j$ the executions ρ_j and ρ_i have a common prefix of length i which is a path fragment in $\mathcal{T}_{\mathsf{red}}$

$$\text{execution } \rho \text{ in } \mathcal{T} \quad \rightsquigarrow \quad \begin{array}{c}\text{execution } \rho_{red} \text{ in } \mathcal{T}_{red} \\ \text{s.t. } \rho \triangleq \rho_{red}\end{array}$$

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$$\rho \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow \ldots$$

where for $i < j$ the executions ρ_j and ρ_i have a common prefix of length i which is a path fragment in $\mathcal{T}_{red}$, i.e., ρ_i has the form

$$\rho_i = \underbrace{s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i}_{\text{in } \mathcal{T}_{red}} \underbrace{\rightarrow s_{i+1} \rightarrow s_{i+2} \rightarrow \ldots}_{\text{in } \mathcal{T}}$$

Stutter trace equivalence of $\mathcal{T}$ and $\mathcal{T}_{red}$

execution ρ in $\mathcal{T}$ $\rightsquigarrow$ execution ρ_{red} in $\mathcal{T}_{red}$ s.t. $\rho \triangleq \rho_{red}$

ρ_{red} results by an infinite sequence application of *cases 0*, *1* and *2*, i.e.,

$$\rho \rightsquigarrow \rho_1 \rightsquigarrow \rho_2 \rightsquigarrow \rho_3 \rightsquigarrow \ldots$$

where

$$\rho_i = s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i \longrightarrow s_{i+1} \longrightarrow s_{i+2} \longrightarrow s_{i+3} \longrightarrow \ldots$$

$$\rho_{i+1} = s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i \Rightarrow s_{i+1} \longrightarrow s_{i+2} \longrightarrow s_{i+3} \longrightarrow \ldots$$

$$\rho_{i+2} = s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_i \Rightarrow s_{i+1} \Rightarrow s_{i+2} \longrightarrow s_{i+3} \longrightarrow \ldots$$

Transformation $\rho \rightsquigarrow \rho_1$

case 0: $\rho = \quad s_0 \xrightarrow{\alpha} s'_0 \longrightarrow \ldots$ with $\alpha \in \text{ample}(s_0)$

---

case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in \text{ample}(s_0)$

$\beta_i \notin \text{ample}(s_0)$

---

case 2: $\rho = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin \text{ample}(s_0)$

Transformation $\rho \rightsquigarrow \rho_1$

case 0: $\rho = \quad s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in \mathrm{ample}(s_0)$

$\rho_1 = \quad s_0 \overset{\alpha}{\Longrightarrow} s_0' \longrightarrow \ldots$

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case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in \mathrm{ample}(s_0)$

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$\rho_1 = \quad s_0 \overset{\alpha}{\Longrightarrow} s_0' \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$

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Transformation $\rho \rightsquigarrow \rho_1$

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$\rho_1 = \quad s_0 \xRightarrow{\alpha} s_0' \longrightarrow \ldots$

---

case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in ample(s_0)$

$\beta_i \notin ample(s_0)$

$\rho_1 = \quad s_0 \xRightarrow{\alpha} s_0' \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$

---

case 2: $\rho = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin ample(s_0)$

$\rho_1 = \quad s_0 \xRightarrow{\alpha} s_0' \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ for some $\alpha \in ample(s_0)$

Transformation $\rho \rightsquigarrow \rho_1$

case 0: $\rho = \quad s_0 \xrightarrow{\alpha} s_0' \longrightarrow \ldots$ with $\alpha \in \text{ample}(s_0)$

$\rho_1 = \quad s_0 \xRightarrow{\alpha} s_0' \longrightarrow \ldots$

case 1: $\rho = \quad s_0 \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \xrightarrow{\alpha} \longrightarrow \ldots$ with $\alpha \in \text{ample}(s_0)$

$\beta_i \notin \text{ample}(s_0)$

$\rho_1 = \quad s_0 \xRightarrow{\alpha} s_0' \xrightarrow{\beta_1} \ldots \xrightarrow{\beta_n} \longrightarrow \ldots$

case 2: $\rho = \quad s_0 \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ with $\beta_i \notin \text{ample}(s_0)$

$\rho_1 = \quad s_0 \xRightarrow{\alpha} s_0' \xrightarrow{\beta_1} \xrightarrow{\beta_2} \xrightarrow{\beta_3} \ldots$ for some $\alpha \in \text{ample}(s_0)$

for the transformation $\rho_1 \rightsquigarrow \rho_2$:

apply case 0,1 or 2 to the suffix starting in state s_0'

Transformation according to cases 1 and 2

$$\rho_0 = \quad s_0 \xrightarrow{\beta_1} \ldots \qquad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

$$\rho_1 = \quad s_0 \overset{\alpha_1}{\Longrightarrow} s_1 \xrightarrow{\beta_1} \ldots \qquad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

$$\rho_2 = \quad s_0 \overset{\alpha_1}{\Longrightarrow} \overset{\alpha_2}{\Longrightarrow} s_2 \xrightarrow{\beta_1} \ldots \qquad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

$$\vdots$$

$$\rho_m = \quad s_0 \overset{\alpha_1}{\Longrightarrow} \overset{\alpha_2}{\Longrightarrow} \ldots \overset{\alpha_m}{\Longrightarrow} s_m \xrightarrow{\beta_1} \quad \ldots \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

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α_i stutter action

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α_i stutter action $\rightsquigarrow \rho_0 \triangleq \rho_1 \triangleq \rho_2 \triangleq \ldots$

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by the cycle condition (A4):

"action β_1 will *not* be postponed forever"

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i.e., there exists some m such that case 0 applies and $\rho_m = \rho_{m+1}$

Transformation according to cases 1 and 2

$$
\begin{array}{ll}
\rho_0 = & s_0 \xrightarrow{\beta_1} \ldots \qquad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
\rho_1 = & s_0 \xRightarrow{\alpha_1} s_1 \xrightarrow{\beta_1} \ldots \qquad \xrightarrow{\beta_{k-1}} \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots \\
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\vdots & \\
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$$\rho_{m+1} = \quad s_0 \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} \ldots \xRightarrow{\alpha_m} s_m \xRightarrow{\beta_1} \quad \ldots \xrightarrow{\beta_k} \xrightarrow{\beta_{k+1}} \ldots$$

by the cycle condition $(\mathrm{A4})$:

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i.e., there exists some m such that case 0 applies and $\rho_m = \rho_{m+1}$

4 conditions for ample sets

(A1) $\emptyset \neq \mathsf{ample}(s) \subseteq Act(s)$

(A2) for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from $\mathsf{ample}(s)$ there is some $i < n$ with $\beta_i \in \mathsf{ample}(s)$

(A3) if $\mathsf{ample}(s) \neq Act(s)$ then all actions in $\mathsf{ample}(s)$ are *stutter actions*

(A4) for each *cycle* $s_0 \Rightarrow s_1 \Rightarrow \ldots \Rightarrow s_n$ in $\mathcal{T}_{red}$ and each action

$$\beta \in \bigcup_{0 \leq i < n} Act(s_i)$$

there is some $i \in \{1, \ldots, n\}$ with $\beta \in \mathsf{ample}(s_i)$

The ample set method for $\text{LTL}_{\setminus\bigcirc}$ model checking

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- on-the-fly DFS-based generatation of $\mathcal{T}_{red}$

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- *exploration* of state s:

  create the states α(s) for $\alpha \in$ ample(s),

The ample set method for $\text{LTL}_{\setminus\bigcirc}$ model checking

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  create the states $\alpha(s)$ for $\alpha \in ample(s)$, but ignore the β-successors of s for $\beta \notin ample(s)$

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  create the states α(s) for $\alpha \in$ ample(s), but ignore the β-successors of s for $\beta \notin$ ample(s)
- *interleave* the generation of $\mathcal{T}_{\text{red}}$ with the product construction $\mathcal{T}_{\text{red}} \otimes \mathcal{A}$

where $\mathcal{A}$ is an NBA for the negation of the formula to be checked

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here: only explanations for reachability analysis

The ample set method for reachability

given: finite transition system $\mathcal{T}$
atomic proposition **a**

goal: on-the-fly construction of $\mathcal{T}_{red}$
abort as soon as a state s with
s $\not\models$ **a** has been generated

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- DFS-stack π

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uses

- **V** = set of states that have been generated so far (organized as a hash table)
- DFS-stack π
- "local" criteria to compute ample(s) from a syntactic representation of the processes P_i

Ample set method (full generation of $\mathcal{T}_{\text{red}}$)

$\pi := \emptyset$; $V := \emptyset$

WHILE $S_0 \not\subseteq V$ **DO**

select an initial state $s \in S_0 \setminus V$; add s to V;

Push(π, s);

OD

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WHILE $\pi \neq \emptyset$ **DO**

$\mathbf{s} :=$ Top(π);

OD

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FI

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        $s := Top(\pi)$;
        IF $\exists \alpha \in ample(s)$ with $\alpha(s) \notin V$
            THEN select such α; add $s' := \alpha(s)$ to V;
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        FI
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     THEN select such α; add $s' := \alpha(s)$ to V;

          Push(π, s'); compute ample(s');

    FI

  OD

OD

Ample set method (full generation of $\mathcal{T}_{\text{red}}$)


```
π := ∅; V := ∅
WHILE S₀ ⊈ V DO
   select an initial state s ∈ S₀ \ V; add s to V;
   Push(π, s); compute ample(s);
   WHILE π ≠ ∅ DO
      s := Top(π);
      IF ∃α ∈ ample(s) with α(s) ∉ V
        THEN select such α; add s' := α(s) to V;
                  Push(π, s'); compute ample(s');
        ELSE  Pop(π)
      FI
   OD
OD
```

The ample set method for reachability

$\pi := \emptyset$; $V := \emptyset$
WHILE $S_0 \not\subseteq V$ **DO**
  select an initial state $s \in S_0 \setminus V$; add s to V;
  $\text{Push}(\pi, s)$; compute $\text{ample}(s)$;
  WHILE $\pi \neq \emptyset$ **DO**
    $s := \text{Top}(\pi)$;
    IF $\exists \alpha \in \text{ample}(s)$ with $\alpha(s) \notin V$
      THEN select such α; add $s' := \alpha(s)$ to V;
        $\text{Push}(\pi, s')$; compute $\text{ample}(s')$;
      ELSE $\text{Pop}(\pi)$
    FI
  OD
OD

Does $\mathcal{T} \models \Box a$ hold?


```
π := ∅; V := ∅
WHILE S0 ⊈ V DO
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  WHILE π ≠ ∅ DO
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      THEN select such α; add s′ := α(s) to V;
                 Push(π, s′); compute ample(s′);
      ELSE   Pop(π)
    FI
  OD
OD
```

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  WHILE $\pi \neq \emptyset$ **DO**
    $s :=$ Top(π);
    IF $s \not\models a$ **THEN** return "NO" **FI**;
    IF $\exists \alpha \in$ ample(s) with $\alpha(s) \notin V$
      THEN ....
      ELSE Pop(π)
    FI
  OD
OD

Does $\mathcal{T} \models \Box a$ hold?

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  WHILE π ≠ ∅ DO
    s := Top(π);
    IF s ⊭ a THEN return "NO" + counterexample FI;
    IF ∃α ∈ ample(s) with α(s) ∉ V
      THEN ....
      ELSE  Pop(π)
    FI
  OD
OD
```

Example: ample set method

full generation of $\mathcal{T}_{\mathsf{red}}$ for $\mathcal{T} = \mathcal{T}_{\mathsf{P}_1 \,|||\, \mathsf{P}_2}$ where

- P_1, P_2 are program graphs with shared variable $\mathsf{b} \in \{0, 1\}$

full generation of $\mathcal{T}_{red}$ for $\mathcal{T} = \mathcal{T}_{P_1 \,|||\, P_2}$ where

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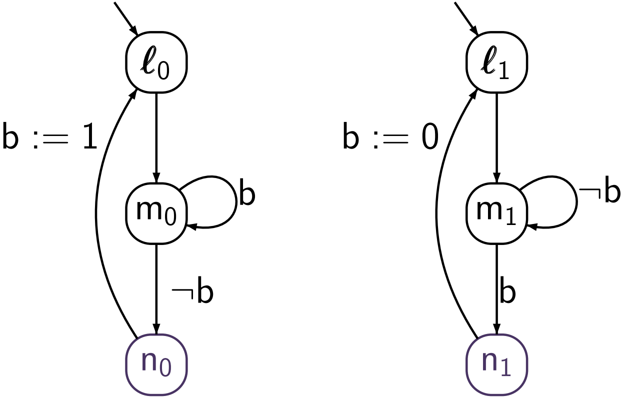

Example: ample set method

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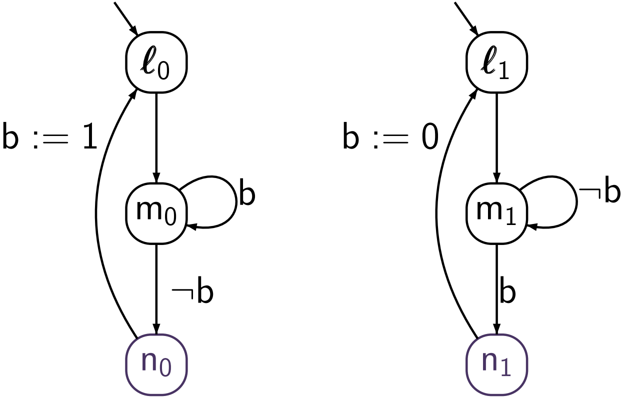
- $AP = \{n_0, n_1\}$



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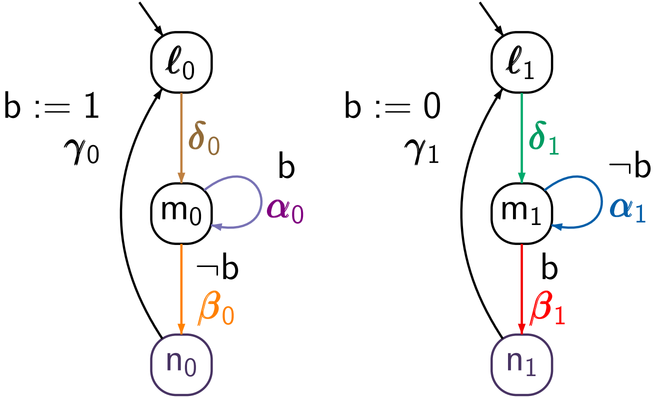
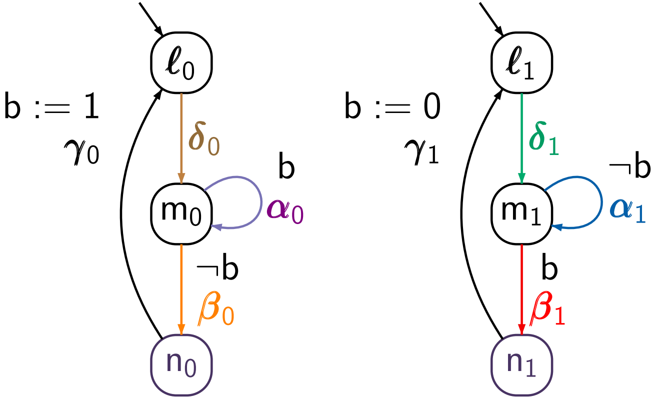
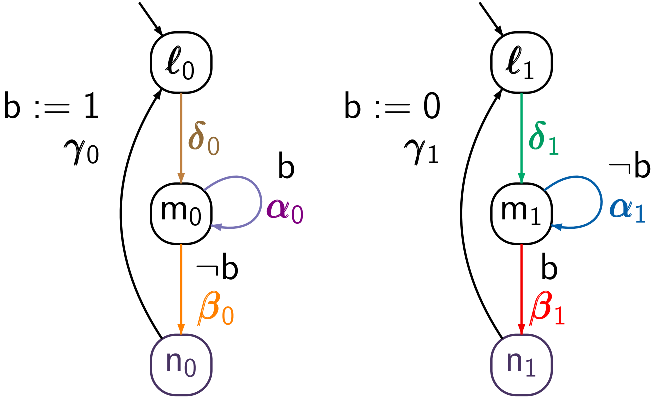




independent actions:

$\delta_0\ \delta_1$ $\quad$ $\delta_0\ \alpha_1$ $\quad$ $\delta_0\ \beta_1$ $\quad$ $\delta_0\ \gamma_1$



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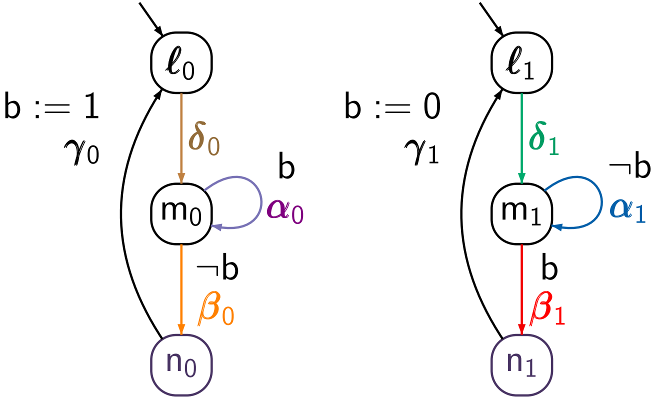
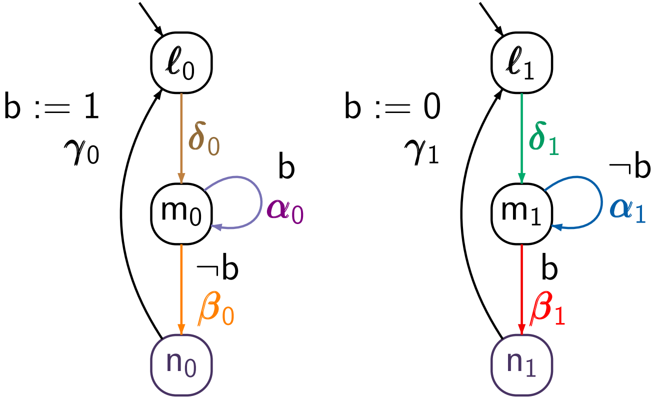


independent actions:

$\delta_0\ \delta_1$	$\delta_0\ \alpha_1$	$\delta_0\ \beta_1$	$\delta_0\ \gamma_1$
$\alpha_0\ \delta_1$		$\alpha_0\ \beta_1$	
$\beta_0\ \delta_1$	$\beta_0\ \alpha_1$	$\beta_0\ \beta_1$	$\beta_0\ \gamma_1$



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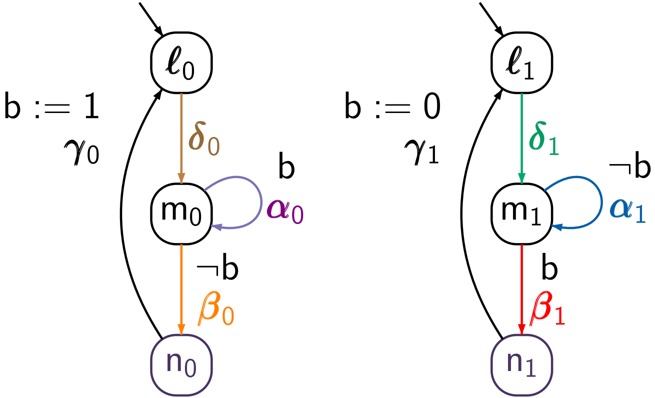
$\alpha_0\ \delta_1$		$\alpha_0\ \beta_1$	
$\beta_0\ \delta_1$	$\beta_0\ \alpha_1$	$\beta_0\ \beta_1$	$\beta_0\ \gamma_1$

β_0 and β_1 are never enabled simultaneously



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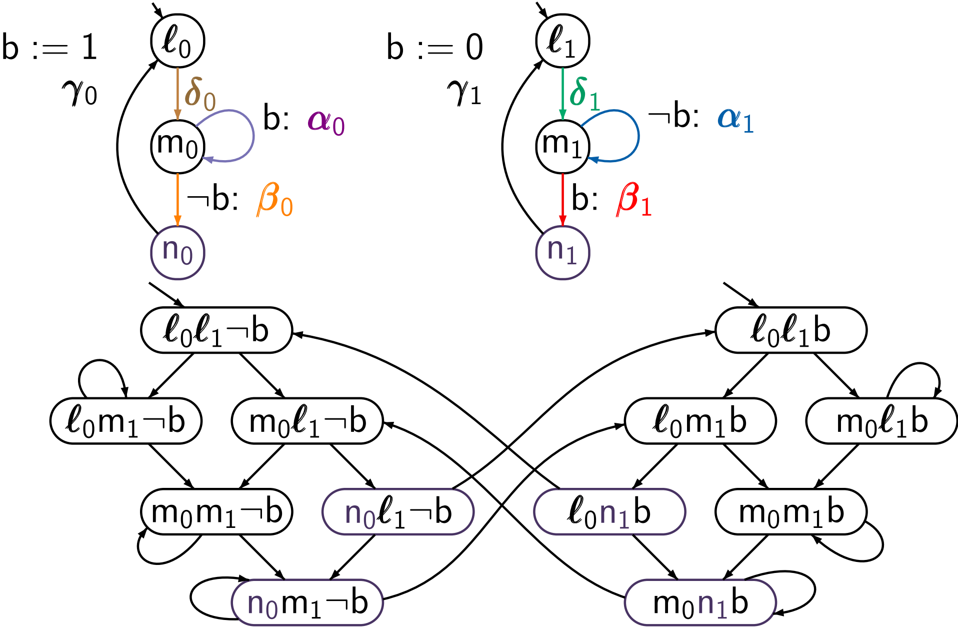
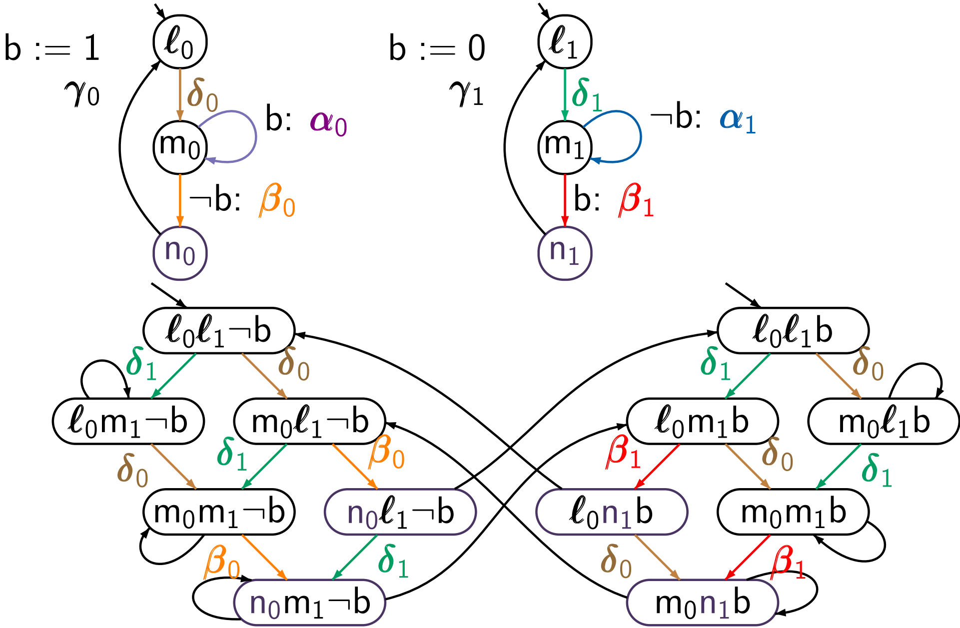
$\delta_0\ \delta_1$	$\delta_0\ \alpha_1$	$\delta_0\ \beta_1$	$\delta_0\ \gamma_1$
$\alpha_0\ \delta_1$		$\alpha_0\ \beta_1$	
$\beta_0\ \delta_1$	$\beta_0\ \alpha_1$	$\beta_0\ \beta_1$	$\beta_0\ \gamma_1$
$\gamma_0\ \delta_1$		$\gamma_0\ \beta_1$	



b := 1
γ_0
ℓ_0
δ_0
m_0
b: α_0
¬b: β_0
n_0
b := 0
γ_1
ℓ_1
δ_1
m_1
¬b: α_1
b: β_1
n_1
$\ell_0\ell_1\neg b$
$\ell_0 m_1 \neg b$
$m_0 \ell_1 \neg b$
$m_0 m_1 \neg b$
$n_0 \ell_1 \neg b$
$n_0 m_1 \neg b$
$\ell_0 \ell_1 b$
$\ell_0 m_1 b$
$m_0 \ell_1 b$
$\ell_0 n_1 b$
$m_0 m_1 b$
$m_0 n_1 b$



b := 1
γ_0
ℓ_0
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b: α_0
¬b: β_0
n_0
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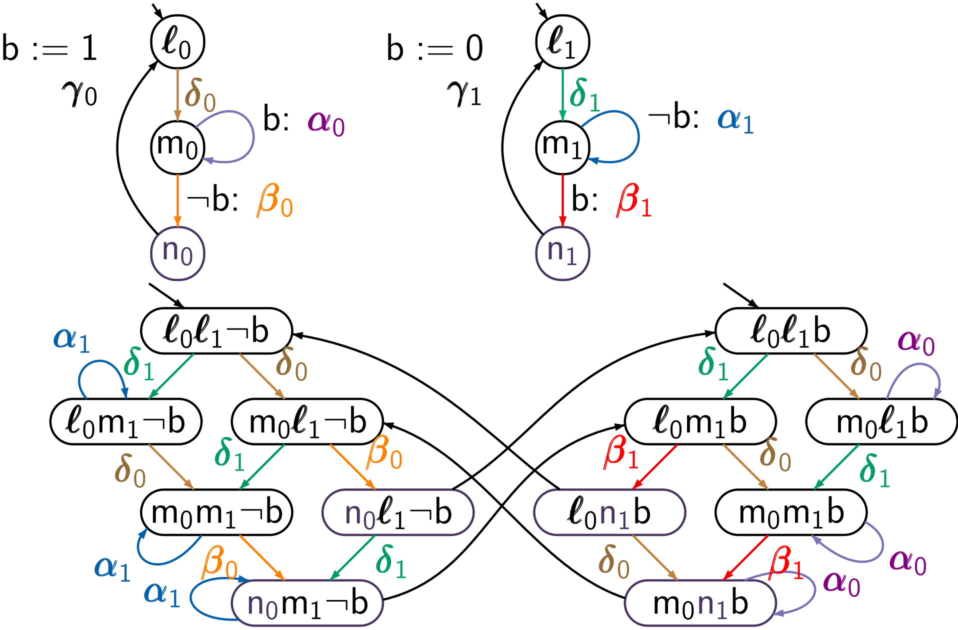
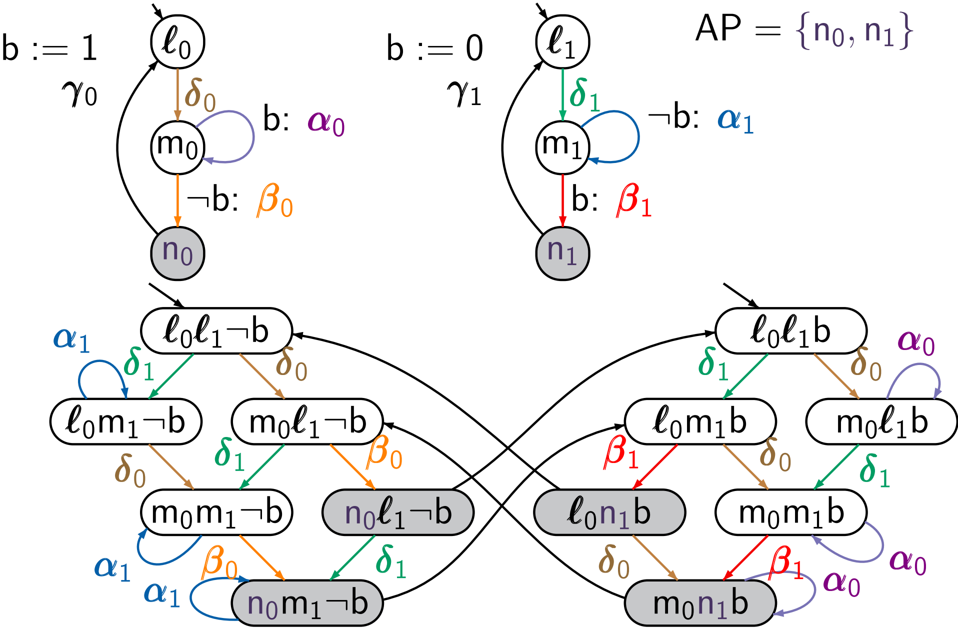
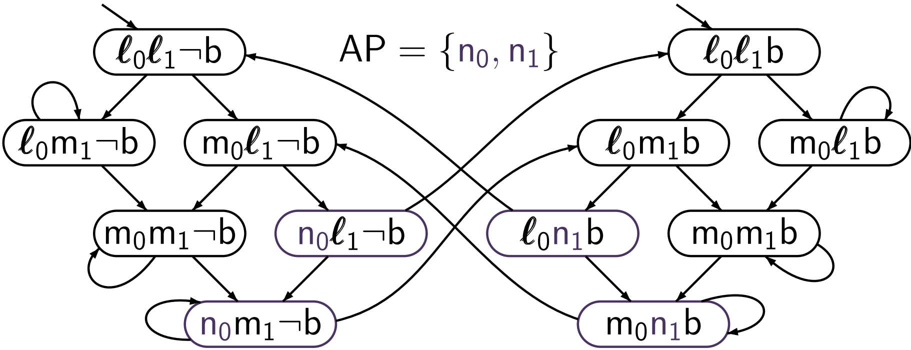
γ_1
ℓ_1
δ_1
m_1
¬b: α_1
b: β_1
n_1
$\ell_0\ell_1\neg b$
$\ell_0 m_1 \neg b$
$m_0 \ell_1 \neg b$
$m_0 m_1 \neg b$
$n_0 \ell_1 \neg b$
$n_0 m_1 \neg b$
$\ell_0\ell_1 b$
$\ell_0 m_1 b$
$m_0 \ell_1 b$
$\ell_0 n_1 b$
$m_0 m_1 b$
$m_0 n_1 b$


b := 1
γ0
ℓ0
δ0
m0
b: α0
¬b: β0
n0
b := 0
γ1
ℓ1
δ1
m1
¬b: α1
b: β1
n1
ℓ0ℓ1¬b
ℓ0m1¬b
m0ℓ1¬b
m0m1¬b
n0ℓ1¬b
n0m1¬b
ℓ0ℓ1b
ℓ0m1b
m0ℓ1b
ℓ0n1b
m0m1b
m0n1b



b := 1
γ0
ℓ0
δ0
m0
b: α0
¬b: β0
n0
b := 0
γ1
ℓ1
δ1
m1
¬b: α1
b: β1
n1
AP = {n0, n1}
ℓ0ℓ1¬b
ℓ0m1¬b
m0ℓ1¬b
m0m1¬b
n0ℓ1¬b
n0m1¬b
ℓ0ℓ1b
ℓ0m1b
m0ℓ1b
ℓ0n1b
m0m1b
m0n1b

Example: on-the-fly generation of $\mathcal{T}_{\text{red}}$



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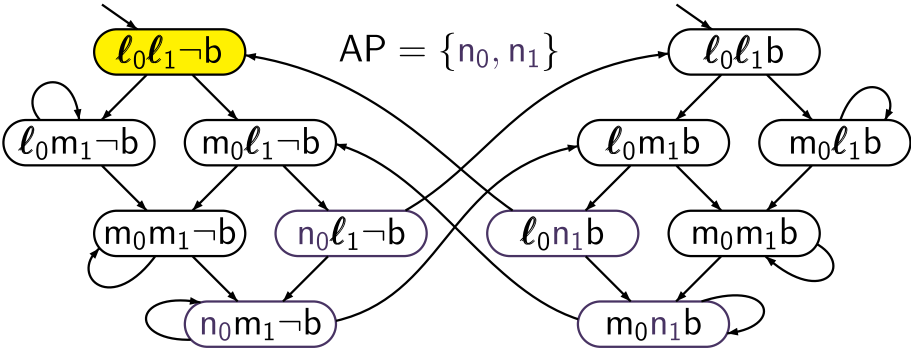

$AP = \{n_0, n_1\}$

$\text{ample}(\ell_0\ell_1\neg b) =$

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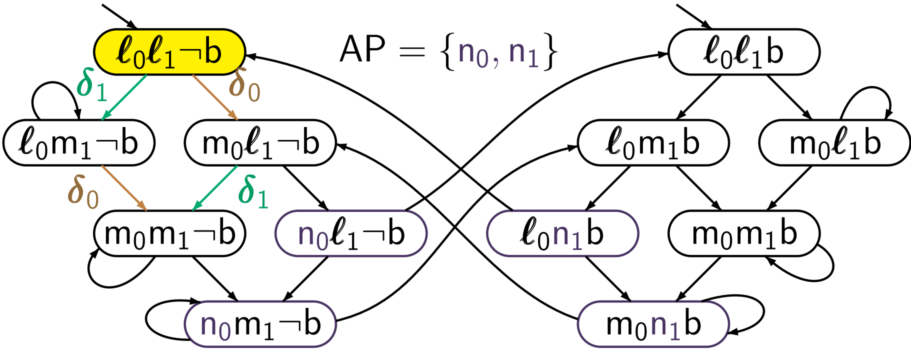

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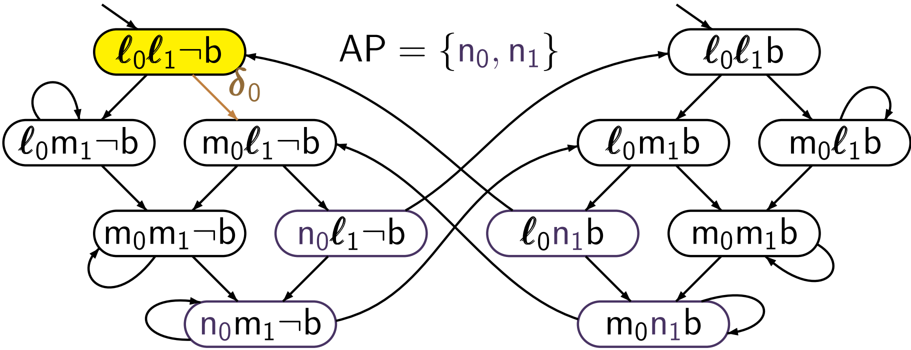
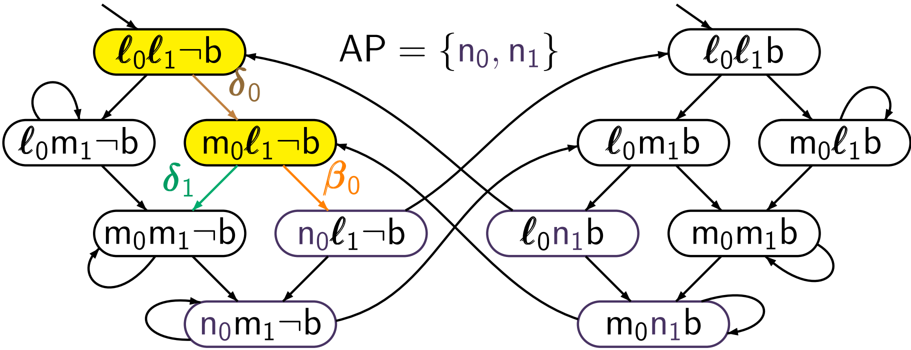


$$\text{ample}(\ell_0\ell_1\neg b) = \{\delta_0\},$$

Example: on-the-fly generation of $\mathcal{T}_{red}$



$\text{ample}(\ell_0\ell_1\neg b) = \{\delta_0\}$, $\quad\text{ample}(m_0\ell_1\neg b) =$

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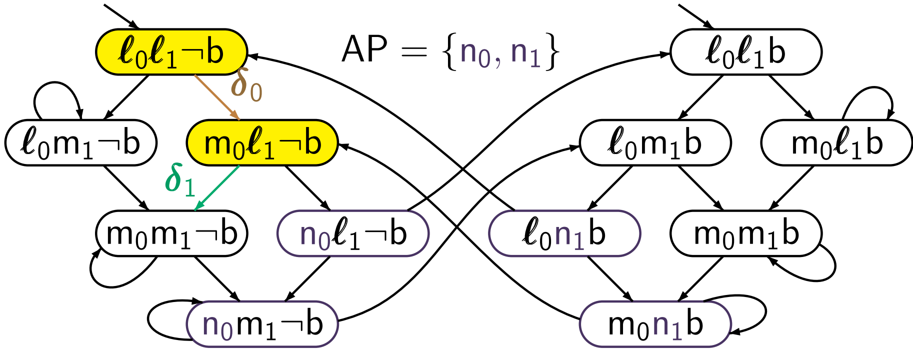

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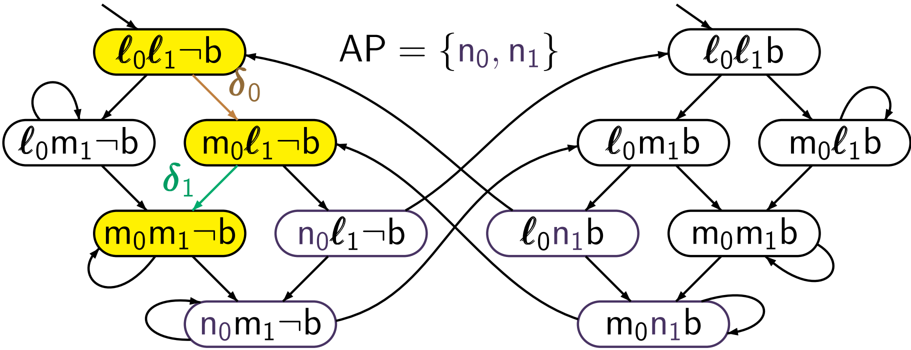

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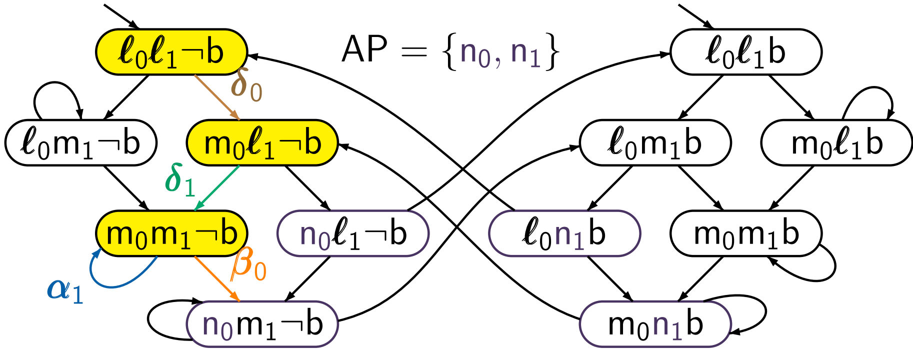

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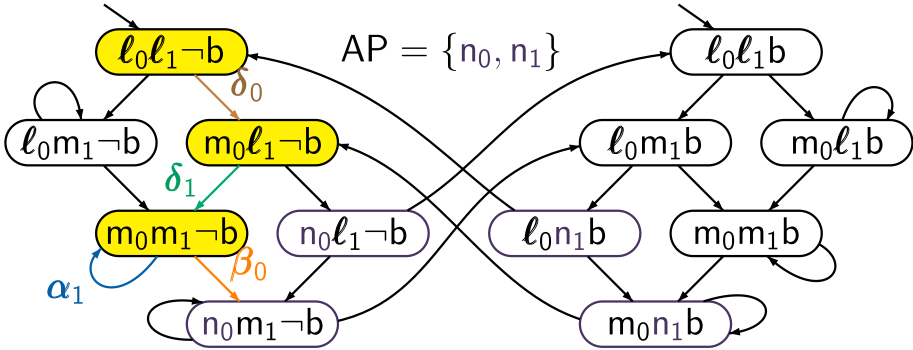

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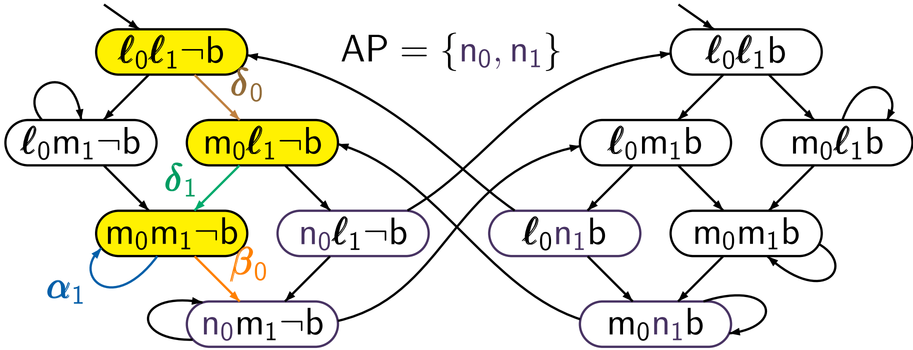

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α_1 closes cycle (A4),

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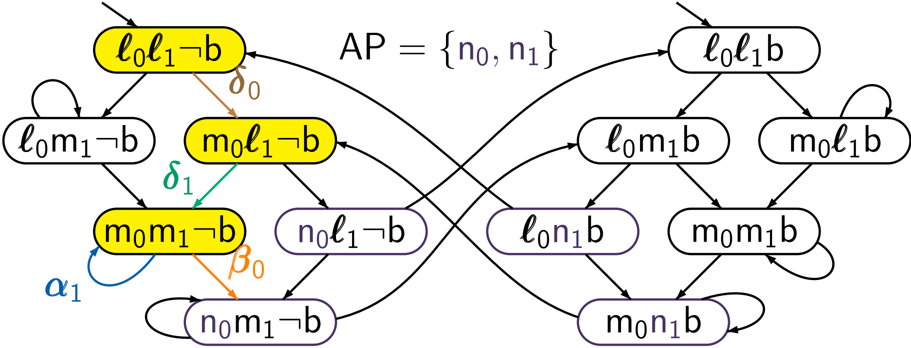

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α_1 closes cycle (A4),

β_0 no stutter action (A3)

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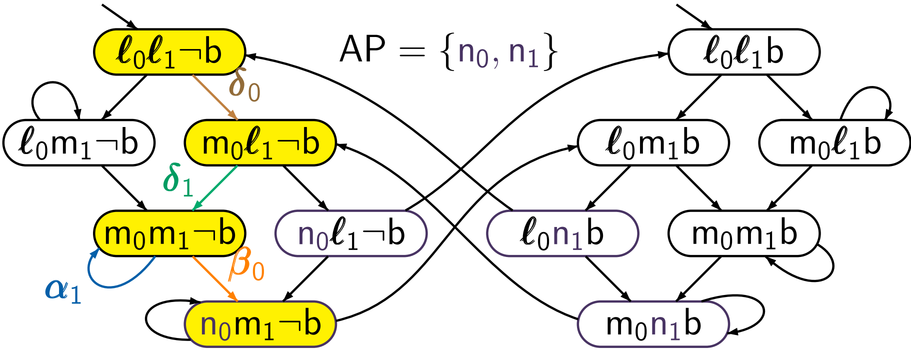

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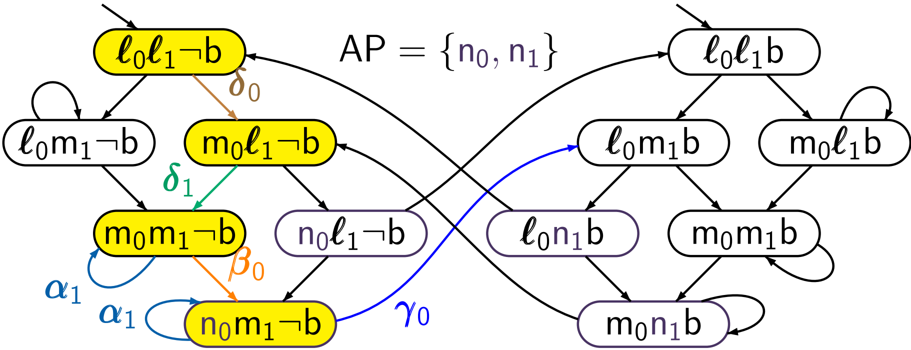

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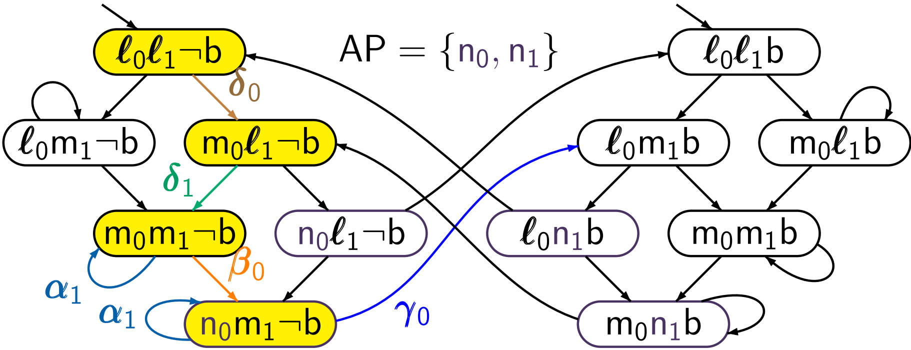

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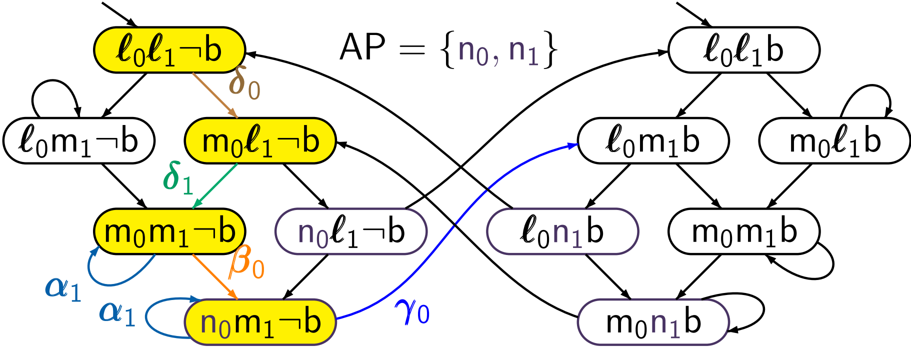

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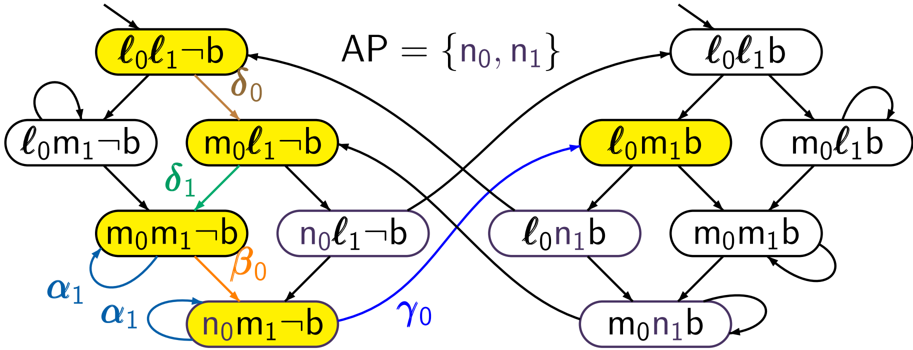

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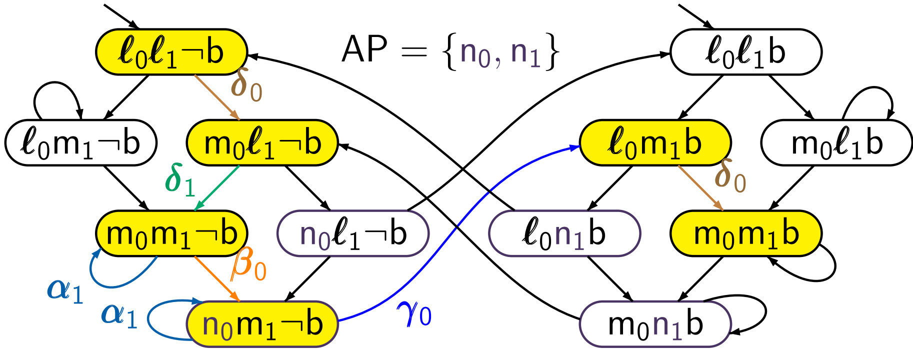

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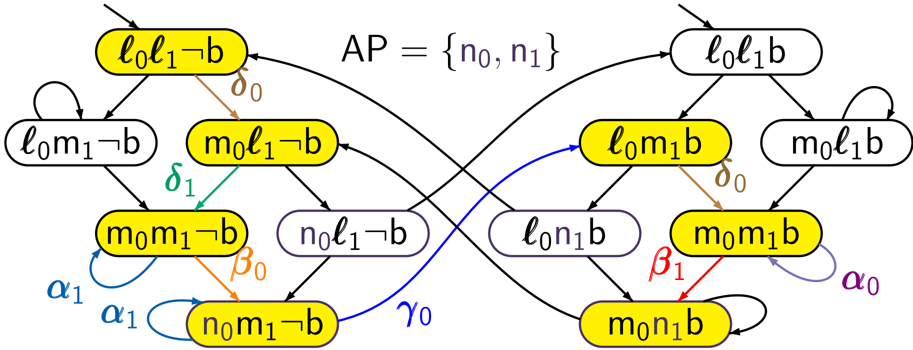

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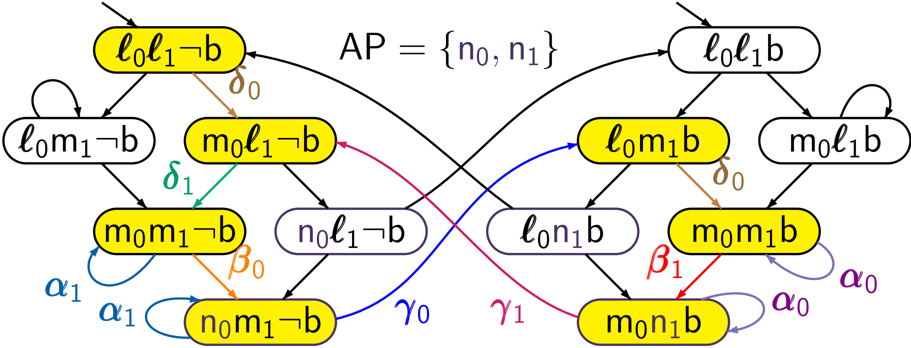

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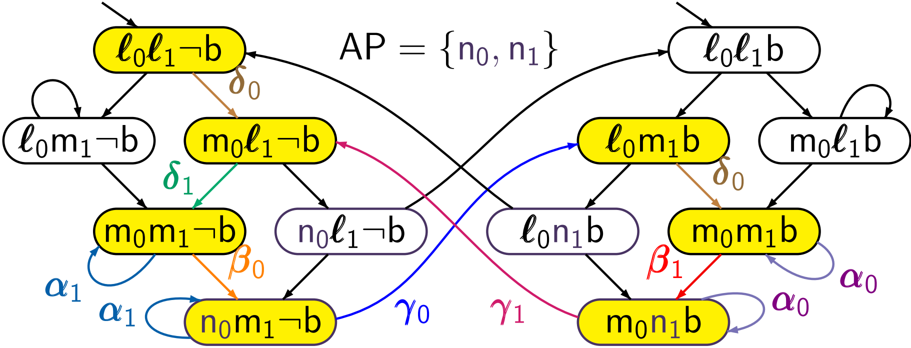

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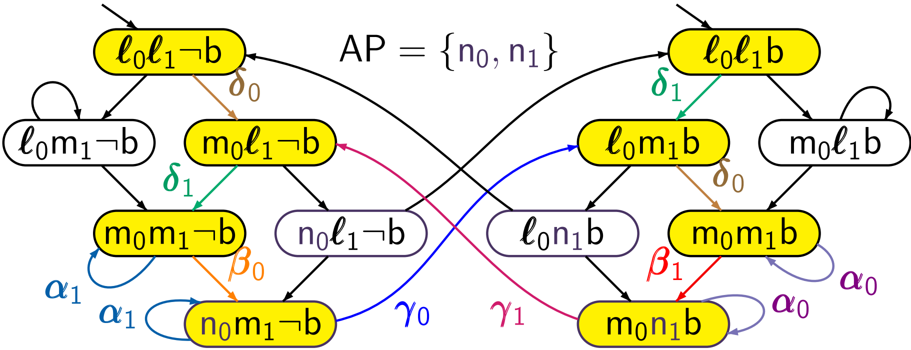

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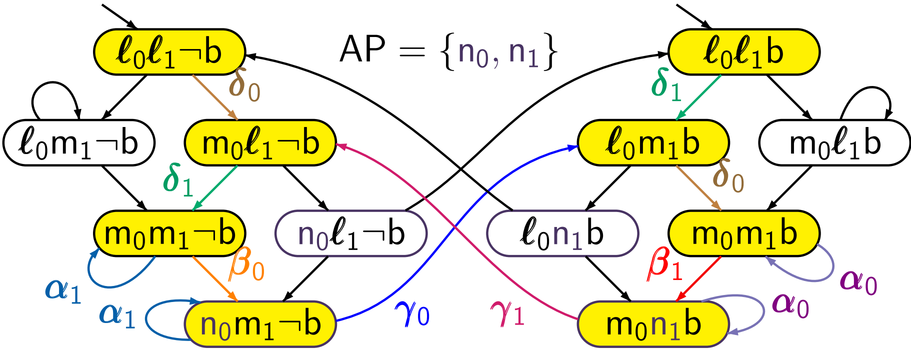
reduction: 8 out of 12 states



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Nested DFS with POR

Nested DFS (standard approach)

remind: nested DFS for checking "$\mathcal{T} \models \Diamond\Box a$?" uses:

outer DFS: visits all reachable states

inner DFS: CYCLE_CHECK(s) searches for a backward edge $s' \to s$

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- early termination

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CYCLE_CHECK(s)

- is called for each state s that violates the persistence condition a
- must not be started before the outer DFS is finished for s
- early termination, e.g., abort with the answer

  $$\text{CYCLE\_CHECK}(s) = \textit{true}$$

  as soon as the inner DFS visits a state in the DFS-stack of the outer DFS

Nested DFS with POR

requirement for the nested DFS in the ample set approach:

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implementation: uses a hash-table for the set of states that have been visited in the outer DFS

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where s is a state and $b, c, a_1, \ldots, a_k$ are bits

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- $b = 1$ iff s has been visited in inner DFS
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- for $Act(s) = \{\alpha_1, \ldots, \alpha_k\}$:

$$a_i = 1 \text{ iff } \alpha_i \in ample(s)$$

On-the-fly construction of $\mathcal{T}_{\text{red}}$

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starting point: syntactic description of the processes $P_1, \ldots, P_n$ of a parallel system

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method: generate the reachable fragment of $\mathcal{T}_{\text{red}}$ in DFS-manner by generating ample sets by means of local conditions that ensure (A1)-(A4)

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$$\text{ample}(s) = \text{set of enabled actions of process } P_i$$

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fulfills (A1), (A2), (A3) and ensure (A4) by searching for backward edges in $\mathcal{T}_{\text{red}}$

Computing the ample set for state s

REPEAT

select a process P_i not considered before

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IF $A \neq \emptyset$ and $(A2)$ is not violated

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and all actions of A are stutter actions

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THEN $ample(s) := A$ **FI**

Computing the ample set for state s

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UNTIL all processes have been considered

or ample(s) is defined;

REPEAT

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$A := \text{action set of } P_i \cap Act(s)$

IF $A \neq \emptyset$ and $(\mathrm{A2})$ is not violated

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UNTIL all processes have been considered

or $ample(s)$ is defined;

IF $ample(s)$ is not yet defined

THEN $ample(s) := Act(s)$ **FI**

Computing the ample set for state s

REPEAT

select a process P_i not considered before

A := action set of $P_i \cap Act(s)$

IF (A1), (A2), (A3) hold **THEN** ample(s) := A **FI**

UNTIL all processes have been considered

or ample(s) is defined;

IF ample(s) is not yet defined

THEN ample(s) := $Act(s)$ **FI**

... consider state $\alpha(s)$ for some $\alpha \in$ ample(s) ...

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IF the expansion of s finds a backwards edge $s' \Longrightarrow s$

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$A :=$ action set of $P_i \cap Act(s)$

IF (A1), (A2), (A3) hold **THEN** $ample(s) := A$ **FI**

UNTIL all processes have been considered

or $ample(s)$ is defined;

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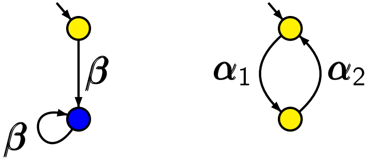
... consider state $\alpha(s)$ for some $\alpha \in ample(s)$ ...

IF the expansion of s finds a backwards edge $s' \Longrightarrow s$

THEN $ample(s) := Act(s)$ **FI**

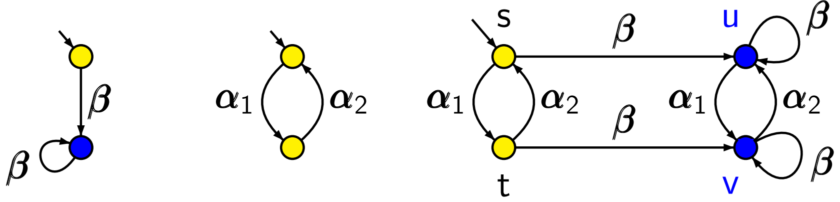
Example: construction of $\mathcal{T}_{\mathsf{red}}$

process 1 process 2

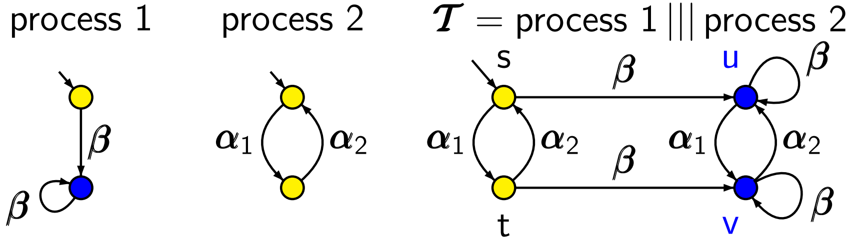


Example: construction of $\mathcal{T}_{\mathsf{red}}$

process 1 process 2 $\mathcal{T}$ = process 1 ||| process 2

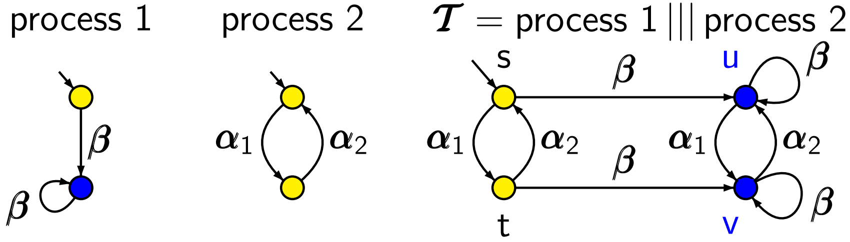


Example: construction of $\mathcal{T}_{\text{red}}$

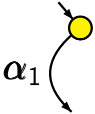


DFS(s)

Example: construction of $\mathcal{T}_{\text{red}}$



DFS(s)
ample(s) $= \{\alpha_1\}$



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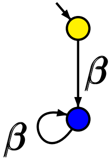
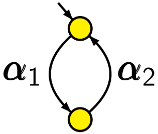
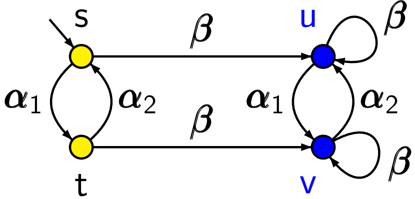




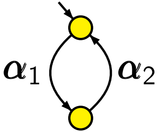


DFS(s)

ample(s) $= \{\alpha_1\}$

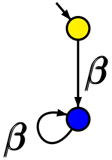
DFS(t)

ample(t) $= \{\alpha_2\}$

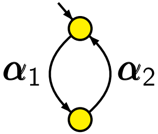


Example: construction of $\mathcal{T}_{\text{red}}$

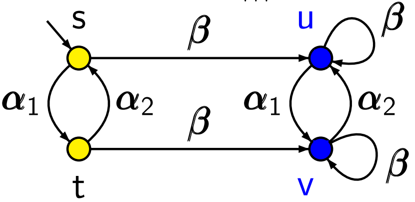
process 1



process 2



$\mathcal{T}$ = process 1 ||| process 2



DFS(s)

ample(s) $= \{\alpha_1\}$

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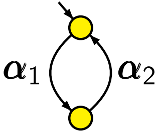
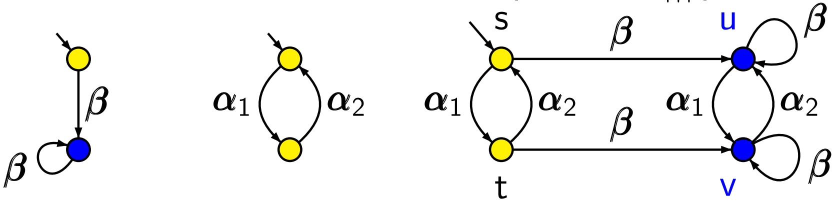
backward edge t $\longrightarrow$ s



Example: construction of $\mathcal{T}_{\text{red}}$

process 1 process 2 $\mathcal{T}$ = process 1 ||| process 2



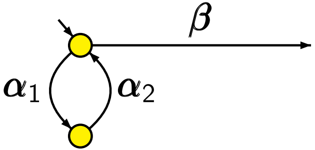
DFS(s)

ample(s) = $\{\alpha_1\} \cup \{\beta\}$

DFS(t)

ample(t) = $\{\alpha_2\}$

backward edge t $\longrightarrow$ s



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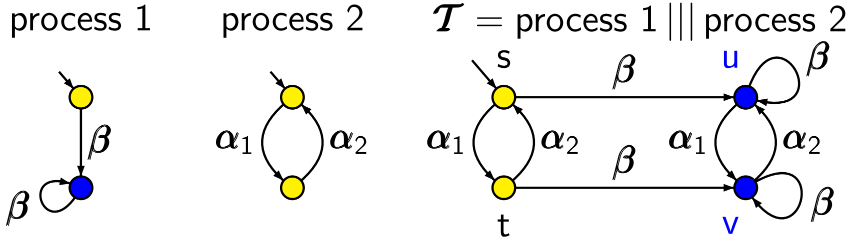

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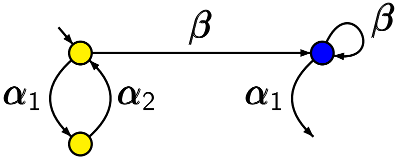
DFS(u) ...



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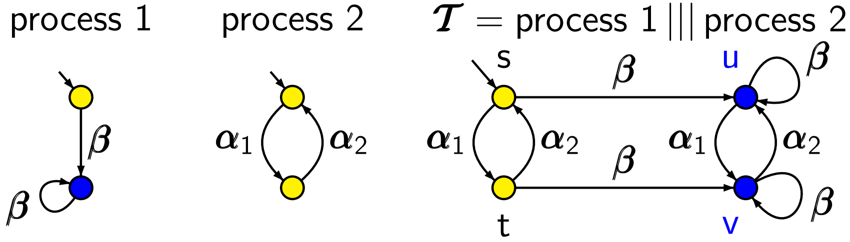

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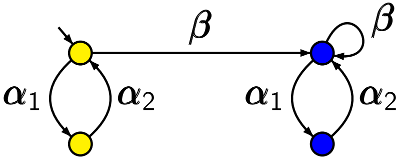
DFS(u) ...

DFS(v) ...



REPEAT

select a process P_i not considered before

$A :=$ action set of $P_i \cap \mathit{Act}(s)$

IF $A \neq \emptyset$ and (A2) holds

and all actions of A are stutter actions

THEN ample$(s) := A$ **FI**

UNTIL all processes have been considered

or ample(s) is defined;

IF ample(s) is not yet defined

THEN ample$(s) := \mathit{Act}(s)$ **FI**

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Checking the dependence condition (A2)?

(A1) nonemptiness condition

(A2) dependence condition:

for each execution fragment in $\mathcal{T}$

$$s \xrightarrow{\beta_1} \xrightarrow{\beta_2} \ldots \xrightarrow{\beta_{i-1}} \xrightarrow{\beta_i} \xrightarrow{\beta_{i+1}} \ldots \xrightarrow{\beta_{n-1}} \xrightarrow{\beta_n}$$

such that β_n is *dependent* from ample(s) there is some $i < n$ with $\beta_i \in$ ample(s)

(A3) stutter condition

(A4) cycle condition

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checking (A2) is as hard as the unreachability problem

given: finite transition system $\mathcal{T}$, **a** $\in$ AP

question: does $\mathcal{T} \not\models \exists\Diamond a$ hold?

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Algorithmic difficulty of checking (A2)

show that the unreachability problem

given: finite transition system $\mathcal{T}$

$\mathbf{a} \in AP$

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given: finite transition system $\mathcal{T}'$, ample sets for $\mathcal{T}'$

question: does (A2) hold?
i.e., does for each execution fragment in $\mathcal{T}'$

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such that β_n is *dependent* from ample(s)
there is some $i < n$ with $\beta_i \in$ ample(s)?

Algorithmic difficulty of checking (A2)

show that the unreachability problem

given: finite transition system $\mathcal{T}$ and initial state s_0
$a \in AP$

question: does $s_0 \not\models \exists\Diamond a$ hold?

is polynomially reducible to the problem of checking (A2)

given: finite transition system $\mathcal{T}'$, ample sets for $\mathcal{T}'$

question: does (A2) hold?
i.e., does for each execution fragment in $\mathcal{T}'$

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unreachability problem	$\leq_{\mathsf{poly}}$	problem of checking $(\mathrm{A2})$

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finite TS $\mathcal{T}$ + state s_0 + atomic prop. a $\rightsquigarrow$ finite TS $\mathcal{T}'$ + ample sets

Algorithmic difficulty of checking (A2)

unreachability problem	$\leq_{\text{poly}}$	problem of checking (A2)

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$\mathcal{T}'$ results from $\mathcal{T}$ by adding two fresh actions α, β s.t.

- α are β are dependent
- α is independent from all actions in $\mathcal{T}$
- β is enabled exactly in the states t with $t \models a$

	finite TS $\mathcal{T}$ + state s_0		finite TS $\mathcal{T}'$
	+ atomic prop. a	$\rightsquigarrow$	+ ample sets
s.t.	$s_0 \not\models \exists \Diamond a$	iff	$(A2)$ holds

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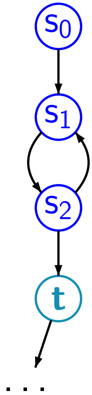

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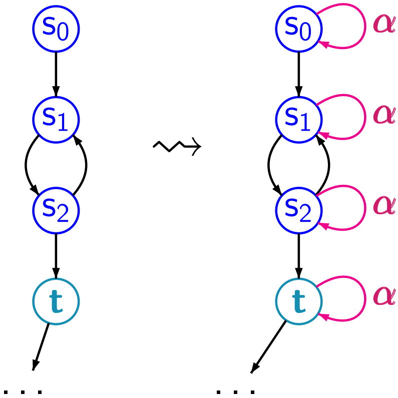

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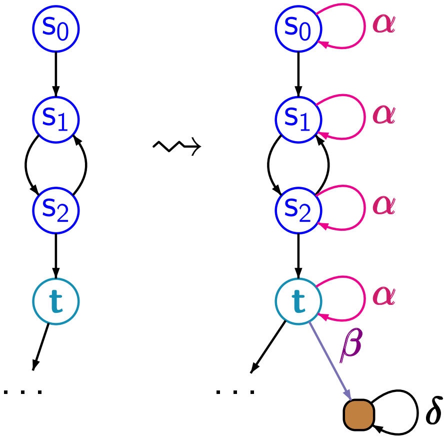

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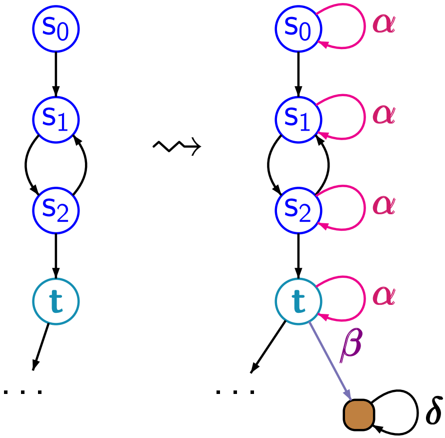

α, β dependent

α independent from all other actions

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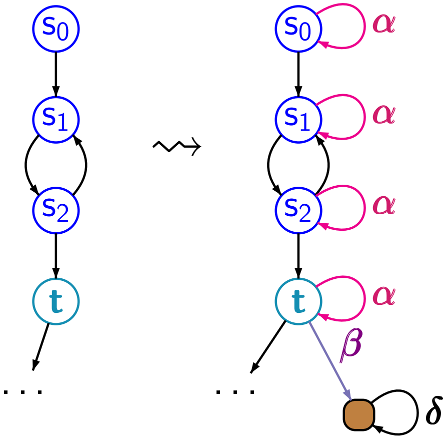

α, β dependent

α independent from all other actions

$ample(s_0) = \{\alpha\}$

$ample(u) = Act(u)$ for all other states u

Local criterion for condition (A2)

idea: replace the global dependency condition (A2) by
a stronger local condition

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e.g., the P_i's are given as program graphs of a channel system.

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e.g., the P_i's are given as program graphs of a channel system. Then: each state s has the form

$$s = \langle \boldsymbol{\ell}_1, \ldots, \boldsymbol{\ell}_n, \eta, \xi \rangle$$

where $\boldsymbol{\ell}_i$ is a location of process P_i, η a variable evaluation, ξ a channel evaluation

Local criterion for condition (A2)

Let Act_i denote the set of actions of process P_i.

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For state s:

$$
\begin{aligned}
Act_i(s) &= Act_i \cap Act(s) \\
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&\quad\ \text{that are enabled in } s
\end{aligned}
$$

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$$\begin{aligned} Act_i(s) &= Act_i \cap Act(s) \\ &= \text{set of actions of process } P_i \\ &\quad\ \text{that are enabled in } s \end{aligned}$$

Provide local criteria such that $ample(s) = Act_i(s)$ fulfills the dependency condition (A2)

Local criterion for condition $(A2)$

Let $\mathsf{s} = \langle \boldsymbol{\ell}_1, \ldots, \boldsymbol{\ell}_{\mathsf{i}-1}, \boldsymbol{\ell}_{\mathsf{i}}, \boldsymbol{\ell}_{\mathsf{i}+1}, \ldots, \boldsymbol{\ell}_{\mathsf{n}}, \ldots \rangle$.
Suppose that

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(A2.1) all actions of P_j, $j \neq i$, are independent from $Act_i(s)$

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(A2.2) there is no action γ of a process P_j where $j \neq i$ s.t. γ can enable an action $\beta \in Act_i \setminus Act(s)$ from some state s' with location ℓ_i for process P_i

Local criterion for condition (A2)

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$$\langle \ldots h_j \ldots \ell_i \ldots \rangle \xrightarrow{\gamma} \langle \ldots k_j \ldots \ell_i \ldots \rangle \xrightarrow{\beta}$$
$$\beta \not\downarrow$$

for some $\beta \in Act_i \setminus Act(s)$

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Then (A2) holds for $ample(s) = Act_i(s)$.

Heuristics for condition (A2)

⋮ expansion of state $s = \langle \ldots \ell_j \ldots \ell_i \ldots \rangle$

Heuristics for condition (A2)

$\vdots$ expansion of state $s = \langle \dots \ell_j \dots \ell_i \dots \rangle$

$A := Act_i(s)$

Heuristics for condition $(A2)$

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check if for all other processes P_j the following holds:

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for some $\beta \in Act_i \setminus A$

Heuristics for condition (A2)

⋮ expansion of state $s = \langle \ldots \ell_j \ldots \ell_i \ldots \rangle$

$A := Act_i(s)$

⋮

check if for all other processes P_j the following holds:

(A2.1) all actions of P_j are independent from A

(A2.2) there is no action γ of P_j such that

$$\langle \ldots h_j \ldots \ell_i \ldots \rangle \xrightarrow{\gamma} \langle \ldots k_j \ldots \ell_i \ldots \rangle \xrightarrow{\beta}$$

$$\beta \not\downarrow$$

for some $\beta \in Act_i \setminus A$

if yes then set $ample(s) := A$

⋮

Correct or wrong?

Let $\mathcal{T}_1$, $\mathcal{T}_2$ be transition systems with $\mathcal{T}_1 \stackrel{\Delta}{=} \mathcal{T}_2$, and let fair be an **LTL** fairness assumption.

Remind: $\stackrel{\Delta}{=}$ denotes stutter trace equivalence.
E.g., $\mathcal{T}_1 = \mathcal{T}$, $\mathcal{T}_2 = \mathcal{T}_{red}$

Then, for all $\mathbf{LTL}_{\setminus \bigcirc}$ formulas φ:

$$\mathcal{T}_1 \models_{fair} \varphi \quad \text{iff} \quad \mathcal{T}_2 \models_{fair} \varphi$$

Correct or wrong?

Let $\mathcal{T}_1, \mathcal{T}_2$ be transition systems with $\mathcal{T}_1 \triangleq \mathcal{T}_2$, and let fair be an **LTL** fairness assumption.

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correct

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Then, for all **LTL**$_{\setminus\bigcirc}$ formulas φ:

$$\mathcal{T}_1 \models_{\text{fair}} \varphi \quad \text{iff} \quad \mathcal{T}_2 \models_{\text{fair}} \varphi$$

correct, as we have:

$$\mathcal{T}_i \models_{\text{fair}} \varphi \quad \text{iff} \quad \mathcal{T}_i \models \text{fair} \rightarrow \varphi$$

Correct or wrong?

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Then, for all $\textbf{LTL}_{\setminus\bigcirc}$ formulas φ:

$$\mathcal{T}_1 \models_{\text{fair}} \varphi \quad \text{iff} \quad \mathcal{T}_2 \models_{\text{fair}} \varphi$$

correct, as we have:

$$\mathcal{T}_i \models_{\text{fair}} \varphi \;\text{ iff }\; \mathcal{T}_i \models \underbrace{\text{fair} \rightarrow \varphi}_{\textbf{LTL}_{\setminus\bigcirc}\text{ formula}}$$